# Symmetrical periods used as matching layers in multilayer thin film design

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Received June 13, 2006

Properties of symmetrical layers as matching layers in multilayer thin film design were analyzed. A calculation method was presented to derive parameters of desired equivalent refractive index. A harmonic beam splitter was designed and fabricated to test this matching method.

*OCIS codes:* 230.1360, 220.0220, 310.6860.

One of the bigger advantages of symmetrical layer is that it could offer an expected equivalent index and equivalent phase thickness, which can be unrealized in a single material. Symmetrical layers have shown their powerful strength especially in multilayer thin film design, since it could reduce the complexity of computing and analyzing. Thus, optical filters, such as edge filters, band-pass filters, harmonic beam splitter etc., are commonly designed by means of equivalent layers. Since Epstein published the theory of equivalent index in 1952[1], it has been widely adopted to design coatings with symmetrical layers[2,3]. Some graphics were also put forward to visualize the analysis of symmetrical layers[4,5]. However, systematic studies on equivalent refractive indices as functions of parameters of symmetrical layers, to our knowledge, were absent in the literature as yet. In this paper, we analyze the relation between equivalent refractive indices and constructive parameters of symmetrical layers. These results are further applied to design a harmonic beam splitter based on symmetrical layers. Experiment of the coating is carried out by ion beam sputtering (IBS), optical characteristic of which shows good agreement with theoretical design.

Figure 1 shows a symmetrical three-layer coating ABA, where A and B represent non-absorbing and homogeneous films with refractive indices being respective $n_1$ and $n_2$, and optical thicknesses being respective $a$ and $2b$

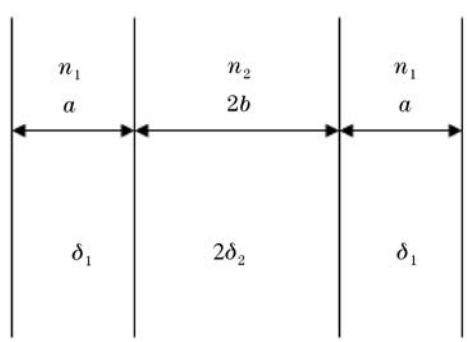


Fig. 1. Symmetrical three-layer coating ABA, where A and B represent non-absorbing and homogeneous films with refractive indices being respectively $n_1$ and $n_2$, and optical thicknesses being respectively $a$ and $2b$ times of $\lambda_0/4$ ($\lambda_0$ is reference wavelength).

times of $\lambda_0/4$ ($\lambda_0$ is reference wavelength). The equivalent refractive index and equivalent phase thickness of this three-layer combination are respectively presented as[6]

$$E = \sqrt{\frac{M_{21}}{M_{12}}} = n_1\sqrt{\frac{\sin(\pi ag)\cos(\pi bg) + p\cos(\pi ag)\sin(\pi bg) - q\sin(\pi bg)}{\sin(\pi ag)\cos(\pi bg) + p\cos(\pi ag)\sin(\pi bg) + q\sin(\pi bg)}}, \quad (1)$$

$$\Gamma = \cos^{-1} M_{11} = \cos^{-1}(\cos(\pi ag)\cos(\pi bg) - p\sin(\pi ag)\sin(\pi bg)), \quad (2)$$

where $p = \frac{1}{2}\left(\frac{n_1}{n_2} + \frac{n_2}{n_1}\right)$, $q = \frac{1}{2}\left(\frac{n_1}{n_2} - \frac{n_2}{n_1}\right)$, and $g = \frac{\lambda_0}{\lambda}$.

From Eqs. (1) and (2), it is seen that both equivalent refractive indices and equivalent phase thicknesses are actually functions of $a$, $b$, $n_1$, $n_2$, and g. If this three-layer combination is supposed to behave as a matching layer, it has to meet[2]

$$N_{\mathrm{m}} = (N_{E1}N_{E2})^{1/2}, \quad (3)$$

$$\Gamma_{\mathrm{m}} = (2\nu + 1)\cdot 90^\circ, \quad \nu = 0, 1, 2, \cdots, \quad (4)$$

where $N_{\mathrm{m}}$ is the expected equivalent refractive index of a matching layer sandwiched between two layers with different refractive indices being respective $N_{E1}$ and $N_{E2}$. From Eq. (4), it can be seen that $\Gamma_{\mathrm{m}} = 90^\circ$, $270^\circ$, $450^\circ$, $\cdots$. If we set $a + b = 1$, which means, those $N_E$ corresponding to $g = 0.5$, 1.5, 2.5, $\cdots$ could be used as matching refractive indices in symmetrical layers.

Equivalent refractive index as a function of $g$ is analyzed for different values of $n_1$, $n_2$ or $a$, $b$. Figure 2 shows the values of equivalent refractive indices of a matching layer 0.5LH0.5L when $n_1$ is 1.46 and $n_2$ is 2.4, 2.2, 2, 1.8, 1.6 respectively. Moreover, Table 1 shows the values of equivalent indices when $g = 0.5$, 1.5, 2.5. Figure 3 shows the values of equivalent refractive indices of 0.3L1.4H0.3L, 0.4L1.2H0.4L, 0.5LH0.5L, 0.6L0.8H0.6L, and 0.7L0.6H0.7L respectively wherein L represents material whose refractive index is $n_1 = 1.46$, while $n_2 = 2.2$.

