Slant path average intensity of finite optical beam propagating in turbulent atmosphere

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The average intensity of finite laser beam propagating through turbulent atmosphere is calculated from the extended Huygens Fresnel principle. Formulas are presented for the slant path average intensity from an arbitrarily truncated Gaussian beam. The new expressions are derived from the modified von Karman spectrum for refractive-index fluctuations, quadratic approximation of the structure function, and Gaussian approximation for the product of Gaussian function and Bessel function. It is shown that the form of average intensity is not a Gaussian function but a polynomial of the power of the binomial function, Gaussian function, and the incomplete gamma function. The results also show that the mean irradiance of a finite optical beam propagating in slant path turbulent atmosphere not only depends on the effective beam radius at the transmitting aperture plane, propagation distance, and long-term lateral coherence length of spherical wave, but also on the radius of emit aperture.

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Wireless laser communication between the ground and satellite has advantages in terms of bandwidth, reduced interference, and having small and lightweight equipment compared with the conventional communication systems. Therefore wireless optical telecommunication systems have become one of the most widely used alternatives among broadband access applications. However, during light waves propagating through atmospheric turbulence, random fluctuations in atmospheric refractive index introduce random phase perturbations across the propagating wave front. After propagating some distances from the point of introduction, these phase perturbations develop into random intensity fluctuations, beam broadening, and beam wandering. Since beam spread and its associated on-axis irradiance degradations are of practical interest, many analyses have been reported[1−9]. Andrews *et al.*[1,2] studied the beam spread of zero-order Gaussian beams in weak regimes of atmospheric fluctuations. Yura[4] investigated the short-term average optical-beam spread in atmospheric turbulence based on the extended Huygens-Fresnel principle and the short-term average mutual coherence function of a spherical wave. Tavis *et al.*[9] gave the short-term average irradiance profile of a focused laser beam transmitting through a homogeneous-isotropic atmosphere. However, the closed-form solution for the mean irradiance of Gaussian beams propagating in all regimes of atmospheric turbulence has not been obtained[2,7]. The previous analysis of the ensemble-averaged irradiance was based on the assume that the mean irradiance sketch of a light propagation in turbulence is Gaussian function[5,7,10].

This paper develops the expressions for the ensemble-averaged irradiance based on the extended Huygens-Fresnel principle, quadratic approximation of the structure function, the modified von Karman spectrum for refractive-index fluctuations, and Gaussian approximation for the product of Gaussian function and Bessel function.

It is shown that for optical propagation in an inhomogeneous, no absorbing medium, sufficiently small scattering angle, the field at an observation point $\mathbf{P}_1$ due to a Gaussian amplitude aperture disturbance $U_0(\mathbf{r}_1)$ can be written as[7]

$$U(\mathbf{P}_1) = \frac{-\mathrm{i}k}{2\pi} \int_{\Sigma} G(\mathbf{P}_1, \mathbf{r}_1)\, U_0(\mathbf{r}_1) \mathrm{d}^2\mathbf{r}_1, \qquad (1)$$

where $k = 2\pi/\lambda$ is the wavenumber, λ is the wavelength, $\mathbf{r}_1$ is the coordinate vector in the aperture plane, $\sum$ is the area of the circular aperture (see Fig. 1(a)). The

