Numerical simulation of isopachic parameter determined by phase-stepping interferometric photoelasticity: errata

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The authors regret that errors occurred in the previous paper[1].

If the polarizer and analyzer are generally represented by $\mathbf{P}_\beta$ with arbitrary angle β to the reference axis in Fig. 1, original Eq. (8) can be rewritten as

$$\mathbf{E}_1 = \mathbf{P}_\beta \mathbf{Q}_\gamma \mathbf{R}_{\delta,\theta} \mathbf{Q}_{\pi/4} \mathbf{P}_\beta. \tag{8}$$

It is noted that the polarizer and analyzer always keep same polarized direction. The last interfered Jones vector $\mathbf{E}_{\text{total}}$ is revised as

$$\mathbf{E}_{\text{total}} = \mathbf{E}_1 + \mathbf{P}_\beta. \tag{9}$$

The final intensities I_i for four configurations are listed as Table 1 (original Eq. (11) is deleted).

Table 1. Light Intensities I_i for Analyzing Isopachics

γ	β	I_i
0	0	$I_1 = \{3 - \sin\delta_\text{d} \sin 2\theta\}/2 + \cos(\delta_\text{d}/2)[\cos(\delta_\text{s}/2) - \sin(\delta_\text{s}/2)] - \sin(\delta_\text{d}/2)\{\sin[(\delta_\text{s} + 4\theta)/2] + \cos[(\delta_\text{s} + 4\theta)/2]\}$
0	$\pi/2$	$I_2 = (3 - \sin\delta_\text{d} \sin 2\theta)/2 + \cos(\delta_\text{d}/2)[\cos(\delta_\text{s}/2) + \sin(\delta_\text{s}/2)] + \sin(\delta_\text{d}/2)\{\sin[(\delta_\text{s} - 4\theta)/2] - \cos[(\delta_\text{s} - 4\theta)/2]\}$
$\pi/2$	0	$I_3 = (3 - \sin\delta_\text{d} \sin 2\theta)/2 + \cos(\delta_\text{d}/2)[\cos(\delta_\text{s}/2) + \sin(\delta_\text{s}/2)] + \sin(\delta_\text{d}/2)\{\cos[(\delta_\text{s} + 4\theta)/2 - \sin[(\delta_\text{s} + 4\theta)/2]]\}$
$\pi/2$	$\pi/2$	$I_4 = (3 - \sin\delta_\text{d} \sin 2\theta)/2 + \cos(\delta_\text{d}/2)[\cos(\delta_\text{s}/2) - \sin(\delta_\text{s}/2)] + \sin(\delta_\text{d}/2)\{\cos[(\delta_\text{s} - 4\theta)/2 + \sin[(\delta_\text{s} - 4\theta)/2]]\}$

Therefore, four step phase-shifting formula is obtained as

$$\tan(\delta_\text{s}/2) = \frac{I_1 - I_2 + I_3 - I_4}{I_1 + I_2 - I_3 - I_4} \quad \text{for} \quad \cos(\delta_\text{d}/2)\cos 2\theta \neq 0. \tag{12}$$

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A phase-stepping interferometric photoelasticity method is proposed to determine whole-filed sum of principal stresses. The four phase steps are introduced by rotating quarter-wave plate and analyzer at definite optical arrangements. Light intensities and phase-stepping formula for the proposed method are derived using Jones calculus. Simulations of a circular disk under diametral compression demonstrate the feasibility of the proposed method.

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Photomechanics method in experimental stress analysis such as photoelasticity can lead to full-field information that relates stress distribution in a specimen. As well as the difference of principal stresses obtained by isochromatics from a circular polariscope, the sum of principal stresses can be obtained by isopachics from a holographic photoelasticity and is helpful to the separation work of stress components. Since isochromatics and isopachics couple each other, subsequent data processing and interpretation have been so difficult to proceed further. With the development of digital photoelasticity combined with digital imaging techniques[1,2], it gets possiblility to automatically determine full-field fringe order of isochromatics, the difference of principal stresses and isoclinics data can be obtained with the methods such as phase-shifting[3−5], color-field phase-shifting[6,7], load-stepping[8−10] and so on. Then a conventional shear difference method or analytic separation method has been used for the stress separation in photoelasticity[11]. If the sum of principal stresses can be got from experiments, the stress separation can be finished wholly based on photoelastic experiments. Yoneyama *et al.* used a Mach-Zehnder type interferometer combined with a circular polariscope for stress separation in two-dimensional (2D) interferometric photoelasticity, where the isopachic phase was analyzed from images obtained by shifting a mirror in the interferometer[12].

