Two-dimensional novel optical lattices with multi-well traps for cold atoms or molecules

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We propose some new schemes to constitute two-dimensional (2D) array of multi-well optical dipole traps for cold atoms (or molecules) by using an optical system consisting of a binary π-phase grating and a 2D array of rectangle microlens. We calculate the intensity distribution of each optical well in 2D array of multi-well traps and its geometric parameters and so on. The proposed 2D array of multi-well traps can be used to form novel 2D optical lattices with cold atoms (or molecules), and form various novel optical crystals with cold atoms (or molecules), or to perform quantum computing and quantum information processing on an atom chip, even to realize an array of all-optical multi-well atomic (or molecular) Bose-Einstein condensates (BECs) on an all-optical integrated atom (or molecule) chip.

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In 1986, the first optical dipole trap for cold Na atoms with a focused red-detuned Gaussian beam was demonstrated by Chu *et al.*[1]. After then, people proposed a variety of optical dipole traps for cold atoms and successfully realized optical trap of neutral atoms. Such as, Takekoshi *et al.* in 1995 proposed a new scheme to trap cold ^{133}Cs atoms by using a focused CO_2 laser beam with a giant red-detuning[2]. In 1998, Yin *et al.* proposed a new gravito-optical trap (GOT) scheme to trap and cool neutral atoms by use of a blue-detuned hollow laser beam (HLB) and the HLB-induced intensity-gradient cooling[3]. In 2001, Birkl *et al.* proposed some new optical microtraps and two-dimensional (2D) array of microtraps for cold atoms on an atom chip[4]. Recently, a red-detuned CO_2 or YAG laser is used to realize all-optical atomic Bose-Einstein condensates (BECs), quantum degenerate Fermi gases, all-optical molecular BEC, and all-optical output of an atom laser[5−10].

On the other hand, controllable double-well or multi-well optical traps for cold atoms (or molecules) and novel optical lattices with a large lattice constant have wide applications in the field of atom, molecular, and optical physics, such as the studies of cold collisions between two atomic or molecular samples[11], matter-wave interference of trapped atoms[12,13], quantum entanglement between two assemble of atoms, and all-optical realization of double-well or multi-well BEC and 2D array of BECs, and so on. In 2004, we proposed a controllable double-well trap for cold atoms (or cold molecule) and its 2D array of double-well traps, and studied the relationships between the characteristic parameters of each optical well and the parameters of the optical system[14]. In this paper, we report a novel 2D array of controllable multi-well traps on a chip, and discuss the potential applications of our novel optical lattices in the fields of atom, molecule and quantum optics.

As shown in Fig. 1(a), when a π-phase plate is extended along both the x and y directions, a 2D array of π-phase plates (i.e., a 2D π-phase grating) will be formed. When a plane light wave with a wavelength of λ goes through an optical system composed of a 2D π-phase grating and a 2D array of planar micro-lenses, a 2D array of four-well optical traps will be formed near the focal plane of micro-lens array because of the complete destructive-interference effect at the light axis of each lens, which is shown in Fig. 1(b).

For a 2D π-phase grating, its amplitude transmission function is given by

$$g(x,y)=\sum_{m=-(M+1)/2}^{(M+1)/2}(-1)^{m+1}\mathrm{rect}\left[\frac{x-(2m-1)a}{2a}\right]$$

$$\times\sum_{n=-(N+1)/2}^{(N+1)/2}(-1)^{n+1}\mathrm{rect}\left[\frac{y-(2n-1)a}{2a}\right],\qquad(1)$$

where rect(x) or rect(y) is the rectangle function, M and N are the numbers of micro-lenses in the x and y directions, respectively. When the center of 2D array of $M\times N$ micro-lenses is set at the origin (0,0) of the x and y coordinates, the coordinates of the center of mnth lens