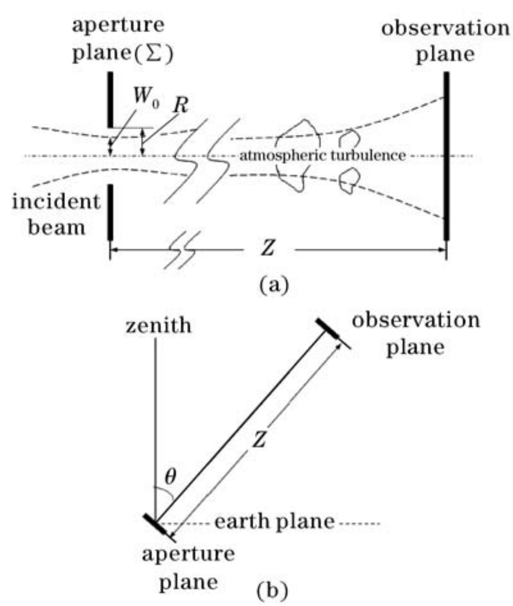

Fig. 1. (a) Gaussian beam propagation in turbulent atmosphere; (b) geometry of slant path propagation. W_0 is the effective beam radius in transmitting aperture plane (or beam waist), R is the radius of emit aperture, Z is the propagation distance, and θ is the zenith angle.

amplitude aperture disturbance $U_0(\mathbf{r}_1)$ is given by

$$U_0(\mathbf{r}_1) = U_0 \exp\left[-\frac{r_1^2}{2W_0^2} - \frac{\mathrm{i}kr_1^2}{2F_0}\right], \quad |\mathbf{r}_1| \le R, \tag{2}$$

where F_0 is the radius of curvature, W_0 is the effective beam radius in transmitting aperture plane, U_0 is the constant amplitude of beam, and $2R$ is the diameter of the circular aperture.

The term $G(\mathbf{P}_1, \mathbf{r}_1)$ is the field at $\mathbf{P}_1$ of a spherical wave propagating due to the atmospheric turbulence from a unite point source at $\mathbf{r}_1$, i.e.,

$$G(\mathbf{P}_1, r_1) = \exp\left(\mathrm{i}k\,|\mathbf{P}_1 - \mathbf{r}_1| + \psi\right) / \,|\mathbf{P}_1 - \mathbf{r}_1|\,, \tag{3}$$

where $\psi(\mathbf{p}_1, \mathbf{r}_1, z) = \chi(\mathbf{p}_1, \mathbf{r}_1, z) + \mathrm{i}S(\mathbf{p}_1, \mathbf{r}_1, z)$[11] is the random part of the complex phase of a spherical wave propagating in the turbulent atmosphere from the point $(\mathbf{r}_1)$ to the point $(\mathbf{p}_1, z)$. And we define $\mathbf{p}_1$ as the normal from the z axis of symmetry to the observation point $\mathbf{P}_1$. By Eqs. (1), (2), and the paraxial approximation, we obtain[7]

$$U(\mathbf{p}_1, z) = \frac{-\mathrm{i}k}{2\pi z}\exp(\mathrm{i}kz) \times \int_{\Sigma} \exp\left[\frac{\mathrm{i}k(\mathbf{p}_1 - \mathbf{r}_1)^2}{2z} + \psi(\mathbf{p}_1, \mathbf{r}_1, z)\right] U_0(\mathbf{r}_1)\,\mathrm{d}^2\mathbf{r}_1, \tag{4}$$

and

$$U(\mathbf{p}_1, z)U^*(\mathbf{p}_2, z) = \left(\frac{k}{2\pi z}\right)^2 \iint_{\Sigma} \exp\left(\frac{\mathrm{i}k}{2z}\right)\left[(\mathbf{p}_1 - \mathbf{r}_1)^2 - (\mathbf{p}_2 - \mathbf{r}_2)^2\right] \times \exp\left(\psi(\mathbf{p}_1, \mathbf{r}_1, z) + \psi(\mathbf{p}_2, \mathbf{r}_2, z)^*\right) \times U_0(\mathbf{r}_1)\,U_0^*(\mathbf{r}_2)\,\mathrm{d}^2\mathbf{r}_1\mathrm{d}^2\mathbf{r}_2. \tag{5}$$

