## Denoising lidar signal by combining wavelet improved threshold with wavelet domain spatial filtering

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Lidar is an effective tool for remotely monitoring target or object, but the lidar signal is often affected by various noises or interferences. Therefore, detecting the weak signals buried in noises is a fundamental and important problem in the lidar systems. In this paper, an effective noise reduction method combining wavelet improved threshold with wavelet domain spatial filtration is presented to denoise pulse lidar signal and is investigated by detecting the simulating pulse lidar signals in noise. The simulation results show that this method can effectively identify the edge of signal and detect the weak lidar signal buried in noises. *OCIS codes:* 010.3640, 070.6020, 070.4560.

Lidar is an effective tool for remotely monitoring target or object, and it determines the range of a target by analyzing the laser pulse reflected from the target. In real applications there are many sources of noises and interferences, which always badly affect the lidar signal and limit the effective range.

Recently, the wavelet transform (WT) has become an increasingly important tool in signal processing field. WT are multi-resolution decompositions that can be used to analyze signals<sup>[1]</sup>. Donoho *et al.*<sup>[2-4]</sup> proposed</sup>wavelet shrinkage method, which becomes the common method for denoising signal. However, hard threshold and soft threshold have their inherent disadvantages, so this method is not suited for all kinds of signals. Xu et al.<sup>[5]</sup> introduced a spatially selective noise filtration algorithm, which is based on the direct spatial correlation of the WT at two adjacent scales. A high correlation is used to infer that there is a significant feature at the position that should be passed through the filter. However, this algorithm was not entire, it did not give the condition to finish the iterative process. Based on the spatial filtration, Yi et al.<sup>[6]</sup> proposed an effective imagefiltering algorithm by combining it with soft threshold.

In this paper, we presented wavelet improved thresholding method, which overcomes the disadvantages of hard and soft thresholds. Then, we used the three-order correlation of wavelet coefficients to distinguish edge signal from noise, and proposed a effective signal denoising algorithm by combining wavelet improved threshold with wavelet domain spatial filtering to denoise pulse lidar signal in CO<sub>2</sub> differential absorption lidar (DIAL). Considering the redundance of stationary wavelet transform (SWT)<sup>[7]</sup>, we use SWT to decompose the lidar signal. Compared the wavelet shrinkage method, this algorithm is superior in detecting the edge of signal and reducing the noise.

Donoho and Johnstone proposed two kinds of thresholds for wavelet shrinkage: hard threshold and soft threshold. Let  $\lambda$  denote the given threshold, the hard threshold can be defined as

$$w'_{\rm H} = \begin{cases} w, & |w| \ge \lambda \\ 0, & |w| < \lambda \end{cases}, \tag{1}$$

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the soft threshold can be defined by

$$w'_{\rm S} = \begin{cases} \operatorname{sgn}(w)(|w| - \lambda), & |w| \ge \lambda \\ 0, & |w| < \lambda \end{cases} .$$
(2)

The hard threshold is discontinuous, when the estimated wavelet coefficients are used to reconstruct, there are some artificial noises in the reconstructed signal. The soft threshold is continuous function, but there are inherent differences between those estimated w' and true w, which lose some high-frequency information, so that reduce the accuracy of reconstructed signal and blur the edge of signal. In order to overcome the disadvantages of hard and soft thresholds, the improved threshold is proposed as

$$w'_{\rm I} = \begin{cases} w - w \left| \frac{\lambda}{w} \right|^{\frac{|w|}{\lambda}}, & |w| \ge \lambda\\ 0, & |w| < \lambda \end{cases}$$
(3)

The relationship of the improved threshold with the hard and soft thresholds is given in Fig. 1. It can be clearly seen that the improved threshold is continuous and lies between the hard and soft thresholds. When |w| is small, w is mostly composed by noise, the improved threshold is close to the soft threshold. When |w| is big, the main of w is the high-frequency information of signal, the improved threshold is close to the hard threshold.



Fig. 1. Relationship of the improved threshold with the hard and soft thresholds.

Therefore, the improved threshold overcomes the discontinuous of the hard threshold and losing high-frequency information of the soft threshold, and incorporates the advantage of the hard and soft thresholds. The improved threshold combined with the following wavelet domain spatial filtering will be suited for pulse signal buried in noise.

The key to wavelet domain spatial filtering is to identify edges. Edges are identified as features that have signal peaks across many scales. Direct spatial correlations of the WT at different scales are used to identify the edges; the small-scale data are passed at positions where the correlation is large and suppressed if the correlation is small. We use the direct multiplication of WT data at adjacent scales to distinguish important edges from noise and accomplish the task of removing noise from signals.