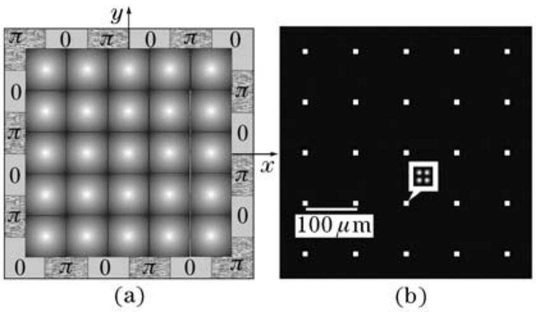

Fig. 1. Schematic diagram of a 2D array of four-well optical traps. (a) Top view of our optical system to generate a 2D array of four-well traps, and (b) intensity-density distribution of 2D array of four-well optical traps on the focal plane of the lens array for $a = 50$ μm, $f = 250$ μm, $\lambda = 1.06$ μm, and the YAG laser power is $p_{mn} = 100$ mW for each lens.

in 2D planar lens array are $x_m = 2ma$ and $y_n = 2na$, respectively, and then the amplitude transmission function of the mnth lens can be described as (here the constant phase factors are neglected)

$$t_{mn}(x,y) = \mathrm{rect}\left[\frac{x-x_m}{2a}, \frac{y-y_n}{2a}\right] \times \exp\left\{\frac{i\pi}{\lambda f}[(x-x_m)^2 + (y-y_n)^2]\right\}. \tag{2}$$

When $a^2 \gg f\lambda$, the distribution of diffraction light intensity is mainly located near the focal point of each lens, and the interference effect among the diffraction light waves from adjacent lenses can be neglected. In this case, when the optical system composed of a 2D π-phase grating and a 2D array of micro-lenses is illuminated by a plane light wave, the distribution of diffraction light intensity of each lens is identical, so we only need to discuss the intensity distribution of a single lens. The distribution of the diffraction field of the mnth lens in 2D planar lens array is given by

$$U_{mn}(x_0, y_0, z) = \frac{A}{i\lambda z}\iint_{-\infty}^{\infty} t_{mn}(x,y) g(x,y) \times \exp\left[\frac{i\pi}{\lambda z}(x-x_0)^2 + (y-y_0)^2\right]\mathrm{d}x\mathrm{d}y, \tag{3}$$

and the corresponding intensity distribution is written by $I_{mn}(x_0, y_0, z) = |U_{mn}(x_0, y_0, z)|^2$. In particular, the light intensity distribution in the focal plane of each lens can be described by

$$I_{f(x_0,y_0)mn} = 4P_n(\beta/\lambda)^2 \frac{\sin^4[\pi(x_0-2ma)\beta/\lambda]}{[\pi(x_0-2ma)\beta/\lambda]^2} \times \frac{\sin^4[\pi(y_0-2na)\beta/\lambda]}{[\pi(y_0-2na)\beta/\lambda]^2}. \tag{4}$$

From Eq. (3), we calculate the intensity distribution of 2D array of four-well optical traps in the focal plane of 2D array of 5×5 micro-lenses illuminated by a YAG laser, and its intensity-density distribution is shown in Fig. 1(b), and know that when the average laser power of each lens is 100 mW ($\lambda = 1.06$ μm), the side length and focal length of each square lens are $a = 50$ μm and $f = 250$ μm, the maximum and average intensities of each optical well are $I_{\max} = 3.93$ GW/m^2 and $\bar{I} = 1.45$ GW/m^2, and the corresponding maximum and mean optical trapping potential for ^{85}Rb atoms in each well are about 596 and 220 μK respectively, which are high enough to trap cold ^{85}Rb atoms with a temperature of ~ 20 μK from a standard optical molasses.

When $a = 50$ μm, $f = 250$ μm, $\lambda = 1.06$ μm, and $P_{mn} = 100$ mW, the transverse and longitudinal geometric sizes of each optical well in the 2D array of four-well traps are 3.34 and 58.20 μm, which are the typical transverse and longitudinal sizes of BEC atomic cloud, respectively. Moreover, the maximum intensity gradients and its curvatures are more than 7.7×10^{14} W/m^3 and 1.69×10^{19} W/m^4, respectively. Obviously, such a 2D array of controllable four-well optical traps is suitable to study an array of four-well atomic (or molecular) BECs, or to form a novel optical lattices with a controllable four-well, even to realize a 2D array of all optical four-well atomic (or molecular) BECs by using optical-potential evaporative cooling.

