A position sensor based on grating projection with spatial filtering and polarization modulation

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A position sensor based on grating projection with spatial filtering and polarization modulation is presented. A grating is projected onto the object to be measured through a $4f$ optical system with a spatial filter. After reflected by the object, the grating projection is imaged on a detection grating through another $4f$ optical system to form moiré fringes. The polarization modulated moiré signal is detected to obtain the position information of the object. In the position sensor, the moiré signal varies sinusoidally with the position of object. The measurement is independent of the incident intensity on the projection grating and the reflectivity of the object to be measured. In experiments, the effectiveness of the position sensor is proved, and the root mean square (RMS) error at each measurement position is less than 13 nm.

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Position measurements have been playing an important role in many fields such as fabrication, biophysics, and autocontrol[1]. In methods of position measurement, optical method holds more interest because of its high accuracy and non-contact measurement[2-6]. In the common optical methods such as image measurement[7-10], interferometry[11], and direct position detection using position sensitive detector (PSD)[12], the measuring beam is in the direction perpendicular to the surface of the object to be measured. When there exists a block in the direction perpendicular to the object[13,14], the common optical methods cannot be applied. To solve the problem, a new position sensor based on grating projection with grazing incidence is presented in this paper.

The position sensor is schematically shown in Fig. 1. A projection grating is illuminated by a collimated laser beam and placed in the front focal plane of lens L_1. Lenses L_1 and L_2 form $4f$ system 1. In this $4f$ system, an aperture is placed in the frequency plane. The projection grating is projected onto the object to be measured through the $4f$ system 1 with grazing incidence. The $4f$ system 2 composed of lenses L_3 and L_4 has the same optical parameters with $4f$ system 1. The detection grating is placed in the back focal plane of L_4. After reflected by the object, the image of the projection grating is imaged onto the detection grating through the $4f$ system 2. Due to the overlapping of the detection grating and the image of the projection grating, the moiré fringes are produced. The moiré signal varying with the position of the object is detected by a detector through lens L_5. A polarizer and a Savart plate are placed between L_4 and the detection grating. The rays forming the image of the projection grating become linearly polarized light through the polarizer. Passing the Savart plate, the linearly polarized light is split into two orthogonally polarized and mutually sheared beams. An analyzer is placed between the detection grating and L_5. When the analyzer is rotated around the optical axis of $4f$ system 2, the polarizer, Savart plate and analyzer form a polarization modulator. With the polarization modulation, the moiré signal is insensitive to the change of light intensity on the projection grating. Thus the position of the object can be measured accurately by detecting the modulated signal.

In Fig. 1, the optical axes of the $4f$ systems are defined as z axis. In the projection grating plane, the directions perpendicular and parallel to the groove of the projection grating are defined as directions of x_0 and y_0 axes, respectively. Similarly, x_2, y_2 axes and x_3, y_3 axes are defined on the image planes of the projection grating. In the frequency plane of the $4f$ system 1, x_1, y_1 axes are parallel to x_0, y_0 axes in the projection grating plane, respectively. The center of the projection grating is placed at the origin of x_0 axis. The line-to-space ratio of the projection grating is 1:1. The period and width of the projection grating are represented by d and B, respectively. The amplitude distribution through the projection grating is expressed as

$$E(x_0) = \sqrt{I_1}\,\mathrm{rect}\left(\frac{x_0}{B}\right)\sum_{k=-\infty}^{\infty}\mathrm{rect}\left(\frac{x_0 - kd}{d/2}\right), \quad (1)$$