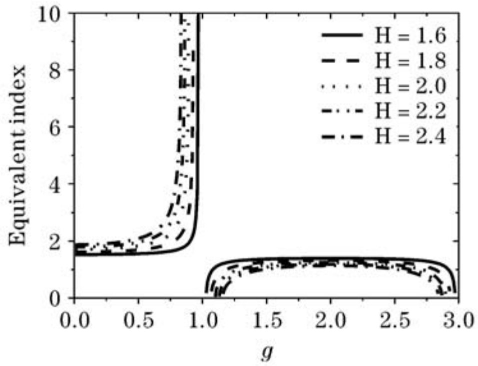


Fig. 2. Curves of equivalent refractive indices of (0.5LH0.5L) when the refractive index $n_1 = 1.46$, while the refractive indices $n_2 = 2.4, 2.2, 2, 1.8, 1.6$.

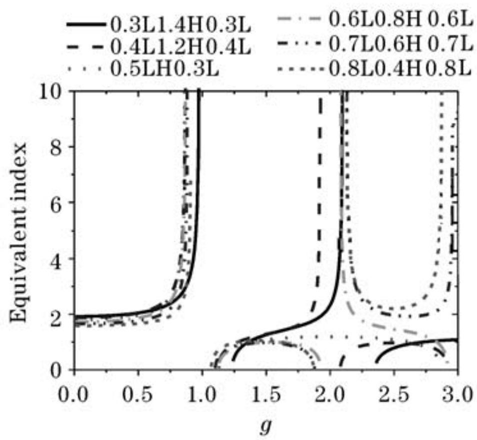


Fig. 3. Curves of equivalent refractive indices of 0.3L1.4H0.3L, 0.4L1.2H0.4L, 0.5LH0.5L, 0.6L0.8H0.6L, 0.7L0.6H0.7L, and 0.8L0.4H0.8L respectively wherein $n_1 = 1.46$ and $n_2 = 2.2$.

**Table 1. Equivalent Refractive Indices of (0.5LH0.5L) as $n_2$ and $g$ are variable**

| $\alpha = n_2/n_1$ | $g = 0.5$ | $g = 1.5$ | $g = 2.5$ |
|---|---|---|---|
| 1.096 = 1.6/1.46 | 1.5577 | 1.3684 | 1.3684 |
| 1.233 = 1.8/1.46 | 1.6939 | 1.2584 | 1.2584 |
| 1.37 = 2.0/1.46 | 1.8273 | 1.1665 | 1.1665 |
| 1.507 = 2.2/1.46 | 1.9593 | 1.0880 | 1.0880 |
| 1.644 = 2.4/1.46 | 2.0907 | 1.0196 | 1.0196 |

**Table 2. Equivalent Refractive Indices of (0.5LH0.5L) as $a$ and $g$ are Variable**

| aL2bHaL | $g = 0.5$ | $g = 1.5$ | $g = 2.5$ |
|---|---|---|---|
| 0.3L1.4H0.3L | 2.0458 | 1.2592 | 0.7973 |
| 0.4L1.2H0.4L | 2.0411 | 1.2821 | 0.9689 |
| 0.5L1.0H0.5L | 1.9593 | 1.0880 | 1.0880 |
| 0.6L0.8H0.6L | 1.8663 | 0.9875 | 1.46 |
| 0.7L0.6H0.7L | 1.7662 | 0.9735 | 1.9593 |
| 0.8L0.4H0.8L | 1.6626 | 1.0443 | 2.2 |

Relevant equivalent refractive indices when $g = 0.5$, 1.5, 2.5 are given in Table 2.

From discussions above, we can obtain refractive indices of certain ranges by changing parameters of symmetrical stack. Therefore, it can be used as matching layers to obtain the expected refractive index and phase thickness.

In following sections, we will apply the theory of equivalent refractive index to design a harmonic beam splitter. The harmonic beam splitter is one of optical components in diode pumped Nd:$YVO_4$ frequency doubling lasers[7] which provides high reflectance at the fundamental wavelength ($\lambda_0 = 1064$ nm) and high transmittance at the doubled one ($\lambda_{\rm m} = 532$ nm)[7]. Usually, half-wave hole and high ripples induced by thickness error in the fabrication process cannot be avoided in the transmittance curve. Though the half-wave hole could be shifted away from the desired wavelength by changing the control wavelength, it cannot be eliminated completely. Moreover, there are still high ripple in the pass-band around 532 nm. An admittance-matching method and half-wavelength control were employed to resolve these difficulties fundamentally.

The basic stack is selected as (L2HL) with the reference wavelength of 532 nm. The high refractive index material is $Ta_2O_5$ ($n_2 = 2.108$@532 nm) and low refractive index material is $SiO_2$ ($n_1 = 1.477$@532 nm), the refractive indices of incident medium and substrate are $n_0 = 1$ and $n_g = 1.52$, respectively. The dispersion curves of $Ta_2O_5$ and $SiO_2$ are shown in Fig. 4.