In this letter, a digital phase-stepping interferometric photoelastic method is proposed for full-field phase map of isopachics. Four isochromatic images coupled with isopachics can be obtained from an interferometric polariscope through rotating analyzer and quarter-wave plate at definite optical arrangements. Light intensities and phase-stepping formula are derived to obtain full-field phase map of isopachics using Jones calculus. Simulations of a circular disk under diametral compression demonstrate the feasibility of the proposed method.

An interferometric photoelastic optical setup (Fig. 1), which consists of a circular polariscope and an interferometer light, is used for the analysis of isopachics by a phase-stepping method. A laser light passed through a polarizer ($P_{\pi/2}$) whose principal axis subtended an angle $\pi/2$ with a reference axis. Then the linearly polarized light was divided into two beams (paths I and II) by a beam splitter (BS1). The beam which passed through the first quarter-wave plate ($Q_{\pi/4}$), a birefringent specimen ($R_{\delta,\theta}$), the second quarter-wave plate (Q_γ), and an analyzer (P_β) on path I interfered with the beam which passed through path II at another BS (BS2). Consequently, the isochromatic and isopachic fringes were observed simultaneously by a charge-coupled device (CCD) camera.

The Jones matrix for a birefringent plate (specimen) $R_{\delta,\theta}$ whose fast axis subtends an isoclinic angle θ with the reference axis is expressed as

$$\mathbf{R}_{\delta,\theta} = \begin{bmatrix} e^{i\delta_1}\cos^2\theta + e^{i\delta_2}\sin^2\theta & (e^{i\delta_1} - e^{i\delta_2})\cos\theta\sin\theta \\ (e^{i\delta_1} - e^{i\delta_2})\cos\theta\sin\theta & e^{i\delta_2}\cos^2\theta + e^{i\delta_1}\sin^2\theta \end{bmatrix}, \tag{1}$$

where δ_1 and δ_2 are the phase retardations along the fast and slow principal directions of the specimen. Those retardations relate the difference and the sum of the principal stresses as

$$\delta_d = \delta_1 - \delta_2 = 2\pi t(\sigma_1 - \sigma_2)/f_d, \tag{2}$$

$$\delta_s = \delta_1 + \delta_2 = 2\pi t(\sigma_1 + \sigma_2)/f_s, \tag{3}$$

where f_d and f_s are the photoelastic material fringe constants respectively corresponding to isochromatics and isopachics, and t represents the thickness of the specimen.

The normalized Jones vector $\mathbf{P}_{\pi/2}$ for the linearly polarized light is expressed as

$$\mathbf{P}_{\pi/2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{4}$$

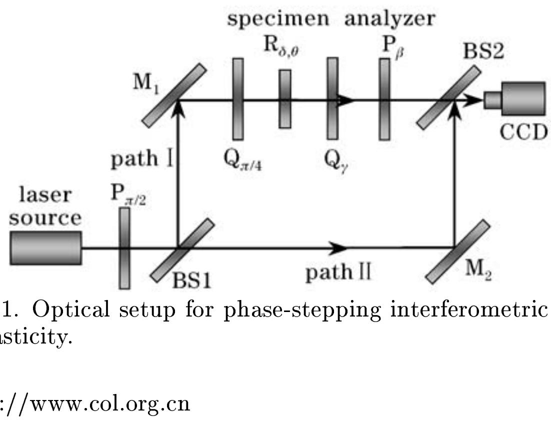

Fig. 1. Optical setup for phase-stepping interferometric photoelasticity.

