A spectroscopic method for determining thickness of quartz wave plate

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A spectroscopic method to determine thickness of quartz wave plate is presented. The method is based on chromatic polarization interferometry. With the polarization-resolved transmission spectrum (PRTS) curve, the phase retardation of quartz wave plate can be determined at a wide spectral range from 200 to 2000 nm obviously. Through accurate judgment of extreme points of PRTS curve at long-wave band, the physical thickness of quartz wave plates can be obtained exactly. We give a measuring example and the error analysis. It is found that the measuring precision of thickness is mainly determined by the spectral resolution of spectrometer.

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An optical wave plate, usually made of crystal quartz or mica, is one of the most commonly used optical elements in the modern polarization technique and laser application technique. In many applications and exact measurements, crystal quartz is preferable owing to its many advantages such as high mechanical robustness, wide range of transmitted spectrum, high damage threshold, stable physical-chemical nature, and even texture without overlapped zones. Wavelength-dependence is the most distinguished characteristic of wave plate. For example, a 1/2 wave plate at λ_1 may be a 1/4 one for λ_2. Some papers[1−4] reported how to evaluate the phase retardation of a wave plate only at a definite wavelength instead of that at a wide spectral range. In addition, the ambient temperature also affects the performance of a wave plate. The temperature effect on the phase retardation of a wave plate, especially on the multi-order wave plate, is too serious to be neglected. But few measuring methods take temperature into account.

In this paper, we present a simple method for simultaneous determination of both the thickness of wave plate and its phase retardation at a wide spectral range. Our method uses dispersion equation of refractive-index difference of quartz with temperature item as input and is based on chromatic polarization interferometry[5]. The method has the advantages of high measurement accuracy and straightforwardness, which is especially useful for the manufacturing process of quartz wave plates.

Let us consider a quartz wave plate of thickness l, sandwiched between two parallel linear polarizers, and the direction of the initial polarizer making an angle of $\pi/4$ with the rapid axis of the quartz wave plate. A monochromatic light beam of wavelength λ is incident normally on the sandwiched group. The linearly polarized light can be resolved into two components travelling along the same path through the crystal but vibrating at right angles to each other. The ordinary ray vibrates in a direction perpendicular to the optic axis while the extraordinary ray vibrates in a direction parallel to the optical axis. Upon entering the analyzer, the two vertical vibrating components issued from the quartz wave plate interfere in a direction parallel to the transmission of the analyzer.

The polarizers' transmission axes are all in y-axis. With unit incident intensity $I_0 = 1$, the Jones vector of the emergent light from the sandwiched group is given by

$$E = \begin{bmatrix} E_x \\ E_y \end{bmatrix} = M_{\mathrm{A}} M_{\mathrm{W}} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos[\delta(\lambda)/2] & i\sin[\delta(\lambda)/2] \\ i\sin[\delta(\lambda)/2] & \cos[\delta(\lambda)/2] \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ \cos[\delta(\lambda)/2] \end{bmatrix}, \tag{1}$$

where M_{A}, M_{W} are the Jones matrices of the analyzer and wave plate, respectively, $\delta(\lambda)$ is the phase retardation of the wave plate at wavelength λ. The emergent intensity I_t is

$$I_t(\lambda) = E^{\dagger} E,$$

then the transmission T is given by

$$T(\lambda) = \frac{I_t}{I_0} = \frac{E^{\dagger} E}{I_0} = \cos^2[\delta(\lambda)/2]. \tag{2}$$

The phase difference $\delta(\lambda)$ between the ordinary and the extraordinary light on propagation through the quartz wave plate is given by[6]

$$\delta(\lambda) = \frac{2\pi l [n_{\mathrm{e}}(\lambda) - n_{\mathrm{o}}(\lambda)]}{\lambda}, \tag{3}$$

where $n_o(\lambda)$, $n_e(\lambda)$ are the principal refractive indices of the ordinary and extraordinary rays at wavelength λ, respectively. By inserting Eq. (3) into Eq. (2), it can be got that

$$T(\lambda) = \cos^2\left\{\frac{\pi l[n_e(\lambda) - n_o(\lambda)]}{\lambda}\right\}. \tag{4}$$

When the angle between the optic axis of wave plate and the transmission axis of polarizer is $-\pi/4$ instead, we have the same transmission expression as Eq. (4).