In addition, we also study the evolution of the four-well array, and find that when the π-phase grating is moved along the x or y direction relative to the lens array by the moving distance $t = a$, the 2D array of four-well traps will be evolved as a 2D array of double-well traps. While the π-phase grating is moved along the diagonal direction of the x and y coordinates by the moving distance $t = \sqrt{2}a$, the 2D array of four-well traps will be evolved into a 2D array of single-well traps. The corresponding maximum intensities of each optical well in 2D array of double-well and single-well traps are 7.48 and 14.3 GW/m^2, respectively.

From Fig. 1(b), we can see that when the cold atoms (or molecules) are loaded into a 2D array of four-well YAG laser traps, a novel 2D four-site optical lattice with a lattice constant of 100 μm will be formed, but its lattice constant $2a = 100$ μm is far larger than one of a standing-wave optical lattice with a lattice constant of $\lambda/2 = 0.53$ μm. This is the case of single-beam illumination. However, if a few incoherent laser beams with different incident angles are used to illuminate the optical system composed of a 2D π-phase grating and a 2D array of micro-lenses, some novel four-site optical lattices with a smaller lattice constant and different lattice types can be obtained near the focal plane of the lens array. For example, 1) Fig. 2(a) shows the case of two-beam illumination, if two incoherent YAG laser beams with an incident angle $i_{1,2} = \pm\mathrm{tg}^{-1}(a/2f)$ are used to illuminate symmetrically the optical system, and when the angle between the incident plane of the two laser beams and the xoz (or yoz) plane is 0°, a 2D optical lattice as shown in Fig. 2(a) will be formed, and its lattice constants along

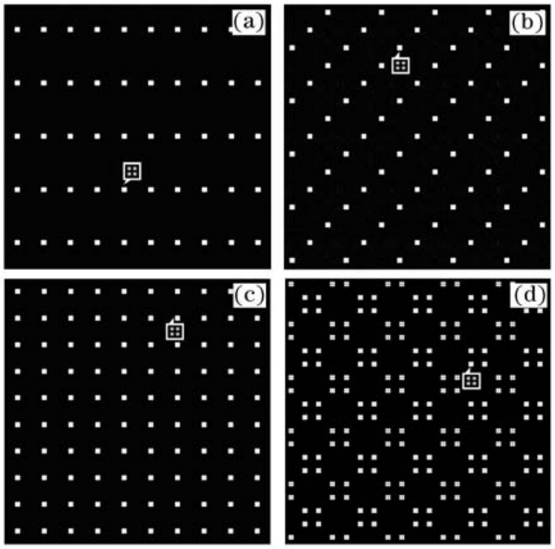

Fig. 2. 2D double-site optical lattices formed by using (a) two, (b) three, (c) four, and (d) eight incident beams to illuminate the optical system composed of a 2D π-phase grating with (0, π) equapartition areas and a 2D array of spherical microlens for $a = 50$ μm, $f = 250$ μm, $\lambda = 1.06$ μm, and a YAG laser power $P_{mn} = 100$ mW of each lens.

the x and y directions are 50 and 100 μm, respectively. 2) Figure 2(b) shows the case of three-beam illumination, two incoherent YAG laser beams with an incident angle $i_{1,2} = \pm\text{tg}^{-1}(\sqrt{8}a/3f)$, combined with the third laser beam propagating along the z direction, are used to illuminate symmetrically the optical system, and when the angle between the incident plane of the two laser beams and the xoz (or yoz) plane is 45°, a triangle lattice will be produced, and its lattice constants along the y and x directions are 33.3 and 100 μm, respectively. 3) Figure 2(c) shows the case of four-beam illumination, four incoherent YAG laser beams with an incident angle $i_{1,2} = \pm\text{tg}^{-1}(\sqrt{2}a/2f)$ are used to illuminate symmetrically the optical system, and when the angle between the incident plane of two laser beams and the xoz (or yoz) plane is $+45°$, and the angle between the incident plane of other two beams and the xoz (or yoz) plane is $-45°$, a 2D square optical lattice with a lattice constant of 50 μm will be generated. 4) Figure 2(d) is the case of eight-beam illumination, a new lattice with a smaller lattice constant is formed by the illumination of eight incoherent beams in which the incident angles of four beams are $i_{1,2} = \pm\text{tg}^{-1}(\sqrt{2}a/4f)$ and $i_{3,4} = \pm\text{tg}^{-1}(\sqrt{18}a/4f)$ respectively, and the angle between the incident plane and the xoz plane is 45°; the incident angles of the other four beams are also $i_{5,6} = \pm\text{tg}^{-1}(\sqrt{2}a/4f)$ and $i_{7,8} = \pm\text{tg}^{-1}(\sqrt{18}a/4f)$, but the angle between the incident plane and the xoz plane is $-45°$. In this illumination case, each lens will generate eight pairs of four optical wells.