The mutual coherence function which is the ensemble average of Eq. (5) over the different realizations of the refractive index field can be written as[9]

$$\Gamma(\mathbf{p}_1, \mathbf{p}_2, z) = \left(\frac{k}{2\pi z}\right)^2 \iint_{\Sigma} \exp\left(\frac{\mathrm{i}k}{2z}\right)\left[(\mathbf{p}_1 - \mathbf{r}_1)^2 - (\mathbf{p}_2 - \mathbf{r}_2)^2\right] \times \exp\left[-D_\psi(\mathbf{p}_1 - \mathbf{p}_2; \mathbf{r}_1 - \mathbf{r}_2)/2\right] \times U_0(\mathbf{r}_1)\,U_0^*(\mathbf{r}_2)\,\mathrm{d}^2\mathbf{r}_1\mathrm{d}^2\mathbf{r}_2, \tag{6}$$

where $D_\psi(\mathbf{p}_1 - \mathbf{p}_2; \mathbf{r}_1 - \mathbf{r}_2)$ is the wave-structure function.

We introduce sum and difference coordinates

$$\mathbf{p} = \mathbf{p}_1 - \mathbf{p}_2, \quad \mathbf{q} = (\mathbf{p}_1 + \mathbf{p}_2)/2, \quad \boldsymbol{\rho} = \mathbf{r}_1 - \mathbf{r}_2, \quad \mathbf{r} = (\mathbf{r}_1 + \mathbf{r}_2)/2. \tag{7}$$

Hence

$$\Gamma(\mathbf{p}, \mathbf{q}, z) = \left(\frac{k}{2\pi z}\right)^2 \int \exp\left\{\left(\frac{\mathrm{i}k}{2z}\right)\left[(\mathbf{r} - \mathbf{q})\cdot(\boldsymbol{\rho} - \mathbf{p})\right]\right\} \exp\left[-\tfrac{1}{2}D_\psi(\mathbf{p}, \boldsymbol{\rho}, z)\right] U_0\left(\mathbf{r} + \tfrac{1}{2}\boldsymbol{\rho}\right) U_0^*\left(\mathbf{r} - \tfrac{1}{2}\boldsymbol{\rho}\right) \mathrm{d}^2\boldsymbol{\rho}\,\mathrm{d}^2\mathbf{r}, \tag{8}$$

where

$$U_0\left(\mathbf{r} + \tfrac{1}{2}\boldsymbol{\rho}\right) U_0^*\left(\mathbf{r} - \tfrac{1}{2}\boldsymbol{\rho}\right) = U_0U_0^* \exp\left[-\frac{r^2 + \frac{1}{4}\rho^2}{W_0^2} - \frac{\mathrm{i}k\mathbf{r}\cdot\boldsymbol{\rho}}{F_0}\right]. \tag{9}$$

And the wave structure function can be written as[8]

$$D_\psi(\mathbf{p}, \boldsymbol{\rho}, z) = 8\pi^2k^2z \times \int_0^1\int_0^\infty \kappa\phi(\kappa)\left[1 - \mathrm{J}_0\left(\left|(1-\xi)\,\mathbf{q} + \xi\boldsymbol{\rho}\right|\kappa\right)\right]\mathrm{d}\kappa\,\mathrm{d}\xi. \tag{10}$$

The average intensity at $\mathbf{p}_1$ is given for an arbitrary aperture disturbance by

$$I(\mathbf{p}_1, z) = \left(\frac{k}{2\pi z}\right)^2 I_0 \times \iint_{\Sigma} \exp\left\{-\mathrm{i}k\left(\frac{1}{F_0} - \frac{1}{z}\right)\mathbf{r}\cdot\boldsymbol{\rho} - \frac{\mathrm{i}k}{z}\mathbf{p}_1\cdot\boldsymbol{\rho}\right\} \times \exp\left[-\tfrac{1}{2}D_\psi(\boldsymbol{\rho}, z)\right]\exp\left[-\frac{r^2 + \frac{1}{4}\rho^2}{W_0^2}\right]\mathrm{d}^2\boldsymbol{\rho}\,\mathrm{d}^2\mathbf{r}, \tag{11}$$