The Jones matrix $\mathbf{P}_\beta$ for analyzer whose axis subtends an arbitrary angle β is

$$\mathbf{P}_\beta = \begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix}. \quad (5)$$

The Jones matrix $\mathbf{Q}_\gamma$ for the quarter-wave plate whose fast axis subtends an arbitrary angle γ is

$$\mathbf{Q}_\gamma = \begin{bmatrix} 1+i\cos2\gamma & i\sin2\gamma \\ i\sin2\gamma & 1-i\cos2\gamma \end{bmatrix}. \quad (6)$$

When γ is $\pi/4$, the upper matrix becomes

$$\mathbf{Q}_{\pi/4} = \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}. \quad (7)$$

According to the Jones calculus, the Jones vector $\mathbf{E}_1$ of the light emerging from the analyzer is

$$\mathbf{E}_1 = \mathbf{P}_\beta \mathbf{Q}_\gamma \mathbf{R}_{\delta,\theta} \mathbf{Q}_{\pi/4} \mathbf{P}_{\pi/2}. \quad (8)$$

The light emerging from the analyzer (path I) interferes with the light passing through path II at BS2 when the shutter on path II is opened. The Jones vector $\mathbf{E}_{\text{total}}$ and the intensity I of the light emerging from BS2 are

$$\mathbf{E}_{\text{total}} = \mathbf{E}_1 + \mathbf{P}_{\pi/2}, \quad (9)$$

$$I = E_{\text{total}} E^*_{\text{total}}, \quad (10)$$

E^*_{total} is the complex conjugate of $\mathbf{E}_{\text{total}}$. The final intensity I is written as

$$\begin{aligned} I = {} & \frac{1}{2}\left[\cos\left(\frac{\delta_{\rm d}+\delta_{\rm s}-2\beta}{2}\right)+\cos\left(\frac{\delta_{\rm d}-\delta_{\rm s}+2\beta}{2}\right)\right] \\ & +\frac{1}{2}\left[\cos\left(\frac{\delta_{\rm d}-\delta_{\rm s}-2\beta+4\theta}{2}\right) - \cos\left(\frac{\delta_{\rm d}+\delta_{\rm s}+2\beta-4\theta}{2}\right)\right] \\ & +\frac{1}{2}\left[3+\sin\left(\frac{\delta_{\rm d}+\delta_{\rm s}+2\beta-4\gamma}{2}\right) - \sin\left(\frac{\delta_{\rm d}-\delta_{\rm s}-2\beta+4\gamma}{2}\right)\right] \\ & -\frac{1}{2}\left[\sin\left(\frac{\delta_{\rm d}+\delta_{\rm s}-2\beta+4\gamma-4\theta}{2}\right) + \sin\left(\frac{\delta_{\rm d}-\delta_{\rm s}+2\beta-4\gamma+4\theta}{2}\right)\right] \\ & +\frac{1}{4}\left[\sin\left(\delta_{\rm d}+2\beta-2\gamma\right)-\sin\left(\delta_{\rm d}-2\beta+2\gamma\right)\right] \\ & +\frac{1}{8}\left[\cos\left(\delta_{\rm d}-2\beta+2\theta\right)-\cos\left(\delta_{\rm d}+2\beta-2\theta\right)\right] \\ & +\frac{1}{8}\left[\cos\left(\delta_{\rm d}+2\beta-4\gamma+2\theta\right)-\cos\left(\delta_{\rm d}-2\beta+4\gamma-2\theta\right)\right]. \end{aligned} \quad (11)$$

Table 1. Light Intensities for Analyzing Isopachics

γ	β	I
0	$\pi/4$	$\{3+\cos\delta_{\rm d}+2\sqrt{2}\cos(\delta_{\rm d}/2)[\cos(\delta_{\rm s}/2)+\sin(\delta_{\rm s}/2)]\}/2$
$\pi/4$	$\pi/2$	$[3+\cos\delta_{\rm d}+4\cos(\delta_{\rm d}/2)\sin(\delta_{\rm s}/2)]/2$
$\pi/2$	$3\pi/4$	$\{3+\cos\delta_{\rm d}+2\sqrt{2}\cos(\delta_{\rm d}/2)[-\cos(\delta_{\rm s}/2)+\sin(\delta_{\rm s}/2)]\}/2$
$3\pi/4$	0	$[3+\cos\delta_{\rm d}+4\cos(\delta_{\rm d}/2)\cos(\delta_{\rm s}/2)]/2$

Four light intensities listed in Table 1 are derived from Eq. (11) when the analyzer and Q_γ are rotating at definite arrangements in interferometric photoelastic setup. Then the phase value of the isopachics parameter can be calculated by the following phase-stepping equation,

$$\tan(\delta_{\rm s}/2) = \frac{\sqrt{2}(I_2-I_4)+I_1-I_3}{I_1-I_3} \quad \text{for} \quad \cos(\delta_{\rm d}/2)\neq 0, \quad (12)$$

where the phase value can be determined for the range of $[-\pi,\pi]$. At last, using unwrapping technique, the full-field phase map of isopachics can be obtained. If $\cos(\delta_{\rm d}/2)=0$, numerator and denominator in Eq. (12) are all equal to 0. This is to say that there are not any usable isopachic data at the centers of isochromatic fringes.