If a plane light wave passes through a π-phase plate as shown in Fig. 3(a) in which the phases of two adjacent areas are 0 and π respectively and are focused by a lens, an optical trap with eight micro-wells near the focal plane of the lens will be formed due to the complete destructive-interference effect between the light fields from the 0 and π phase regions, and the calculated results are shown in Figs. 3(b) and (c).

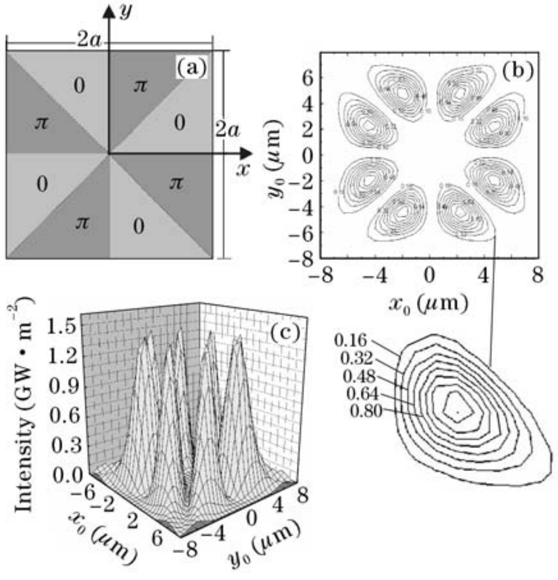

Fig. 3. Schematic diagram of an eight-well optical tarp. (a) A binary π-phase plate with eight $(0, \pi)$ equipartition areas, (b) the intensity contours, and (c) the 2D intensity distribution on the focal plane of the lens.

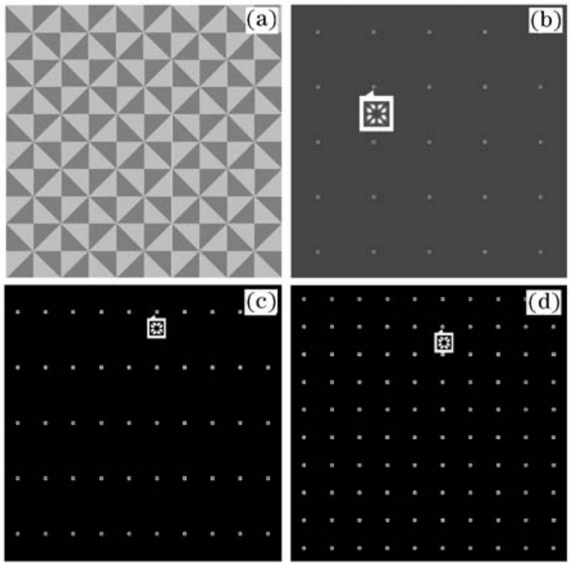

Fig. 4. Schematic diagram of a 2D array of eight-well optical traps. (a) Top view of the optical system to generate a 2D array of eight-well traps, and (b) intensity-density distribution of 2D array of eight-well optical traps on the focal plane of the lens array for $a = 50$ μm, $f = 250$ μm, $\lambda = 1.06$ μm, and the YAG laser power is $P_{mn} = 100$ mW for each lens. 2D eight-site optical lattices formed by using (c) two and (d) four incident beams to illuminate the optical system composed of a 2D π-phase grating with eight $(0, \pi)$ equapartition areas and a 2D array of spherical microlens for $a = 50$ μm, $f = 250$ μm, $\lambda = 1.06$ μm, and a YAG laser power $P_{mn} = 100$ mW of each lens.

