

Complex-wave retrieval based on blind signal separation

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In the process of the reconstruction of digital holography, the traditional methods of diffraction and filtration are commonly adopted to recover the original complex-wave signal. Influenced by twin-image and zero-order terms, the above-mentioned methods, however, either limit the field of vision or result in the loss of the amplitude and phase. A new method for complex-wave retrieval is presented, which is based on blind signal separation. Three frames of holograms are captured by a charge coupled device (CCD) camera to form an observation signal. The term containing only amplitude and phase of complex-wave is separated, by means of independent component analysis, from the observation signal, which effectively eliminates the zero-order term. Finally, the complex-wave retrieval of pure phase wavefront is achieved. Experimental results show that this method can better recover the amplitude and phase of the original complex-wave even when there is a frequency spectrum mixture in the hologram.

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In the process of reconstruction of digital holography, there are three main methods: diffraction based on Fresnel transform, filtration based on Fourier transform, and complex-wave retrieval based on Fresnelets. The first method simulates a wave's propagation and diffraction by the holograms, so the twin-image and zero-order terms which influence the quality of image and the size of vision field are generated. The second method is Fourier transform, a linear technique that selectively filters the relevant information in the Fourier domain. In fact, it is formally equivalent to the way recent zero-order and twin-image-removal algorithms for digital holography operation. But with a frequency spectrum mixture, they cannot be separated effectively, so the loss of the high-frequency component and the phase of the complex-wave occur. To solve the problem, Cuhe *et al.*^[1-4] adopted several methods to suppress the zero-order terms, especially in 2003 and 2004, Cuhe used digital hologram captured by charge coupled device (CCD) to show the sample of biologic cell, then did complex-wave retrieval with multiresolution for the digital hologram by using Fresnelets^[5-7] to avoid the zero-order terms, obtained a better result. Therefore, existence of the zero-order terms is the main factor to influence the reconstruction of digital holography in interference hologram. This thesis presents a new method for complex-wave retrieval, which is based on blind signal separation.

We consider that the course of capturing digital hologram using CCD is random, in other words, there must be tiny difference between the phases of the holograms which have more than two frames and captured by CCD from the same object. Therefore, this group of holograms can be considered as combination of some independent components. Independent component analysis (ICA) in the signal process can be used in the reconstruction of digital holography. The object processed by ICA is a mixed signal obtained by the linear combination of a group of signal sources which are statistical independent each other, finally we can pick up the independent sig-

nal component from the mixed signal. The separation of mixed signal using ICA based on statistical independent is better than that using traditional method based on irrelevant measurement. The whole course of processing is done in space domain. It separates the zero-order terms $|R(x, y)|^2$, $|O(x, y)|^2$, and $|R(x, y)| \cdot |O(x, y)| \cos(\theta_o - \theta_R)$ which contain nothing but complex-wave, preserve the amplitude and phase of the complex-wave effectively. Instead of frequency shift and filtration, this method can better recover the amplitude and phase of the complex-wave even when there is a frequency spectrum mixture and noise.

In recent years, blind signal separation is becoming a hotspot which is commonly interested by both neural network and signal processing domains. Blind signal separation ensures a transformation based on mixed data matrix observed, finally picks up each independent signal component from the mixed signal to recover original signal or signal source. The core of blind signal separation is the learning algorithm of separating matrix. It belongs to unsupervised learning, the basic ideal is to abstract the characteristic of the statistical independent as the denotation of input without the loss of information. Blind signal separation can be described as

$$X(t) = AS(t), \quad (1)$$

where $S(t) = [s_1(t), s_2(t), \dots, s_n(t)]^T$ is a n -dimensional vector that is consisted of N source signals; where $X(t) = [x_1(t), x_2(t), \dots, x_m(t)]^T$ is a m -dimensional observed data vector, its elements are the outputs of each sensor; Where A is $m \times n$ dimensional mixed matrix, its elements represent the mixture of signals. To obtain the m -dimensional observed data vector, mixing the n source signals is the meaning of Eq. (1). The mention of blind signal separation is to determine separating matrix W based on observed data vector $X(t)$ only, without knowing the mixing matrix A and source signal, making the output after transformation. The copy or estimation of

