Definition and measurement of the beam propagation factor M^2 for chromatic laser beams

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The concept of the beam propagation factor M^2 is extended for chromatic laser beams. The definition of the beam propagation factor can be generalized with the weighted effective wavelength. Using the new definition of factor M^2, the propagation of chromatic beams can be analyzed by the beam propagation factor M^2 as same as that of monochromatic beams. A simple method to measure the chromatic beam factor M^2 is demonstrated. The chromatic factor M^2 is found invariable while the laser beam propagates through the dispersion-free $ABCD$ system.

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The beam propagation factor M^2 is one of the most important parameters for laser technology and its applications[1−3]. The most attractive feature of the factor M^2 is that the propagation of general optical beams can be traced with the ray matrix ($ABCD$) formalism when the second-moment definition of beam width is used[4]. The common definition of the factor M^2 is only valid for monochromatic laser beams[5,6]. In practice, broadband or multi-wavelength laser beams are widely applied[7−9]. Furthermore, many commercial M^2-factor meters do not constrain their use to monochromatic lasers. The definition of the factor M^2 has been extended to chromatic beams[10], however, it is controversial whether the chromatic beam factor M^2 obeys the $ABCD$ formalism[11]. No experiment has been reported to measure the chromatic beam factor M^2. In this letter, the factor M^2 for chromatic beams is given with more practical definition. A simple method is demonstrated to measure the chromatic beam propagation factor M^2. It is shown that the effective beam widths of chromatic beams obey the same propagation rule as that of monochromatic beams. In addition, the factor M^2 remains invariable while the chromatic beam propagates through $ABCD$ systems.

Consider a chromatic beam consisting of a continuous spectrum over a finite range

$$E_\lambda = \int e_\lambda u_\lambda \mathrm{d}\lambda, \tag{1}$$

where u_λ is the transverse amplitude distribution at the wavelength λ; e_λ is the normalized coefficient with $\int |e_\lambda|^2 \mathrm{d}\lambda = 1$. The propagation of each spectral component from (x_1, z_1) to (x_2, z_2) can be described by the Huygen's integral

$$u_\lambda(x_2, z_2) = \left(\frac{j}{\lambda}\right)^{1/2} \int u_\lambda(x_1, z_1) \times \exp\left[-\frac{j\pi}{\lambda z}(x_1^2 - 2x_1x_2 + x_2^2)\right] \mathrm{d}x_1. \tag{2}$$

For the sake of simplicity, the calculation is limited to one transverse dimension. The general second moment of the beam profile or intensity variance should be integral over all spectral range

$$\sigma^2(z) = \int |e_\lambda|^2 \mathrm{d}\lambda \int x^2 |u_\lambda(x, z)|^2 \mathrm{d}x, \tag{3}$$

and the beam width is then given by $d(z) = \sqrt{4\sigma^2(z)}$. Substituting Eq. (2) into Eq. (3), and using the characteristics of the Fourier integral, one can obtain the conclusion after some algebraic manipulations[4]

$$\sigma^2(z) = \sigma^2(0) \times \left[1 + \left(\frac{z - z_0}{z_\mathrm{R}}\right)^2\right]. \tag{4}$$

The minimum intensity variance $\sigma^2(0)$ locates at the beam waist z_0

$$\sigma^2(0) = \sigma_\lambda^2(z_1) - \left(\int T_\lambda(z_1)\lambda |e_\lambda|^2 \mathrm{d}\lambda\right)^2 / 4 \int \Theta_\lambda^2(z_1)\lambda^2 |e_\lambda|^2 \mathrm{d}\lambda, \tag{5}$$

and the effective Raylaigh range is

$$z_\mathrm{R} = \frac{4\pi\sigma^2(0)}{\left(16\pi^2\sigma^2(0) \int \Theta_\lambda^2(0)\lambda^2 e_\lambda^2 \mathrm{d}\lambda\right)^{1/2}}, \tag{6}$$

$$\Theta_\lambda^2(z_1) = \frac{1}{4\pi^2} \int \left|\frac{\partial u_1}{\partial x_1}\right|^2 \mathrm{d}x_1, \tag{7}$$

$$T_\lambda(z_1) = \frac{1}{2\pi} j \int \left(\frac{\partial u_1}{\partial x_1} u_1^* - \frac{\partial u_1^*}{\partial x_1} u_1\right) \mathrm{d}x_1. \tag{8}$$