The classical measurements of Macè de Lèpinary on the variation of quartz birefringence $(n_e - n_o)$ with wavelength λ in μm and temperature Γ in degree centigrade, can be expressed by the empirical equation[7,8]

$$\begin{aligned}10^3[n_e(\lambda) - n_o(\lambda)] &= 8.86410 + 0.107057\lambda^{-2} \\ &+0.0019893\lambda^{-4} - 0.17175\lambda^2 \\ &-10^{-3}\Gamma(1 + \Gamma/900)(1.01 + 0.2\lambda^2).\end{aligned} \tag{5}$$

It is noted that the effect of temperature is taken into account in Eq. (5). The values obtained by Eq. (5) coincide very well with the experimental values shown in the Table of quartz refractive indices[9] at a wide spectral region of 0.2—2.0 μm, which can be seen clearly in Fig. 1(a). We define the function, $f(\lambda) = \frac{n_e(\lambda) - n_o(\lambda)}{\lambda}$, and perform the first and second derivatives of $f(\lambda)$. We can deduce that $\frac{df(\lambda)}{d\lambda} < 0$ and $\frac{d^2 f(\lambda)}{d\lambda^2} > 0$ in a wide region of 0.2—2.0 μm at 25 °C, which can be illustrated by Figs. 1(b), (c), and (d).

Consider now a beam of white light normally incident through the sandwiched group. The wavelength-dependent optical transmission $T(\lambda)$ varies with the physical thickness of quartz wave plate, expressed as Eqs. (4) and (5). We can achieve a certain and single polarization-resolved transmission spectrum (PRTS) curve for a quartz wave plate of fixed thickness. For example, the theoretical PRTS curves of a 300-μm-thick and a 500-μm-thick quartz wave plates at 25 °C are shown in Fig. 2. We also analyze the temperature effect on the multi-order quartz wave plate. The PRTS curves of a 1500-μm-thick quartz wave plate at 25 and 20 °C are shown in Fig. 3. There is a spectral shifting of ~ 2 nm owing to a temperature difference of ~ 5 °C.

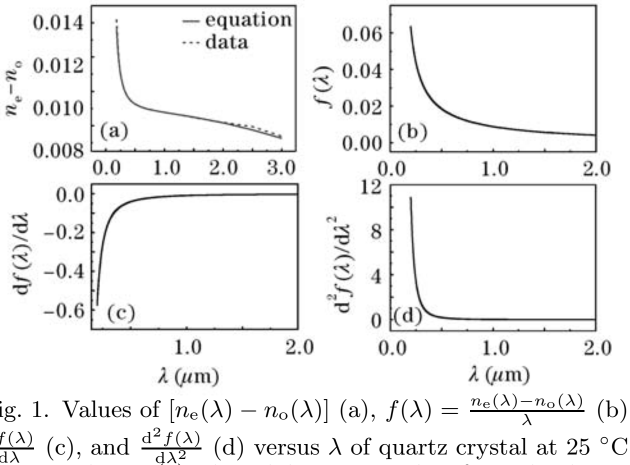

Fig. 1. Values of $[n_e(\lambda) - n_o(\lambda)]$ (a), $f(\lambda) = \frac{n_e(\lambda) - n_o(\lambda)}{\lambda}$ (b), $\frac{df(\lambda)}{d\lambda}$ (c), and $\frac{d^2 f(\lambda)}{d\lambda^2}$ (d) versus λ of quartz crystal at 25 °C, respectively. In (a), the solid curve is taken from the dispersion equation; the dotted curve from the table of refractive indices[9].

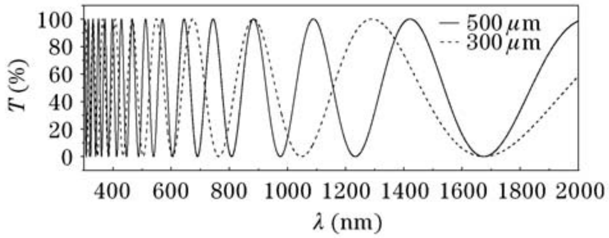

Fig. 2. Theoretical PRTS curves of two quartz wave plates sandwiched between two parallel polarizers at 25 °C with plate thicknesses of 500 and 300 μm.