the source signal vector $S(t)$ ^[8] is obtained as

$$S(t) = WX(t). \tag{2}$$

The basic purpose of ICA is to ensure a linear conversion matrix W which can make the output component $S(t)$ as possible as statistical independent. The key is to set up an objective function which can measure the independence of the separation result and a separating algorithm. Several objective functions and algorithms have been proposed from different aspects. This thesis is based on a criterion of negentropy and a very effective ICA algorithm: Fast ICA.

Known from center limit theorem, if a stochastic variable consists of the sum of many stochastic variables which is independent each other, it must approach to Gaussian distribution without reference to the distribution each independent variable belongs to, as long as they have limit means and variances. The observed signal x approaches to Gaussian distribution better than s in Eq. (1), in other words, s has better non-Gaussianity than x . Therefore we can measure the non-Gaussianity characteristic of s during the separation process. The separation of each independent component is completed when the measurement of non-Gaussianity is maximal.

Assuming that the probability density function of a stochastic variable y is $p(y)$, the definition of entropy is

$$H(y) = - \int p(y) \lg p(y) dy. \tag{3}$$

Based on information theory, among the stochastic variable with the same variance, the ones with Gaussian distribution have the max information entropy. The greater the non-Gaussianity is, the smaller the information entropy is. A more reasonable measure function of non-Gaussianity is

$$Ng(y) = H(y_{\text{Gauss}}) - H(y). \tag{4}$$

In Eq. (4), y_{Gauss} is a stochastic variable with Gaussian distribution, and has the same variance of y . $Ng(y) = 0$, when y has the Gaussian distribution, the greater the non-Gaussianity of y is, the greater the value of $Ng(y)$ is. A. Hyvärinen brings a approximate formula to measure the non-Gaussianity

$$Ng(y) \propto [E\{G(y)\} - E\{G(y_{\text{Gauss}})\}]^2, \tag{5}$$

where $G(\cdot)$ can choose $G_1(u) = \lg \cos(a_1 u)$, $G_2(u) = -\exp(-u^2/2)$.

Fast ICA algorithm adopts the objective function defined to measure the non-Gaussianity of the separation s , and adjusts the separation matrix W . The adjustment formula of Fast ICA^[9] is

$$\begin{aligned} \widetilde{W}_i(n+1) &= E\{\widetilde{X}G'(\widetilde{W}_i^T(n)\widetilde{X})\} \\ &\quad - E\{G''(\widetilde{W}_i^T(n)\widetilde{X})\}\widetilde{W}_i(n). \end{aligned} \tag{6}$$

The iteration process terminates when there is no variance or the variance is very small between two successive $\widetilde{W}_i(n)$. If want to know more about the theory and method of ICA, you can refer to Ref. [8].

Interference hologram is the non-linear combination of object wavefront and reference wave. Although ICA is only fit for the linear combination of signals, the complex-wave retrieval based on blind signal separation does not directly separate the object wavefront and reference wave, but separates the independent component expanded. Taking off-axis Fresnel hologram as example, assuming the complex amplitude of object wavefront is $O(x, y)e^{i\varphi(x, y)}$, the complex amplitude of reference wave is $R(x, y)e^{i\theta(x, y)}$, the intensity of interference hologram is

$$I = \left| O(x, y)e^{i\varphi(x, y)} + R(x, y)e^{i\theta(x, y)} \right|^2. \tag{7}$$

Expanding the Eq. (7), obtain

$$\begin{aligned} I &= |O(x, y)|^2 + |R(x, y)|^2 \\ &\quad + 2 \cdot O(x, y) \cdot R(x, y) \cdot \cos(\theta - \varphi), \end{aligned} \tag{8}$$

where $|R(x, y)|^2$ and $|O(x, y)|^2$ constitute the zero-order terms, $|R(x, y)| \cdot |O(x, y)| \cos(\theta - \varphi)$ constitutes the information term which contains the amplitude and phase of complex-wave. If $|R(x, y)| \cdot |O(x, y)| \cos(\theta - \varphi)$ can be separated, then we can recover the complex amplitude of object wavefront.