The equivalent refractive index of stack (L2HL) at 532 nm is $E = 1.2363$. $n = 14$ is chosen as period number for basic stack. Expected matching refractive index between (L2HL) and the incident medium is $E_0 = \sqrt{E \cdot n_0} = 1.1119$; similarly, expected matching refractive index between (L2HL) and substrate is $E_g = \sqrt{E \cdot n_g} = 1.3708$. From the relation between equivalent refractive index $E$, $a$ and $g$ (see in Fig. 5), we can pick out the satisfactory structural values of matching

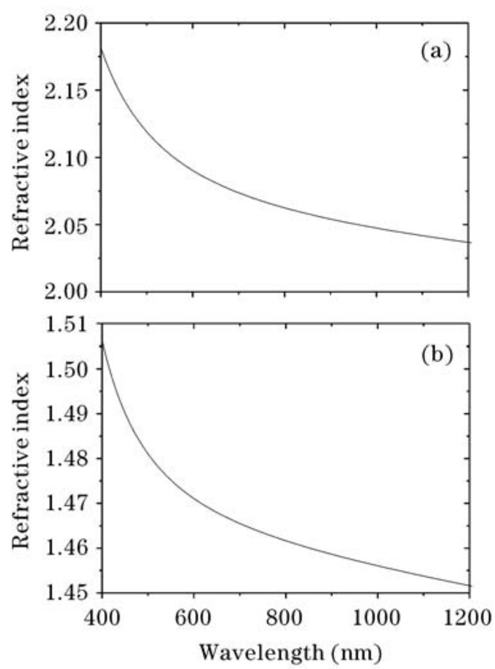


Fig. 4. Dispersion curves of $Ta_2O_5$ (a) and $SiO_2$ (b).

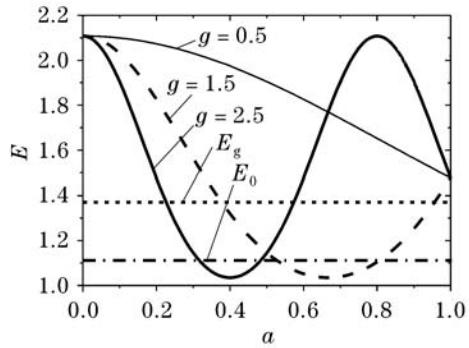


Fig. 5. Relation between equivalent refractive index $E$ and $a$, $g$.

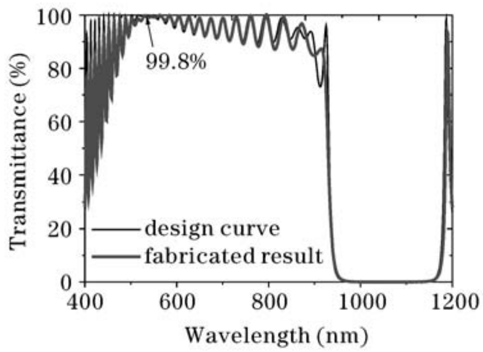


Fig. 6. Performance of harmonic beam splitter.

layer. For equivalent refractive index $E_0$, if we set $g = 1.5$, $a$ can be selected as 0.38 or 0.81; if we set $g = 2.5$, $a$ can be set as 0.32 or 0.485. For equivalent refractive index $E_g$, $a$ can be selected as 0.53 or 0.96 when $g$ is 1.5, and 0.225 or 0.575 when $g$ is 2.5. Take $g = 2.5$, $a = 0.575$ for example, thus $b = 1 - a = 0.425$ for $E_0$. Similarly, choose $g = 2.5$, $a = 0.485$, so $b = 0.515$ for $E_g$. Thus, a final stack 2.5(0.575L0.425H0.575L) $(\mathrm{L2HL})^{14}$ 2.5(0.485L0.515H0.485L) is obtained. Figure 6 shows the optical performance of the design. The transmittance at 532 nm is 99.999%. Average transmittance from 520 to 550 nm is 99.86% and the lowest is about 99.75% in this region.

This splitter was prepared by IBS. Detailed description about IBS was presented in Ref. [7]. The thickness of each layer was controlled by time. The sputtering rates for $Ta_2O_5$ and $SiO_2$ were 0.28 and 0.23 $\mathrm{nm{\cdot}s^{-1}}$. The substrate was BK7 with refractive index of 1.52. The optical property of the sample was characterized with a Lambda 900 spectrophotometer, as shown in Fig. 6. The transmittance is 99.8% at 532 nm.

In conclusion, relations between equivalent refractive indices and parameters of symmetrical layers were analyzed. These analytic results based on symmetrical layers were then used to design a harmonic beam splitter which shows excellent optical characteristic. The designed harmonic beam splitter was prepared by IBS and showed excellent agreement with theoretical design.

The authors thank Dr. Shijie Liu, Degang Deng, Lei Yuan for their kindhearted opinions. D. Wang's e-mail address is sunnywd@mail.siom.ac.cn.

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