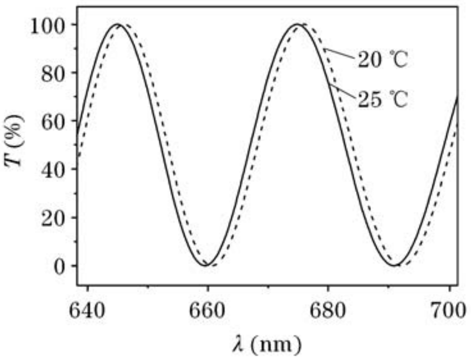

Fig. 3. PRTS curves of a 1500-μm-thick quartz wave plate at 25 and 20 °C.

With Eq. (4), we can get the phase retardation $\delta(\lambda)$ of the quartz wave plate

$$\delta(\lambda) = 2\cos^{-1}[T(\lambda)^{1/2}]. \tag{6}$$

Equation (6) shows that the phase retardation $\delta(\lambda)$ can be obtained readily with the transmission $T(\lambda)$ of the sandwiched group. As an example, the PRTS curve of a 500-μm-thick quartz wave plate and its corresponding phase retardation $\delta(\lambda)$ at 500—700 nm are shown in Fig. 4. It is found that the same wave plate acts as a half-wave plate at wavelengths with minimum transmission (hereafter called valley wavelength), a full-wave plate at wavelengths with maximum transmission (hereafter called peak wavelength), and a quarter-wave plate at wavelengths with half-maximum transmission.

We can extract the physical thickness of the quartz wave plate by exactly picking up two wavelengths from the PRTS curve. We generally choose two peak or valley wavelengths for simplicity and higher accuracy. For example, we choose two valleys to extract the thickness. The physical thickness of a half-wave plate corresponding to the valleys of the PRTS curve satisfies

$$m + 1/2 = \frac{n_e(\lambda) - n_o(\lambda)}{\lambda} l = f(\lambda_m) l. \tag{7}$$

The function $f(\lambda)$ is a decrement function $\left(\frac{df(\lambda)}{d\lambda} < 0\right)$, so the variable m, which is defined as the order of the wave plate corresponding to every valley wavelength and is a positive integer, decreases with increasing valley wavelength λ_m, and the order difference

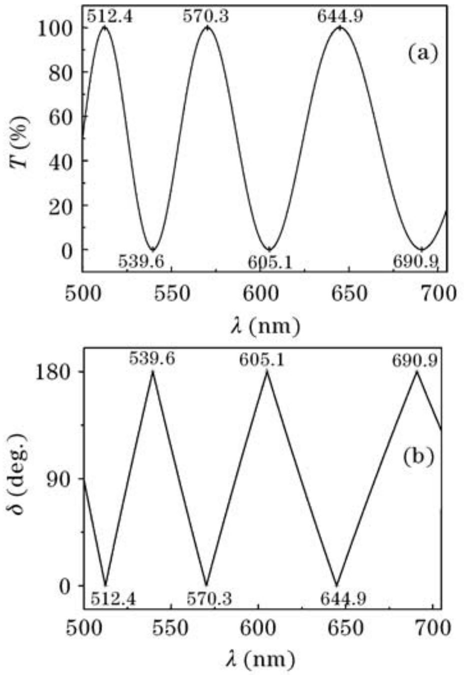

Fig. 4. (a) Theoretical PRTS curve $T(\lambda)$ and (b) phase retardation curve $\delta(\lambda)$ at 500—700 nm for a 500-μm-thick quartz wave plate sandwiched between two parallel polarizers at 25 °C.

of the adjacent peaks is 1. Then the thickness l can be given by

$$l = \frac{i-j}{f(\lambda_i) - f(\lambda_j)} = \frac{\Delta m}{f(\lambda_i) - f(\lambda_j)}, \quad (i \neq j), \tag{8}$$

where i and j are the orders of the same wave plate at the peak wavelengths of λ_i and λ_j, respectively. We can get the value of the order difference Δm of the two valleys by counting the number of peaks m' between the two peaks we choose and $\Delta m = m' + 1$.

On the other hand, if the wave plate is sandwiched between a pair of crossed linear polarizers, and the angle between the optical axis of the wave plate and the transmission direction of the two polarizers is $\pi/4$, the transmission T is then given by

$$T(\lambda) = \sin^2\left[\frac{\pi t(n_{\mathrm{e}}(\lambda) - n_{\mathrm{o}}(\lambda))}{\lambda}\right]. \tag{9}$$

Differently, the wave plate acts as a full-wave plate at valley wavelengths, and a half-wave plate at peak wavelengths. But we can also extract the physical thickness of wave plates by the same way as we do when the two polarizers are parallel to each other.