where $I_0 = U_0U_0^*$. By the Bessel relationship $\mathrm{J}_0(z) = \frac{1}{2\pi}\int_0^{2\pi}\exp[\text{-i}z\cos\alpha]\mathrm{d}\alpha$, Eq. (11) becomes

$$I(\mathbf{p}_1, z) = (k/z)^2 I_0 \iint_{\Sigma} \mathrm{J}_0[k\eta r\rho/z]\mathrm{J}_0[kp_1\rho/z] \times \exp\left[-\frac{r^2 + \frac{1}{4}\rho^2}{W_0^2}\right]\exp\left[-\tfrac{1}{2}D_\psi(\boldsymbol{\rho}, z)\right]\rho\mathrm{d}\rho r\mathrm{d}r, \tag{12}$$

where $\eta = (z/F_0 - 1)$. Based on the modified von Karman spectrum[7]

$$\phi_n(\kappa, L_0, l_0) = 0.033C_n^2\exp\left(-\kappa^2/\kappa_m^2\right)\left(\kappa_0^2 + \kappa^2\right)^{-11/6},$$

where $\kappa_0 = 2\pi/L_0$, $\kappa_m = 5.92/l_0$, L_0 and l_0 are the outer scale and inner scale of atmospheric turbulence, respectively. C_n^2 is the refractive index structural characteristic. One of the most widely used models is the

Hufnagel-Velley model described by[12]

$$C_n^2(z\xi\cos\theta) = 0.00594\,(\upsilon/27)^2\left(z\xi\cos\theta\times10^{-5}\right)^{10}$$
$$\times\exp\left(-z\xi\cos\theta/1000\right)$$
$$+2.7\times10^{-16}\exp\left(-z\xi\cos\theta/1500\right)$$
$$+C_n^2(0)\exp\left(-z\xi\cos\theta/100\right),$$

where z is the propagation distance measured from the transmitter z_0 along a slant path to the satellite z_1 in meters, $\upsilon = 2.1$ m/s is the root-meat-square (RMS) wind speed, and $C_n^2(0)\mathrm{A} = 1.7\times10^{-14}$ or $C_n^2(0) = 3\times10^{-13}$ $\mathrm{m}^{-2/3}$ is the refractive index structural characteristic of ground, θ is the zenith angle (see Fig. 1(b)). And under the quadratic approximation[8], the long-term wave structure function can be written as

$$D_{\Psi LT}(\boldsymbol{\rho}, z)$$
$$\approx 2\left(1.45k^2\sec\theta\int_{z_0}^{z_1}C_n^2(z)\left(\frac{z_1-z}{z_1-z_0}\right)^{5/3}\mathrm{d}z\right)^{6/5}$$
$$\times\left[1-0.715\kappa_0^{1/3}\right]\rho^2 = 2\frac{\rho^2}{\tilde{\rho}_0^2}, \qquad (13)$$

where $\rho_0^2 = \left(1.45k^2\sec\theta\int_{z_0}^{z_1}C_n^2(z)\left(\frac{z_1-z}{z_1-z_0}\right)^{5/3}\mathrm{d}z\right)^{-6/5}$ is the long-term lateral coherence length of a spherical wave and $\tilde{\rho}_0^2 = \rho_0^2\left[1-0.715\kappa_0^{1/3}\right]^{-1}$. The short-term wave structure function can be written as[4]

$$D_{\psi ST}(\boldsymbol{\rho}, z) = 2\left[1-(\rho_0/2W_0)^{1/3}\right]\rho^2/\tilde{\rho}_0^2. \qquad (14)$$