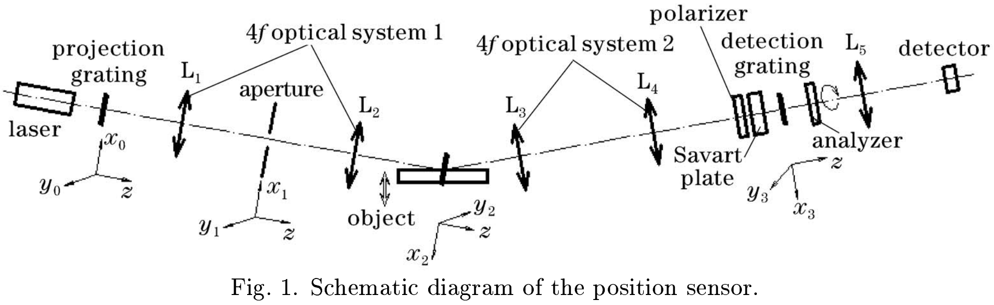

Fig. 1. Schematic diagram of the position sensor.

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where I_1 is the intensity on the projection grating. With Fourier transform of lens L_1, the frequency distribution in the frequency plane of the $4f$ system 1 is given as

$$\widetilde{E}(u) = B\sqrt{I_1}\left\{\frac{1}{2}\mathrm{sinc}(Bu) + \frac{1}{\pi}\mathrm{sinc}[B(u-\frac{1}{d})] + \frac{1}{\pi}\mathrm{sinc}[B(u+\frac{1}{d})] + \cdots\right\}_{u=\frac{x_1}{\lambda f}}, \tag{2}$$

where λ is the wavelength of the laser and f is the focal length of L_1. In the frequency plane, the aperture is used as a spatial filter and its filtering function is written as

$$\widetilde{t}(u) = \mathrm{rect}\left(\frac{u}{4/d}\right)_{u=\frac{x_1}{\lambda f}}. \tag{3}$$

Hence only 0, ±1 diffraction orders pass the aperture and the frequency distribution through the aperture is written as

$$\widetilde{E}'(u) = \widetilde{E}(u)\cdot\widetilde{t}(u) = B\sqrt{I_1}\left\{\frac{1}{2}\mathrm{sinc}(Bu) + \frac{1}{\pi}\mathrm{sinc}[B(u-\frac{1}{d})] + \frac{1}{\pi}\mathrm{sinc}[B(u+\frac{1}{d})]\right\}. \tag{4}$$

With Fourier transform of L_2, the amplitude distribution $E(x_2)$ in the back focal plane of L_2 is given by

$$E(x_2) = \sqrt{I_1}\mathrm{rect}\left(\frac{x_2}{B}\right)\left[\frac{1}{2} + \frac{2}{\pi}\cos\left(\frac{2\pi}{d}x_2\right)\right]. \tag{5}$$

Thus the intensity distribution $i(x_2)$ of the image of projection grating is expressed as

$$i(x_2) = |E(x_2)|^2 = I_1\mathrm{rect}\left(\frac{x_2}{B}\right)\left[\frac{1}{4} + \frac{2}{\pi^2} + \frac{2}{\pi}\cos\left(\frac{2\pi}{d}x_2\right) + \frac{2}{\pi^2}\cos\left(\frac{4\pi}{d}x_2\right)\right]. \tag{6}$$

The object to be measured is placed near the back focus of L_2. The front focus of L_3 is superposed with the back focus of L_2 and $4f$ systems 1 and 2 are laid symmetrically relative to the vertical. After reflected by the object, the projection is imaged on the detection grating through the $4f$ system 2. The horizontal plane where the front focus of L_3 is placed is defined as the zero plane of the position sensor. If there exists a distance x between the object and the sensor's zero plane, the image of the projection grating on the detection grating will be moved with displacement

$$\Delta x_3 = 2x\sin\theta, \tag{7}$$

where θ is the angle of grazing incidence. The intensity distribution of the image of the projection grating on the detection grating is expressed as

$$i(x_3) = RI_1\mathrm{rect}\left(\frac{x_3-\Delta x_3}{B}\right)\left\{\frac{1}{4} + \frac{2}{\pi^2} + \frac{2}{\pi}\cos\left[\frac{2\pi}{d}(x_3-\Delta x_3)\right] + \frac{2}{\pi^2}\cos\left[\frac{4\pi}{d}(x_3-\Delta x_3)\right]\right\}, \tag{8}$$

where R is the reflectivity of the object to be measured.