In the process of interference, the course of capturing digital hologram using CCD is random, in other words, there must be tiny difference between the holograms which have more than two frames and captured by CCD from the same object. Assuming that in the process of capturing three holograms, all have tiny differences between the intensities and phases. The intensities of the three holograms are

$$\begin{aligned} I_1 &= \left| k_1 \cdot O(x, y)e^{i\phi_1(x, y)} + k_2 \cdot R(x, y)e^{i\theta_1(x, y)} \right|^2 \\ &= k_1^2 |O(x, y)|^2 + k_2^2 |R(x, y)|^2 \\ &\quad + 2 \cdot k_1 k_2 O(x, y) \cdot R(x, y) \cdot \cos(\alpha_1), \end{aligned} \tag{9}$$

$$\begin{aligned} I_2 &= \left| k_3 \cdot O(x, y)e^{i\phi_2(x, y)} + k_4 \cdot R(x, y)e^{i\theta_2(x, y)} \right|^2 \\ &= k_3^2 |O(x, y)|^2 + k_4^2 |R(x, y)|^2 \\ &\quad + 2 \cdot k_3 k_4 O(x, y) \cdot R(x, y) \cdot \cos(\alpha_2), \end{aligned} \tag{10}$$

$$\begin{aligned} I_3 &= \left| k_5 \cdot O(x, y)e^{i\phi_3(x, y)} + k_6 \cdot R(x, y)e^{i\theta_3(x, y)} \right|^2 \\ &= k_5^2 |O(x, y)|^2 + k_6^2 |R(x, y)|^2 \\ &\quad + 2 \cdot k_5 k_6 O(x, y) \cdot R(x, y) \cdot \cos(\alpha_3). \end{aligned} \tag{11}$$

In Eqs. (9)—(11), k_i ($i = 1, 2, 3, 4, 5, 6$) are the variables, α_i ($i = 1, 2, 3$) are the phase displacement, and $\alpha_1 \neq \alpha_2 \neq \alpha_3$, then the holograms captured can be considered the linear combination of $|R(x, y)|^2$, $|O(x, y)|^2$, and $|R(x, y)| \cdot |O(x, y)| \cos(\theta - \varphi)$. The object dealt with ICA is a mixed signal obtained by the linear combination of a group of signal sources which are independent of each other, finally we can pick up the independent signal component from the mixed signal. The separation of mixed

signal using ICA based on independent has a better effect than that using traditional method based on irrelevant measurement. From the experiments, we can see that the zero-order term has deterministic statistical independence, ICA based on blind signal separation can separates $|R(x, y)|^2$, $|O(x, y)|^2$, and $|R(x, y)| \cdot |O(x, y)| \cos(\theta - \varphi)$.

The hologram captured by CCD is a kind of digital hologram, we constitute the observed signal X with the hologram obtained which has three frames, secondly do whitening process with X . The whitening process can remove the correlation between the observed signals, simplify the independent component extraction algorithm followed. Thirdly matrix \tilde{W} is separate to implement the extraction of independent component (\tilde{W} here is for the signal done by whitening process). The calculation of the separation matrix \tilde{W} adopts Fast ICA algorithm. We can obtain the separation matrix \tilde{W} , mixing matrix A , and independent component S by doing iterative processing according to Eq. (6), then S contains $|R(x, y)|^2$, $|O(x, y)|^2$, $|R(x, y)| \cdot |O(x, y)| \cos(\theta - \varphi)$.