By Eqs. (11) and (13), we have the long-term average intensity

$$I(\mathbf{p}_1, z) = (k/2z)^2 I_0\iint_{\Sigma}\mathrm{J}_0\left[\frac{k\eta r\rho}{z}\right]\mathrm{J}_0\left[\frac{kp_1\rho}{z}\right]$$
$$\times\exp\left[-\frac{r^2+\frac{1}{4}\rho^2}{W_0^2}\right]\exp\left[-\frac{\rho^2}{\tilde{\rho}_0^2}\right]\mathrm{d}\rho^2\mathrm{d}r^2. \qquad (15)$$

Dre'ge *et al.* have shown that the Bessel function in the product $\wp = \exp\left(-a_{\mathrm{eff}}^2\rho^2\right)\mathrm{J}_0\left(kp_1\rho/z\right)$ can be approximated as Gaussian function[10], i.e.,

$$\exp\left(-a_{\mathrm{eff}}^2\rho^2\right)\mathrm{J}_0\left(kp_1\rho/z\right)$$
$$\approx\exp\left(-a_{\mathrm{eff}}^2\rho^2\right)\exp\left[-k^2p_1^2\rho^2/(Kz)^2\right], \qquad (16)$$

where $K\approx1.5257$. Substituting Eq. (16) into Eq. (15) gives

$$I(\mathbf{p}_1, z) = \left(\frac{k}{2z}\right)^2 I_0\iint_{\Sigma}\exp\left(-\left(\left(\frac{k\eta\rho}{zK}\right)^2+\frac{1}{W_0^2}\right)r^2\right)$$
$$\times\mathrm{J}_0\left[\frac{kp_1\rho}{z}\right]\exp\left[-\left(\frac{1}{4W_0^2}+\frac{1}{\tilde{\rho}_0^2}\right)\rho^2\right]\mathrm{d}\rho^2\mathrm{d}r^2. \qquad (17)$$

The integral in Eq. (17) for r can be analytically evaluated to obtain

$$I(\mathbf{p}_1, z) = \left(\frac{k}{2z}\right)^2\int_{\Sigma}\frac{I_0}{\left(\left(\frac{k\eta\rho}{zK}\right)^2+\frac{1}{W_0^2}\right)}$$
$$\times\left[1-\exp\left(-\left(\left(\frac{k\eta\rho}{zK}\right)^2+\frac{1}{W_0^2}\right)R^2\right)\right]$$
$$\times\mathrm{J}_0\left[\frac{kp_1\rho}{z}\right]\exp\left[-\left(\frac{1}{4W_0^2}+\frac{1}{\tilde{\rho}_0^2}\right)\rho^2\right]\mathrm{d}\rho^2. \qquad (18)$$

Substituting Eq. (16) into Eq. (18) gives

$$I(\mathbf{p}_1, z) = \left(\frac{k}{2z}\right)^2\int_{\Sigma}\frac{I_0}{\left(\left(\frac{k\eta\rho}{zK}\right)^2+\frac{1}{W_0^2}\right)}$$
$$\times\left[1-\exp\left(-\left(\left(\frac{k\eta\rho}{zK}\right)^2+\frac{1}{W_0^2}\right)R^2\right)\right]$$
$$\times\exp\left[-\left(\frac{1}{4W_0^2}+\frac{1}{\tilde{\rho}_0^2}+\left[\frac{kp_1}{zK}\right]^2\right)\rho^2\right]\mathrm{d}\rho^2. \qquad (19)$$

By the relationship $e^{-x} = 1-x+x^2/2!-x^3/3!\cdots$, Eq. (19) can be written as

$$I(\mathbf{p}_1, z) = \left(\frac{k}{2z}\right)^2 R^2 I_0$$
$$\times\int_{\Sigma}\sum_{j=1}^{\infty}\frac{(-1)^{j+1}\left[\left(\left(\frac{k\eta\rho}{zK}\right)^2+\frac{1}{W_0^2}\right)R^2\right]^j}{(j)!}$$
$$\times\exp\left[-\left(\frac{1}{4W_0^2}+\frac{1}{\tilde{\rho}_0^2}+\left[\frac{kp_1}{zK}\right]^2\right)\rho^2\right]\mathrm{d}\rho^2. \qquad (20)$$