The angle between the polarizer's transmission axis and the groove direction of the detection grating is 45°. The Savart plate comprises two birefringence plates whose optic axes are orthogonal mutually. It splits the rays that form the image of the projection grating into two orthogonally polarized and mutually sheared rays, namely ordinary and extraordinary rays. Thus two images of the projection grating called ordinary and extraordinary images are formed on the detection grating. The distance between the two images is $d/2$. The intensity distribution of the ordinary and extraordinary images are expressed as

$$i_{\mathrm{o}}(x_3) = \frac{RI_1}{2}\left\{\frac{1}{4} + \frac{2}{\pi^2} + \frac{2}{\pi}\cos\left[\frac{2\pi}{d}(x_3-2x\sin\theta)\right] + \frac{2}{\pi^2}\cos\left[\frac{4\pi}{d}(x_3-2x\sin\theta)\right]\right\}, \tag{9}$$

$$i_{\mathrm{e}}(x_3) = \frac{RI_1}{2}\left\{\frac{1}{4} + \frac{2}{\pi^2} - \frac{2}{\pi}\cos\left[\frac{2\pi}{d}(x_3-2x\sin\theta)\right] - \frac{2}{\pi^2}\cos\left[\frac{4\pi}{d}(x_3-2x\sin\theta)\right]\right\}, \tag{10}$$

respectively, where $-B/2 \le x_3 - 2x\sin\theta \le B/2$.

The detection grating has the same period and line-to-space ratio as the projection grating and its groove is parallel to the groove of the projection grating. The detection grating is placed with $d/4$ offset relative to the origin of axis x_3. The two moiré signals are written as

$$I_{\mathrm{o}} = \int_0^{d/2} i_{\mathrm{o}}(x_3)\mathrm{d}x_3 = \frac{RI_1}{2}\left[\left(\frac{d}{8}+\frac{d}{\pi^2}\right) + \frac{2d}{\pi^2}\sin\left(\frac{4\pi x\sin\theta}{d}\right)\right], \tag{11}$$

$$I_{\mathrm{e}} = \int_0^{d/2} i_{\mathrm{e}}(x_3)\mathrm{d}x_3 = \frac{RI_1}{2}\left[\left(\frac{d}{8}+\frac{d}{\pi^2}\right) - \frac{2d}{\pi^2}\sin\left(\frac{4\pi x\sin\theta}{d}\right)\right]. \tag{12}$$

If the angle between the analyzer's transmission axis and groove direction of the detection grating is α, the two moiré signals through the analyzer are written as

$$I'_{\mathrm{o}} = I_{\mathrm{o}}\cos^2\alpha = \frac{I_{\mathrm{o}}}{2}(1+\cos 2\alpha), \tag{13}$$

$$I'_{\mathrm{e}} = I_{\mathrm{e}}\cos^2(90^\circ-\alpha) = \frac{I_{\mathrm{e}}}{2}(1-\cos 2\alpha). \tag{14}$$

The detector detects the light flux through the analyzer, and the intensity on the detector is given by

$$I = I'_{\mathrm{o}} + I'_{\mathrm{e}} = \frac{RI_1}{2}\left[\left(\frac{d}{8}+\frac{d}{\pi^2}\right) + \frac{2d}{\pi^2}\sin\left(\frac{4\pi x\sin\theta}{d}\right)\cos 2\alpha\right]. \tag{15}$$

When the analyzer is rotated with angle $\alpha(t)$ around the optical axis of the $4f$ system 2, polarization modulation of the moiré signal is realized and the modulated intensity on the detector is expressed as

$$I = \frac{RI_1}{2}\left(\frac{d}{8}+\frac{d}{\pi^2}\right) + \frac{RI_1 d}{\pi^2}\sin\left(\frac{4\pi x\sin\theta}{d}\right)\cos[2\alpha(t)] = I_{\mathrm{DC}} + I_{\mathrm{AC}}\cos[2\alpha(t)]. \tag{16}$$