Accurate measurement of the physical thickness is performed with this method. Figure 5 shows the schematic diagram of an optical system. The collimated white light passes directly through the sandwiched group and then was collected by a miniature grating spectrometer with a high-sensitivity charge-coupled device (CCD) array for quick and easy measurement of a complete spectrum. With accurate rotation mounts, the polarizer, quartz wave plate, and analyzer can be rotated in horizontal and vertical planes, respectively. Simply, we also can place the sandwiched group in a spectraphotometer to obtain the PRTS curve. In fact, we can calculate the thickness of the quartz wave plate as long as we acquire two adjacent peaks or valleys in the PRTS curve. So we can choose a spectrometer with narrow spectrum range but better resolution. Figure 6 shows a typical optical transmission spectrum measured when the two polarizers are parrallel. It is obvious that the wave plate can work as a half-wave plate at 1064.7 nm. In this PRTS curve, the two adjacent peak wavelengths are 993.7 and 1147.8 nm. Calculating with Eq. (8), we get the thickness of the quartz wave plate, 792.84 μm, which coincides well with the value given by the manufacturer, 793 μm.

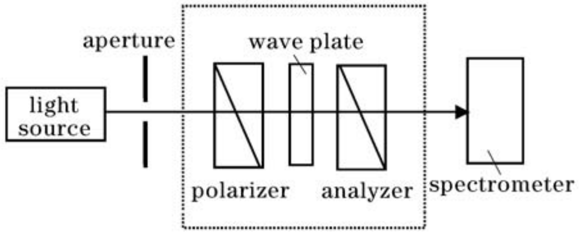

Fig. 5. Schematic diagram of the measuring system of PRTS curve.

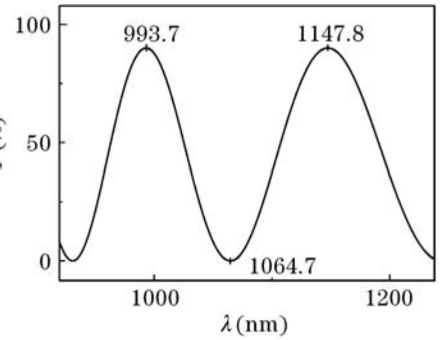

Fig. 6. Optical transmission spectrum of a quartz wave plate sandwiched between two parallel polarizers at 25 °C.

In experiments, Glan-Taylor polarizers are used and the scale accuracy of the rotation stage is 0.1°. The errors of the angle in parallelism of the two polarization axes and the angles between the planes of the two polarizers and ray of the incident light can be well controlled within 0.2°. The errors introduced in the process of picking up peak wavelengths inaccurately follow a particular rule, which will be discussed in detail in the following.

We obtain the physical thickness of the quartz wave plate with Eq. (8). In practical operation, Δm is an integer, the offsets of the two fixed wavelengths of λ_i and λ_j, $\Delta\lambda_i$ and $\Delta\lambda_j$, are determined by the spectral resolution power of the spectrometer $\Delta\lambda$ and will lead to measurement errors. The equation of maximal absolute error can be expressed by

$$\begin{aligned} \Delta l &= \left|\frac{\partial l}{\partial \lambda_i}\right| \Delta\lambda_i + \left|\frac{\partial l}{\partial \lambda_j}\right| \Delta\lambda_j \\ &= \left|-\frac{\Delta m}{[f(\lambda_2) - f(\lambda_1)]^2}\right| \left(\left|\left.\frac{\mathrm{d}f(\lambda)}{\mathrm{d}\lambda}\right|_{\lambda_i}\right| + \left|\left.\frac{\mathrm{d}f(\lambda)}{\mathrm{d}\lambda}\right|_{\lambda_j}\right| \right) \Delta\lambda. \end{aligned} \tag{10}$$

$f(\lambda)$ is a decrement function, $\frac{\mathrm{d}f(\lambda)}{\mathrm{d}\lambda} < 0$, Eq. (10) can be rewritten as

Table 1. Maximal Relative Errors E_l for Different Thicknesses l of Wave Plate at Different Wave Bands with Different Spectral Resolutions of the Spectrometer $\Delta\lambda$ with the Order Difference of Two Wavelength Valleys $\Delta m = 1$