Based on the method above, we simulate an experiment that recovers the complex-wave signal from the

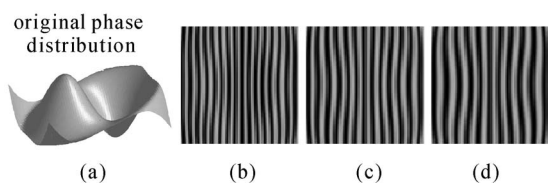


Fig. 1. Pure phase distribution (a), and interference holograms (b), (c), and (d) (64×64 samples).

interferogram, which has pure phase wavefront. In the simplest case the interferogram (hologram) is generated by mixing of an aberrated beam, for example a perfect beam reflected by an imperfect mirror. In Fig. 1(a) the phase distribution $e^{i\varphi(x, y)}$ of an aberrated beam is shown. Assuming that the wavelength $\lambda = 500$ nm, the size of holographic plane $s = 10 \times 10$ (mm), the reference beam is plane wave, angle of incidence $\alpha = 30^\circ$, the amplitudes of object wave and the reference beam vary $\pm 1\%$ (or one of them varies $\pm 1\%$), the phase displacement varies $\pm 1\%$. Figures 1(b), (c), and (d) show the hologram with three frames. For illuminating the separation process of ICA, we can observe the hologram and the separated independent component which all transformed to Fourier domain firstly. Figure 2 shows the spectrum of one hologram after Fourier transform, the spectrum contains zero-order term frequency spectrum and a pair of conjugated frequency spectrum contains object wave information, Fig. 2(b) shows the spectrum of the separated independent components, they are a pair of conjugated frequency spectra which contain object wave only and no zero-order term frequency spectrum, in other words, complex-wave signal can be separated using ICA. Figures 2(a) and (c) show the spectrum of the other two independent components. It becomes very simple to reconstruct the digital hologram using the separated independent component, the reconstructed phase distribution of complex-wave can be obtained by unwrapping the phase of complex-wave captured, showed in Fig. 3 (according to Fig. 2). Figure 3(b) illustrates the retrieval of the phase of original object wave using ICA effectively.

We can also do complex-wave retrieval using ICA for the off-axis Fresnel hologram. Figure 4(a) shows the