The intensity is converted by the detector into electric signal which includes a direct current component I_{DC} and an alternating current component I_{AC}. The electric signal is amplified by an amplifier and its direct and alternating current components are separated by a filter circuit. The position of the object is given by

$$x = \frac{d}{4\pi \sin\theta} \arcsin\left[\frac{I_{\mathrm{AC}}}{I_{\mathrm{DC}}}\left(\frac{\pi^2}{16} + \frac{1}{2}\right)\right], \tag{17}$$

with the variation range

$$-\frac{d}{8\sin\theta} < x < \frac{d}{8\sin\theta}. \tag{18}$$

In Eq. (17), I_{AC} and I_{DC} have common factors I_1 and R, so the distance is independent of the initial intensity I_1 and the reflectivity R. The influence of the output variation of the laser and the reflectivity difference in different objects to be measured can be eliminated in the position sensor.

Using the light path shown in Fig. 1, the experiment was carried out. A projection grating was illuminated by a collimated laser beam whose wavelength was 785 nm. The period of projection grating was 0.1 mm. The line-to-space ratio of projection grating is 1:1. The focal length of lens L_1 was 120 mm. The diameter of the aperture in the frequency plane of the $4f$ system 1 was 3.5 mm. The object to be measured was placed on a manual linear stage supplying up/down movement. The angle between the axis of $4f$ system 1 and the normal of the object was 84.5°. The $4f$ system 2 and the detection grating had same optical parameter as the $4f$ system 1 and the projection grating, respectively. The polarizer and the analyzer were Glan-Taylor prisms. The Savart plate was made of two same quartz prisms whose beam displacement was 0.05 mm. The analyzer was placed in a rotation mount and was rotated to modulate the moiré signal.

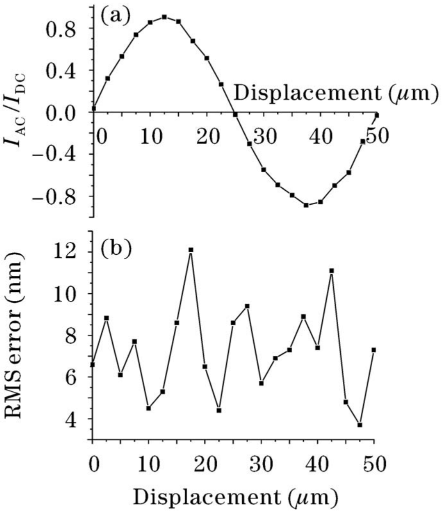

Fig. 2. $I_{\mathrm{AC}}/I_{\mathrm{DC}}$ (a) and RMS error (b) measurement results when vertically moving the object.

In the experiment, the position of the object was changed by moving the manual linear stage step by step with 2.5-μm step size within 50-μm range. At each position, the height of the object was measured 100 times. The experiment results are shown in Fig. 2. Figure 2(a) shows the relation between the $I_{\mathrm{AC}}/I_{\mathrm{DC}}$ and the height of the object. $I_{\mathrm{AC}}/I_{\mathrm{DC}}$ varies sinusoidally with the height. Therefore the measuring principle of the position sensor is verified. Figure 2(b) shows the root mean square (RMS) errors at 21 measurement positions, the RMS errors are less than 13 nm. So the height of object can be measured by the position sensor with high precision.

In conclusion, we have proposed a position sensor based on grating projection with spatial filtering and polarization modulation. The position sensor can be applied when there is a block in the direction perpendicular to the object to be measured. The moiré signal varying sinusoidally with the position of the object is obtained in the position sensor. The position sensor is insensitive to the output variation of light source and the reflectivity difference in objects with the polarization modulation. In experiments, the feasibility of the method was verified.

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