$\Delta\lambda$ (nm)	l (μm)	E_l at 500—620 nm	E_l at 650—900 nm	E_l at 1050—1700 nm
0.1	300	2.03×10^{-3}	1.00×10^{-3}	3.67×10^{-4}
	500	3.06×10^{-3}	1.46×10^{-3}	4.80×10^{-4}
0.01	300	2.00×10^{-4}	1.00×10^{-4}	3.67×10^{-5}
	500	3.20×10^{-4}	1.46×10^{-4}	4.80×10^{-5}

$$\Delta l = \frac{l^2}{\Delta m}\left(-\left.\frac{\mathrm{d}f(\lambda)}{\mathrm{d}\lambda}\right|_{\lambda_i} - \left.\frac{\mathrm{d}f(\lambda)}{\mathrm{d}\lambda}\right|_{\lambda_j}\right)\Delta\lambda. \quad (11)$$

The maximal relative error can be given by

$$E_l = \frac{\Delta l}{l} = \frac{l}{\Delta m}\left(-\left.\frac{\mathrm{d}f(\lambda)}{\mathrm{d}\lambda}\right|_{\lambda_i} - \left.\frac{\mathrm{d}f(\lambda)}{\mathrm{d}\lambda}\right|_{\lambda_j}\right)\Delta\lambda. \quad (12)$$

For a wave plate, the value of coefficient $\frac{l}{\Delta m}$ is a constant. Then the value of E_l is determined by the two items, $\left(-\left.\frac{\mathrm{d}f(\lambda)}{\mathrm{d}\lambda}\right|_{\lambda_i} - \left.\frac{\mathrm{d}f(\lambda)}{\mathrm{d}\lambda}\right|_{\lambda_j}\right)$ and $\Delta\lambda$.

In Eqs. (11) and (12), $\frac{\mathrm{d}^2 f(\lambda)}{\mathrm{d}\lambda^2} > 0$, then the function $\left(-\frac{\mathrm{d}f(\lambda)}{\mathrm{d}\lambda}\right)$ is a decrement one. So the larger the values of the valley wavelengths are, the smaller the value of E_l. Thus we can improve the accuracy by choosing valley wavelength in long-wave band on the PRTS curve. In addition, the error has a further dependence on the spectral resolution power of the spectrometer, that is, the value of E_l is proportional to $\Delta\lambda$. The measuring accuracy can be improved by several orders of magnitude through heightening the spectral resolution power. The error rule is illustrated in Table 1.

In this letter, we present a simple method to measure the thickness of a quartz wave plate and its phase retardation at wide spectral range. In spite of its simplicity, the method allows for maximal relative error of 10^{-3} in the thickness measurement. Higher accuracies are expected by choosing longer wave band for measurement or by using spectrometer with higher spectral resolution. The temperature effect is considered in our method. The attractive aspect of this method lies in the fact that the phase retardation at a wide spectral range can be read out from the PRTS curve, which can help the manufacturer to monitor whether the product can meet user's need. This method can be applied to other uniaxial birefringent crystals on condition that we have obtained their dispersion equation of refractive-index difference. It is easy to go back and forth between the thickness and its refractive-index difference of quartz wave plate. In solid-state optics, characterization of newly developed uniaxial birefringent crystals can benefit from this method, that is, dispersion equation of refractive-index difference of these crystals can be achieved when we know the accurate thickness of crystal plate.

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References

1. Y.-T. Jeng and Y.-L. Lo, Appl. Opt. **45,** 1134 (2006).
2. P. Zhao, F. Wu, D. Hao, R. Wang, and S. Ren, Acta Opt. Sin. (in Chinese) **26,** 379 (2006).
3. Y.-L. Lo, J.-F. Lin, and S.-Y. Lee, Appl. Opt. **43,** 6248 (2004).
4. M.-H. Chiu, C.-D. Chen, and D.-C. Su, J. Opt. Soc. Am. A **13,** 1924 (1996).
5. P. S. K. Lee, J. B. Pors, M. P. Exter, and J. P. Woerdman, Appl. Opt. **44,** 866 (2005).
6. F. Bréhat, B. Wyncke, and A. M. Benoit, J. Phys. D **32,** 227 (1999).
7. M. J. Macè de Lèpinary, J. Phys. **1,** 23 (1892).
8. A. E. Ennos and K. W. Opperman, Appl. Opt. **5,** 170 (1966).
9. V. G. Dmitriev, G. G. Gurzadyan, and D. N. Nikogosyan, in *Handbook of Nonlinear Optical Crystals* A. E. Siegman (ed.) (Springer-Verlag, Berlin, Heidelberg, 1997) p.284.