If a π-phase plate with eight $(0, \pi)$ equipartition areas (see Fig. 3(a)) is extended along both the x and y directions, a 2D π-phase grating as shown in Fig. 4(a) will be formed. When a plane light wave with a wavelength λ goes through an optical system composed of the 2D π-phase grating and a 2D array of micro-lenses, a 2D array of eight-well optical traps will be formed near the focal plane of micro-lens array, and its intensity-density distribution, a novel optical lattice with eight optical micro-wells, is shown in Fig. 4(b). When $a = 100$ μm, $f = 500$ μm, $\lambda = 1.06$ μm, $P_{mn} = 100$ mW, and $I_0 = 2.5 \times 10^6$ W/m^2, the maximum intensity of each well reaches 1.46 GW/m^2.

From Fig. 4(b), we can see that when cold atoms (or cold molecules) are loaded into a 2D array of eight-well YAG laser traps, a novel 2D eight-site optical lattice with a lattice constant of 100 μm can be formed by using a single-beam illumination. Similarly, if a few incoherent laser beams with different incident angles are used to illuminate the optical system composed of a 2D π-phase grating with eight $(0, \pi)$ equapartition areas and a 2D array of planer micro-lenses, some novel eight-site optical lattices with a smaller lattice constant and different lattice types can be obtained near the focal plane of the lens array.

For example, if two incoherent YAG laser beams with an incident angle $i_{1,2} = \pm\text{tg}^{-1}(a/2f)$ are used to illuminate symmetrically the optical system, and when the angle between the incident plane of the two laser beams and the xoz (or yoz) plane is 0°, a 2D square optical lattice as shown in Fig. 4(c) will be formed. In Fig. 4(d), four incoherent YAG laser beams with an incident angle $i_{1,2} = \pm\text{tg}^{-1}(\sqrt{2}a/2f)$ are used to illuminate symmetrically the optical system, and when the angle between the incident plane of the two laser beams and the xoz (or yoz) plane is 45°, and the angle between the incident plane of the other two beams and the xoz (or yoz) plane is $-45°$, a 2D square lattice with a lattice constant of 50 μm will be formed. In this scheme, each lens will generate four pairs of eight optical wells.

It can be seen from Figs. 2 and 4(c,d) that if more incoherent incident beams are used to illuminate the optical system, we can obtain many novel optical lattices with a smaller lattice constant and a more complex lattice microstructure. Also, if a 2D π-phase grating with six or more (0, π) equapartition areas is used, some novel optical lattices with six or more sites will be generated by the optical system and multi-beam illumination. In addition, we know that from ac Stark effect, the optical trapping potential for neutral molecules in a red-detuned optical dipole trap is given by[15]

$$U(r) = -\frac{\alpha}{2\varepsilon_0 c} I(r), \tag{5}$$

where $I(r)$ is the intensity distribution of the optical well, α is the average polarizability of molecules, ε_0 and c are the dielectric constant and the speed of light in the vacuum, respectively. When the 2D array of four-well optical traps in the focal plane of 2D array of 5×5 micro-lenses is illuminated by the YAG laser with a power of 100 W, the maximum intensity of each optical well is $I_{\max} = 157.2$ GW/m^2, and the corresponding maximum optical trapping potential for cold methane (CH_4) molecules in each well is about 578 μK, which is high enough to trap cold molecules with a temperature of lower than 250 μK, which are formed by laser-cooled ultracold atoms by using magnetically-tunable Feshbach resonance or photoassociation spectroscopy, even resulted from Strak deceleration and cooling.