The wavelet domain noise filtration technique is based on the fact that sharp edges have large signal over many wavelet scales, and noise dies out swiftly with increasing scale. We are using the direct spatial correlation  $\operatorname{Corr}_{\mathrm{L}}(j,k)$  of WT contents at several adjacent scales to accurately detect the location of edges or other significant features,

$$\operatorname{Corr}_{L}(j,k) = \prod_{i=0}^{L-1} W(j+i,k), \quad k = 1, 2, \cdots, K, \quad (4)$$

where L is the number of scales involved in the direct multiplication, K is the length of signal, j < J - L + 1, and J is the total number of scales. The normalized correlation coefficient is given by

$$\operatorname{NCorr}_{L}(j,k) = \operatorname{Corr}_{L}(j,k) * \sqrt{\frac{P_{W}(j)}{P_{\operatorname{Corr}_{L}}(j)}},$$
(5)

where  $P_{\mathrm{W}}(j) = \sum_{k} W^2(j,k), P_{\mathrm{Corr}_L}(j) = \sum_{k} \mathrm{Corr}_L^2(j,k).$ 

Comparing the absolute values of normalized correlation coefficient  $|\operatorname{NCorr}_L(j,k)|$  with the absolute value of wavelet coefficient |W(j,k)|, the most important edges at wavelet scale j are identified. If

$$|\mathrm{NCorr}_L(j,k)| > |W(j,k)|, \tag{6}$$

we consider that the wavelet coefficient contains the edge information of signal at this position, and set the wavelet domain space filter  $W_{\rm sf}(j,k)$  to 1, set W(j,k) to 0; otherwise, we consider that the wavelet coefficient is noise, and preserve W(j,k) and set  $W_{\rm sf}(j,k)$  to 0. This procedures of power normalization, data value comparison, and edge information extraction can be iterated many times until  $P_{\rm W}(j)/K$  is nearly equal to the average noise power at each wavelet scale. Thus, we obtain the filters  $W_{\rm sf}(j,k)$  at scale j.

The procedure of entire denoising process is given as follows. Step1: Decompose the signal with SWT, then copy wavelet coefficient W(j,k) to  $W_{\text{origin}}(j,k)$ . Step2: Estimate the average noise power of  $\bar{P}_n$ . Step3: Compute the three-order spatial correlation coefficient  $\text{Corr}_3(j,k)$ of detail signal. Step4: Normalize the correlation coefficient to obtain  $\text{NCorr}_3(j,k)$ , and compare the normalized correlation coefficient with wavelet coefficient, if  $|\text{NCorr}_3(j,k)| > |W(j,k)|$ , set  $W_{\text{sf}}(j,k) = 1$ ; W(j,k) = 0;  $\operatorname{Corr}_3(j,k) = 0$ , else  $W_{\mathrm{sf}}(j,k) = 0$ , preserve W(j,k)and  $\operatorname{Corr}_3(j,k)$ . Step5: Calculate the power  $P_W(j)$ of the preserved W(j,k), if  $P_W(j)/K > \overline{P}_n$ , step4 is iterated. Step6: Re-scale the wavelet domain spatial filters at all scale  $W'_{\mathrm{sf}}(j,k) = W_{\mathrm{sf}}(j,k) \times W_{\mathrm{sf}}(j+1,k)$ , the detail signal is filtered with the obtained  $W'_{\mathrm{sf}}(j,k)$ ,  $W_{\mathrm{new}}(j,k) = W'_{\mathrm{sf}}(j,k) \times W_{\mathrm{origin}}(j,k)$ . Step7: Process wavelet coefficient by the improved threshold, and obtain new wavelet coefficient  $W'_{\mathrm{new}}(j,k)$ . Step8: Obtain denoised signal by reconstructing with  $W'_{\mathrm{new}}(j,k)$ .

The algorithm discussed here was programmed for use in evaluating their performance on simulated topographic pulse lidar data from the return signal statistical model of differential absorption  $lidar^{[8]}$ . At first, we assume that the full-wide at half maximum (FWHM) of transversely excited atmospheric (TEA)  $CO_2$  laser pulse is 60 ns and the sample rate of the A/D converter is 120 MHz, so laser pulse includes about 15 sampling points. If the target range is 5000 m, the simulated received lidar signal is shown in Fig. 2(a). Figure 2(d) is the denoised lidar signal processed by our algorithm. In order to compare the performance, the denoised signals processed by wavelet hard and soft thresholds are respectively given in Figs. 2(b) and (c). In Fig. 2(d), it is obvious to find the received lidar signal at 4000 sampling point, and the result agrees well with that of the simulated target range. However, it is difficult to find the received signal from Figs. 2(a), (b), and (c).

This simulation results show that noise reduction is remarkable, at the same time, it preserves and positions the sharp edges of pulse signal. This algorithm can accurately estimate the arriving time of the received lidar signal and effectively measure the range of target. It is specially suitable for sharp edge pulse due to its low signal-to-noise (SNR) ratio. The performance of detecting edge of signal is superior to the traditional wavelet shrinkage.

Here, we simulate the whole sampling process, the sampling data are large so that it takes long time to calculate and is not suitable for real-time processing, especially for long range. However, using the range gating technique, the sampling data will be reduced evidently so that the processing time is shortened obviously, and this algorithm can be suitable for real-time processing.



Fig. 2. Simulated received lidar signal (a), denoised signals processed by wavelet hard (b) and soft thresholds (c), and by our method (d).

In conclusion, we have introduced an effective method combining wavelet improved threshold with wavelet domain spatial filtering technique into pulse lidar signal to denoise the signal and detect the edge of the received pulse signal. The lidar signal from longer distance is almost buried in the noise. Our method can remove the noise and preserve the edge of the received signal. Consequently, the noise in lidar signal is almost entirely suppressed and the received pulse signal edge is obviously detected, so it can more precisely estimate the arriving time of the received signal and more accurately measure the range of target.

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