Exchanging the integral and sum, we obtain

$$I(\mathbf{p}_1, z) = \left(\frac{k}{2z}\right)^2 R^2 I_0\sum_{j=1}^{\infty}\frac{(-1)^{j+1}}{j!}$$
$$\times\int_{\Sigma}\left[\left(\left(\frac{k\eta\rho}{zK}\right)^2+\frac{1}{W_0^2}\right)R^2\right]^j$$
$$\times\exp\left[-\left(\frac{1}{4W_0^2}+\frac{1}{\tilde{\rho}_0^2}+\left[\frac{kp_1}{zK}\right]^2\right)\rho^2\right]\mathrm{d}\rho^2. \qquad (21)$$

Changing variables $\left[\left(\left(\frac{k\eta\rho}{zK}\right)^2+\frac{1}{W_0^2}\right)R^2\right]=x$ and $\rho^2=\left(x-\frac{R^2}{W_0^2}\right)\left(\frac{zK}{kR\eta}\right)^2$ gives

$$I(\mathbf{p}_1,z)=\left(\frac{K}{2\eta}\right)^2 I_0\sum_{j=1}^{\infty}\frac{(-1)^{j+1}}{j!}$$
$$\times\int x^j \exp\left[-\left(\frac{1}{4W_0^2}+\frac{1}{\tilde{\rho}_0^2}+\left[\frac{kp_1}{zK}\right]^2\right)\right.$$
$$\left.\left(x-\frac{R^2}{W_0^2}\right)\left(\frac{zK}{kR\eta}\right)^2\right]\mathrm{d}x. \quad (22)$$

Using the reference integral

$$\int_0^R x^j\exp\left(-\mu x\right)\mathrm{d}x=\mu^{-j-1}\gamma\left(j+1,\mu R\right), \quad (23)$$

we have the new expression of mean irradiance

$$I(\mathbf{p}_1,z)=\left(\frac{K}{2\eta}\right)^2 I_0$$
$$\exp\left[\left(\frac{1}{4W_0^2}+\frac{1}{\tilde{\rho}_0^2}+\left[\frac{kp_1}{zK}\right]^2\right)\left(\frac{zK}{kW_0\eta}\right)^2\right]$$
$$\times\sum_{j=1}^{\infty}\frac{(-1)^{j+1}}{j!}\left[\left(\frac{1}{4W_0^2}+\frac{1}{\tilde{\rho}_0^2}+\left[\frac{kp_1}{zK}\right]^2\right)\left(\frac{zK}{kR\eta}\right)^2\right]^{-j-1}$$
$$\times\gamma\left(j+1,\left(\frac{1}{4W_0^2}+\frac{1}{\tilde{\rho}_0^2}+\left[\frac{kp_1}{zK}\right]^2\right)R\sqrt{\left(\frac{k\eta R}{zK}\right)^2+\frac{1}{W_0^2}}\right), \quad (24)$$

where $\gamma\left(j,x\right)$ is the incomplete gamma function.

In order to gain an insight into the relationship between the long-term average intensity and the parameters (W_0, z, $\tilde{\rho}_0$, and R) of atmospheric image system, we discuss the focused beam propagation in the slant path turbulent atmosphere. In the focused beam propagation case, $\eta=0$, Eq. (21) or (22) becomes

$$I(\mathbf{p}_1,z)=\left(\frac{k}{2z}\right)^2 W_0^2 I_0\left\{1-\exp\left[-\left(\frac{R^2}{W_0^2}\right)\right]\right\}$$
$$\times\int_R \exp\left[-\left(\frac{1}{4W_0^2}+\frac{1}{\tilde{\rho}_0^2}+\left[\frac{kp_1}{zK}\right]^2\right)\rho^2\right]\mathrm{d}\rho^2.$$