Compared with conventional optical lattices composed of 1D or 2D standing-wave light[16,17], the novel optical lattices have some novel and unique advantages as follows. 1) The lattice constant and trapping volume of each well are far larger than those of the conventional optical lattices with a lattice constant of $\lambda/2$, so the trapped atomic number at each site and its difference of refraction index will far larger than those of the standing-wave lattice, which is very beneficial to preparation of novel photon crystals; 2) The novel optical lattices have multiple optical wells at each site, and these wells with a small period of a few micrometers can be controllable by moving the phase grating, so it is convenient to manipulate and control cold atoms in the multi-well lattice, which may have some potential applications in quantum computing and information processing; 3) The atomic number in each well can reach $10^8 - 10^9$ atoms when its density is $10^{13} - 10^{14}$ atoms/cm^3, so the novel optical lattice can be used to directly realize an all optical array of atomic (or molecular) BECs by using optical-potential evaporative cooling[18]; In addition, it is convenient to form various 2D optical lattices with multiple wells on an all-optical atom (or molecule) chip.

In conclusion, we have proposed some new schemes to constitute 2D array of multi-well optical dipole traps for cold atoms (or molecules) by using an optical system consisted of a binary 2D π-phase grating and a 2D array of rectangle micro-lens, and calculated the intensity-density distributions of various 2D arrays of four- and eight-well optical traps, and studied the optical and geometric parameters of each optical well in the 2D multi-well trap arrays and so on. The study finds that the proposed 2D array of multi-well traps can be used to form novel 2D optical lattices with cold atoms (or molecules) by various illuminations with multiple incoherent beams. It is clear that such 2D optical lattices with controllable multiple sites are not only used to form various novel optical crystals with cold atoms or (cold molecules), or to perform quantum computing and quantum information processing on an atom chip, but also to realize a 2D array of all-optical multi-well atomic (or molecular) BECs on an all-optical integrated atom (or molecule) chip by using optical-potential evaporative cooling, and so on.

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References

1. S. Chu, J. E. Bjorkholm, A. Ashkin, and A. Cable, Phys. Rev. Lett. **57,** 314 (1986).
2. T. Takekoshi and R. J. Knize, Opt. Lett. **21,** 77 (1996).
3. J. Yin and Y. Zhu, Opt. Commun. **152,** 421 (1998).
4. G. Birkl, F. B. J. Buchkremer, R. Dumke, and W. Ertmer, Opt. Commun. **191,** 67 (2001).
5. M. D. Barrett, J. A. Sauer, and M. S. Chapman, Phys. Rev. Lett. **87,** 010404 (2001).
6. T. Weber, J. Herbig, M. Mark, H.-C. Nägerl, and R. Grimm, Science **299,** 232 (2003).
7. S. R. Granade, M. E. Gehm, K. M. O'Hara, and J. E. Thomas, Phys. Rev. Lett. **88,** 120405 (2002).
8. S. Jochim, M. Bartenstein, A. Altmeyer, G. Hendl, S. Riedl, C. Chin, J. H. Denschlag, and R. Grimm, Science **302,** 2101 (2003).
9. M. Greiner, C. Regal, and D. Jin, Nature **426,** 537 (2003).
10. G. Cennini, G. Ritt, C. Geckeler, and M. Weitz, Phys. Rev. Lett. **91,** 240408 (2003).
11. M. Arndt, M. B. Dahan, D. Guéry-Odelin, M. W. Reynolds, and J. Dalibard, Phys. Rev. Lett. **79,** 625 (1997).
12. Y. Shin, M. Saba, T. A. Pasquini, W. Ketterle, D. E. Pritchard, and A. E. Leanhardt, Phys. Rev. Lett. **92,** 050405 (2004).
13. M. Saba, T. A. Pasquini, C. Sanner, Y. Shin, W. Ketterle, and D. E. Pritchard, Science **307,** 1945 (2005).
14. X. Ji and J. Yin, J. Opt. Soc. Am. B **22,** 1737 (2005).
15. T. Liu, T. Zhang, J. Wang, and K. Peng, Acta Phys. Sin. (in Chinese) **53,** 1346 (2004).
16. K. I. Petsas, A. B. Coates, and G. Grynberg, Phys. Rev. A **50,** 5173 (1994).
17. L. Guidoni and P. Verkerk, J. Opt. B **1,** 23 (1999).
18. J. Yin, Phys. Rep. **430,** 1 (2006).