As shown in Eq. (5), the chromatic beam width undergoes a parabolic evolution when the beam passes through its beam waist. Therefore, the propagation of chromatic beam can be characterized by the effective Rayleigh range, which, in turn, links to the beam propagation factor M^2. Compared with the standard expression of the Rayleigh range for monochromatic beams, the denominator in Eq. (6) is M^2-wavelength product, which is related to the beam waist-divergence product in the form

$$(M_\lambda^2 \lambda_{\text{eff}})^2 = \pi^2 d^2 \theta^2, \quad (9)$$

$$\theta^2 = 4 \int \Theta_\lambda^2(0) \lambda^2 |e_\lambda|^2 \, \mathrm{d}\lambda. \quad (10)$$

The propagation factor M_λ^2 for the chromatic beam is then solved as

$$M_\lambda^2 = \frac{\pi}{\lambda_{\text{eff}}} \mathrm{d}\theta. \quad (11)$$

To determine the effective wavelength, we construct the diffraction limited chromatic beam, which has the same spectrum as the measured beam. Assuming $M_\lambda^2 = 1.0$ for this diffraction limited beam, we can obtain the effective wavelength

$$\lambda_{\text{eff}} = \int |e_\lambda|^2 \lambda \mathrm{d}\lambda. \quad (12)$$

When the beam passes through any $ABCD$ system, the effective wavelength is invariant, and the transformation of the field amplitude is given by the Collins equation[12]

$$u_\lambda(x, z_2) = \sqrt{\frac{i}{\lambda B}} \int u_\lambda(x_0, z_1) \times \exp\left[-i\frac{k}{2B}(Ax_0^2 - 2xx_0 + Dx^2)\right] \mathrm{d}x_0. \quad (13)$$

If $ABCD$ matrix is independent of the beam wavelength, we obtain the similar result as given by Eqs. (4)—(11) with the same calculation as for Eq. (2). The chromatic beam factor remains constant after the beam passes through $ABCD$ system.

Experimental measurement of the chromatic beam factor M_λ^2 for a super wideband light is shown in Fig. 1. The light source was an incandescent bulb. The light was collected and collimated by a non-sphere optical system. A spatial filter was used to improve the beam quality of the launched beam from the light source. We could choose the beam quality by adjusting the pin-hole inside the spatial filter. The dispersion of the spatial filter was compensated carefully. A fiber-coupled spectrograph was used as the detector. The accepted area of the detector was about 0.1 mm^2. Although the spectra of the light source covered from 390 to 1500 nm, the detector could respond only from 520.3 to 1184.35 nm. In this case, the measured result was not truly the beam factor of the light source, but the one of the light in the spectra range from 520.3 to 1184.35 nm as shown in Fig. 2. The responsive curve of the spectrograph was calibrated to be independent of wavelength. According to Eq. (12), the effective wavelength was obtained to be 781.56 nm.

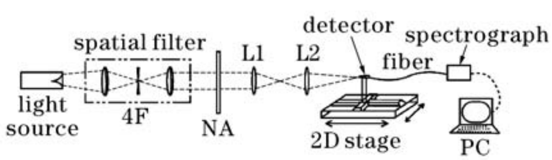

Fig. 1. Experimental setup. L1 and L2: achromatic convex lenses; NA: natural attenuator.