After integration, we obtain a new approximated expression for the long-term average intensity

$$I(\mathbf{p}_1,z)=\frac{W_0^2 I_0\left\{1-\exp\left[-\left(\frac{R^2}{W_0^2}\right)\right]\right\}}{W_e^2\left(1+\left[\frac{2p_1}{W_eK}\right]^2\right)}$$
$$\times\left\{1-\exp\left[-W_e^2\left(1+\left[\frac{2p_1}{W_eK}\right]^2\right)\frac{k^2R^2}{4z^2}\right]\right\}, \quad (25)$$

where $W_e^2=\left(\frac{2z}{k}\right)^2\left(\frac{1}{4W_0^2}+\frac{1}{\tilde{\rho}_0^2}\right)$ is the long-term beam spread for the focused beam[4]. When $R\gg W_0$, Eq. (25) is approximately equal to

$$I(\mathbf{p}_1,z)=\frac{W_0^2 I_0}{W_e^2\left(1+\left[\frac{2p_1}{W_eK}\right]^2\right)}. \quad (26)$$

Under the case $4p_1^2\ll W_e^2K^2$ and to the lowest order in$(2p_1/W_eK)^2$, Eq. (25) takes the approximation form

$$I(\mathbf{p}_1,z)=\frac{W_0^2 I_0}{W_e^2}\left(1-\left[\frac{2p_1}{W_eK}\right]^2\right)$$
$$\sim\frac{W_0^2 I_0}{W_e^2}\exp\left(-\left[\frac{2p_1}{W_eK}\right]^2\right). \quad (27)$$

Equation (26) shows that in limit $4p_1^2\ll W_e^2K^2$ and $R\gg W_0$, apart a constant number $2/K$, the long-term average intensity is approximately given by the well know Gaussian function[2,4].

In conclusion, in this paper we have used the extended Huygens-Fresnel principle to obtain an expression for the mean irradiance in a random medium. We have noted, on the basis of the quadratic approximation of the wave-structure function, the modified von Karman spectrum for refractive-index fluctuations, and Gaussian approximation for the product of Gaussian function and Bessel function, that our solution do not agree with a Gaussian intensity distribution function. Our results show that the mean irradiance of a finite optical beam propagating in slant path turbulent atmosphere not only depends on the W_0, z, and $\tilde{\rho}_0$ but also on the emit aperture R.

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References

1. L. C. Andrews, W. B. Miller, and J. C. Ricklin, J. Opt. Soc. Am. A **11,** 1653 (1994).
2. L. C. Andrews, R. L. Phillips, and C. Y. Hopen, *Laser Beam Scintillation with Applications* (SPIE Press, Bellingham, 2001) pp.31—34.
3. J. Liu, Y. Huang, and Y. Wang, Acta Opt. Sin. (in Chinese) **25,** 1009 (2005).
4. H. T. Yura, J. Opt. Soc. Am. **63,** 567 (1973).
5. M. I. Charnotskii, Proc. SPIE **3125,** 252 (1997).
6. A. J. Masino, C. Y. Young, L. Andrews, K. Swanger, F. S. Vetelino, K. Grant, K. Corbett, and B. Clare, Proc. SPIE **5793,** 178 (2005).
7. R. F. Lutomirski and H. T. Yura, Appl. Opt. **10,** 1652 (1971).
8. H. T. Yura, Appl. Opt. **11,** 1399 (1972).
9. M. T. Tavis and H. T. Yura, Appl. Opt. **15,** 2922 (1976).
10. E. M. Dre'ge, N. G. Skinner, and D. M. Byme, Appl. Opt. **39,** 4918 (2000).
11. R. L. Fante, *Progress in Optics XXII* E. Wolf (ed.) (Elsevier Science, Netherlands, 1985) pp.341—398.
12. L. C. Andrews, R. L. Phillips, and C. Y. Hopen, *Laser Beam Scintillation with Applications* (SPIE Press, Bellingham, 2001) p.8.