It is noted that there exist initial interferometric fringes independent of the load. The initial phase needs to be subtracted from the above isopachic phase. In order to overcome the difficulty, a two-load stepping method is adopted[10]. The phase values of the isopachics are determined for the two different loads of p_1 and p_2 ($p_1 < p_2$). Here, the difference of the phase values is smaller than π. The correct values of the isopachics, $\delta_{\rm sr}$, can be determined by

$$\delta_{\rm sr} = \begin{cases} \frac{p_1}{p_2-p_1}(\delta_{\rm s2}-\delta_{\rm s1}) & \text{if} \quad \delta_{\rm s2}\geq\delta_{\rm s1} \\ \frac{p_1}{p_2-p_1}(\delta_{\rm s2}-\delta_{\rm s1}+2\pi) & \text{if} \quad \delta_{\rm s2}<\delta_{\rm s1} \end{cases}, \quad (13)$$

where $\delta_{\rm s1}$ and $\delta_{\rm s2}$ represent the phase values of the isopachics for the loads p_1 and p_2, respectively. Further phase-unwrapping is not needed.

A circular polycarbonate disk under diametral compression (disk diameter and thickness are $d = 50$ mm and $t = 4.8$ mm, diametric forces are $p_1 = 313.6$ N and $p_2 = 318.6$ N) is simulated to verify the effectiveness of the proposed method. The material fringe constants of isochromatics and isopachics are $f_{\rm d} = 7.6222$ N/(mm·fringe) and $f_{\rm s} = -1.8275$ N/(mm·fringe). The

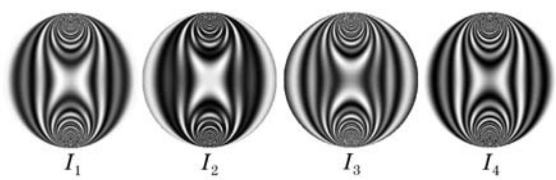

Fig. 2. Four fringes corresponding to Table 1 show isochromatics coupled with isopachics.

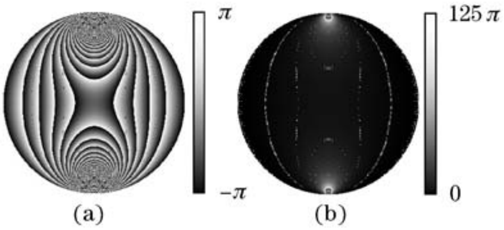

Fig. 3. Wrapped (a) and unwrapped (b) phase maps of isopachics after two-load stepping.

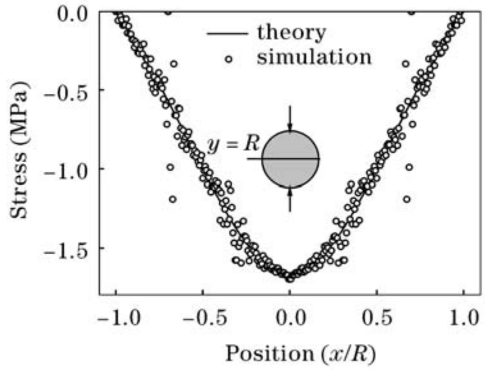

Fig. 4. Sum of principal stresses along a horizontal line of the disk specimen.

simulated image size is 300 × 300 pixels. Isochromatic and isopachic fringes appear simultaneously in four images for Table 1 under the load of 313.6 N, as shown in Fig. 2. The full-field wrapped and unwrapped phase maps (in Fig. 3, 313.6 N) demonstrate the perfect results except for where the stress concentrations are located. Figure 4 shows the results at the disk central horizontal line under the load of 313.6 N, which agree well with the theoretical values. The results under the load of 318.6 N are similar.

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