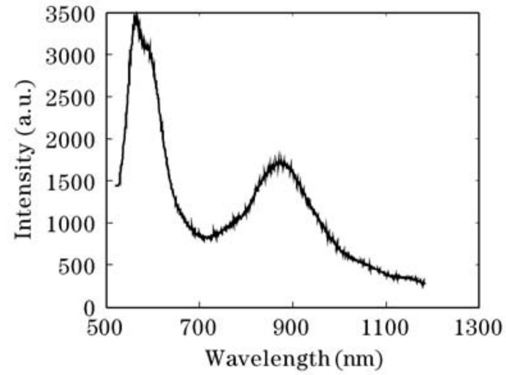

Fig. 2. Measured spectrum of the light source.

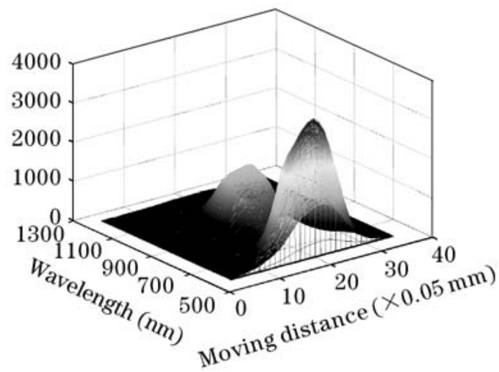

Fig. 3. Recorded spectra at various positions across the beam cross section near the beam focus.

The light was focused by an achromatic convex lens (L1), and the measurement was performed by moving the detector across the beam cross section. The precisions of the moving stage are 0.005 and 0.5 mm in the x- and z-axis directions, respectively. The typical recorded spectra for moving the detector through the beam cross section are given in Fig. 3. The beam width for each spectral component can be determined by the similar procedure as doing for monochromatic beams[13,14]. The chromatic beam width was then achieved by weighted summing these beam widths as indicated by Eq. (3).

The chromatic beam widths were measured at different positions around the beam focus. As shown in Fig. 4, the chromatic beam displays the similar propagation

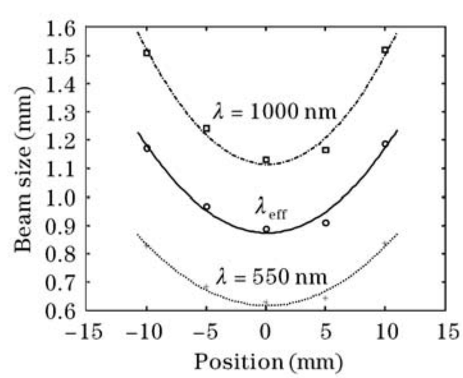

Fig. 4. Beam width for various spectral components as a function of the distance.

behavior as the monochromatic beams. Although the effective wavelength λ_{eff} is 781 nm, it is should noted that the beam width for the wavelength of 781 nm is different from that of the chromatic beam. We fitted the measured data with the parabolic function. The beam waist of 0.870 mm and the divergence angle of 23.8 mrad were obtained. The beam factor was calculated with $M_{\lambda}^2 = 41$. In comparison, the beam factors for 550 and 1000 nm wavelengths were 38 and 40, respectively. Actually, the beam factor for every spectral component was less than 41. The spectral combination mostly leads to a worse beam quality. Another achromatic convex lens (L2) was used to test the beam factor through $ABCD$ system. Similar measurements were performed near the focus of lens L2. The beam factor was almost unchanged. About 1% deviation may be results from higher order dispersion of lens L2.

In conclusion, in the description of laser beam propagation, the beam propagation factor M^2 is a valuable parameter not only for monochromatic beams, but also for chromatic beams. The chromatic beam with the factor M^2 spreads out M^2 times faster than that of the diffraction-limited beam, which has the same spectrum as the measured chromatic beam. The chromatic beam factor of a white light has been measured. It is found that the chromatic beam factor remains constant through the $ABCD$ systems if the dispersion can be ignored.

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