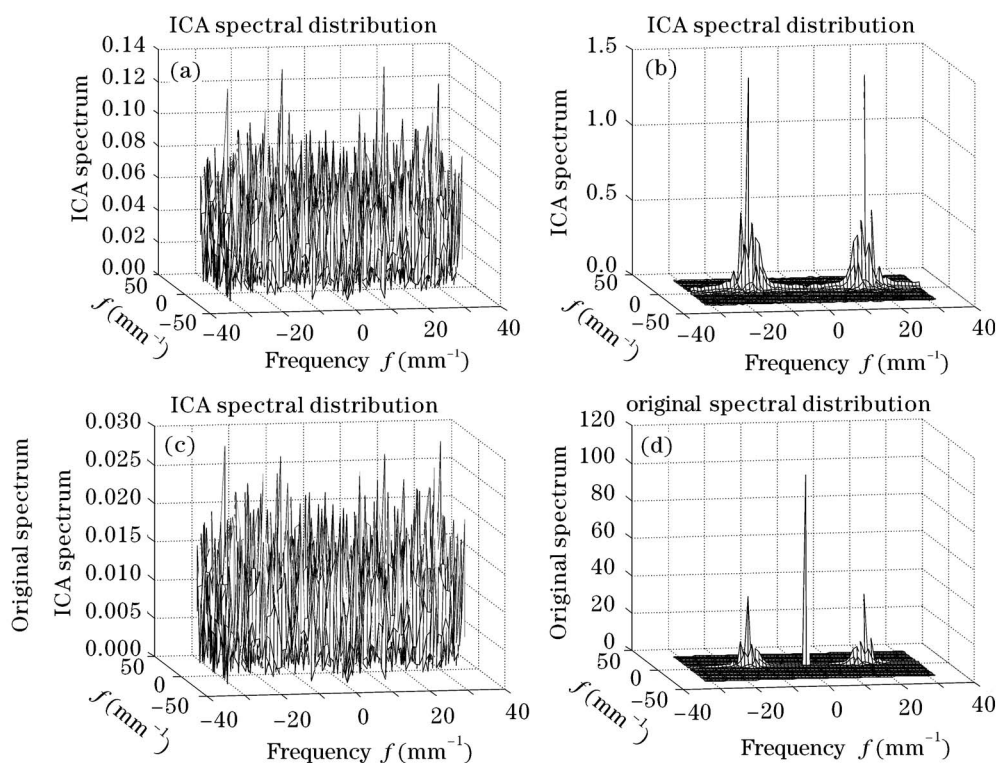


Fig. 2. Separation results of ICA (a), (b), and (c) (64×64 samples) and spectral distribution of original hologram (d).

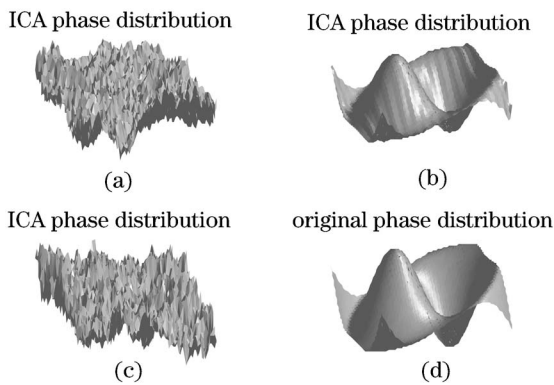


Fig. 3. Phase of complex-wave retrieval with ICA (a), (b), and (c), and phase of original complex-wave (d) (64×64 samples).

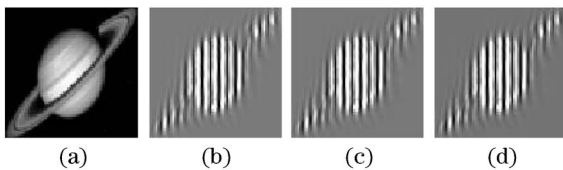


Fig. 4. Original image (a) and Fresnel off-axis holograms (b), (c), and (d) (64×64 sample).

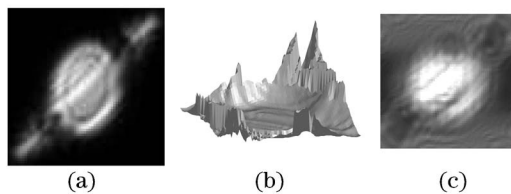


Fig. 5. Amplitude of complex-wave retrieval with ICA (a), phase of complex-wave retrieval with ICA (b), and reconstruction image with amplitude and phase (c).

original picture (original light field), the size of holographic plane $s = 10 \times 10$ (mm), we use the parallel light such as red light irradiate the picture, and let the light transmit 50 cm along the optical axis, then let it do interference with the incidence which is reference and plane wave (the included angle is 10°), the off-axis Fresnel hologram is obtained. Figures 4(b), (c), and (d) show three pictures recorded by CCD. Figure 5 shows the amplitude and phase of complex-wave (hologram plane) obtained by ICA, and the picture reconstructed by the amplitude and phase.

This thesis proposes a method of complex-wave retrieval based on blind signal separation aimed at digital hologram. This method applies ICA in the signal process to the reconstruction of digital holography, separates the complex-wave signal from digital hologram effectively,

and preserves the amplitude and phase of complex-wave well. It is different from the traditional methods of the reconstruction of digital holography, the whole course of processing is done in space domain. Meanwhile, we consider that the method can be implemented in the reconstruction of digital holography, and also has great value in other aspects of optical information processing domain. ICA is not perfect until now, there are still some questions, for example, non-linear, the source signal is complex function, the estimation of objective function and so on. We will research the algorithm related more, recover the amplitude and phase of complex-wave exactly, because the phase is the key of true three-dimensional display based on holography, it is important for the developing of digital holography^[10-12].

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