Stochastic resonance for signal-modulated pump noise in a single-mode laser

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By adopting the gain-noise model of the single-mode laser in which with bias and periodical signals serve as inputs, combining with the effect of coloured pump noise, we use the linear approximation method to calculate the power spectrum and signal-to-noise ratio (SNR) of the laser intensity under the condition of pump noise and quantum noise cross-related in the form of δ function. It is found that with the change of pump noise correlation time, both SNR and the output power will occur stochastic resonance (SR). If the bias signal α is very small, changing the intensities of pump noise and quantum noise respectively does not lead to the appearance of SR in the SNR; while α increases to a certain number, SR appears.

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In the 1990s, nonlinear system, noise, and input signal were thought of as three necessary factors of stochastic resonance (SR) in bistable or multistable system. While the new progress in recent years shows that SR can occur in monostable system. To two-dimensional (2D) system without periodical force, the existence of its intrinsic periodicity also leads to SR. Even in the linear system SR also can occur. The breakthrough lies in that the laser machine always works with the accompaniment of fluctuation, so that when input signal is fed to laser system, the cooperative effect between the noise and signal will lead to the appearance of SR[1−5]. The traditional SR was expressed by the relationship between signal-to-noise ratio (SNR) and noise intensity[6]. In 1998 Barzykin *et al.* proposed that SNR also can be represented by SNR and noise correlation time[7]. In laser communication when laser is modulated by signal, the noise in the laser is modulated too, hence the modulated noise appears[8]. It takes great effect on the statistical properties of laser. In this letter, the input signal is multiplied by the noise, and the bias and periodical signals are used to modulate noise. This research is important to practical applications. In recent years, the effect of the coupling noises on SR arouses great interest in scientific fields[9,10]. We concern the gain-noise model in a single-mode laser in which pump noise and quantum noise have different autocorrelation time and cross-correlation in the form of δ function. Studying its output power function in the steady state can discover that SR occurs in the SNR by changing autocorrelation time of pump noise. The same result is also for the overall output signal power.

The gain-noise model of a single-mode laser with an input bias signal is described by

$$\frac{\mathrm{d}I}{\mathrm{d}t'} = -2KI + \frac{2\Gamma}{1+\beta I}I + D + \frac{2I}{1+\beta I}\left[\alpha + \frac{1}{2}A\cos(\Omega t')\right]\xi(t') + 2\sqrt{I}\eta(t'), \quad (1)$$

where the pump noise $\xi(t')$ and the quantum noise $\eta(t')$ are correlated in the following forms

$$\langle\xi(t')\rangle = \langle\eta(t')\rangle = 0,$$

$$\langle\xi(t')\xi(s)\rangle = \frac{Q}{2\tau_1}\exp\left(-\frac{|t'-s|}{\tau_1}\right),$$

$$\langle\eta(t')\eta(s)\rangle = \frac{D}{2\tau_2}\exp\left(-\frac{|t'-s|}{\tau_2}\right),$$

$$\langle\xi(s)\eta(t')\rangle = \langle\xi(t')\eta(s)\rangle = \lambda\sqrt{DQ}\delta(t'-s), \quad -1 \le \lambda \le 1. \quad (2)$$

In Eqs. (1) and (2), I is the laser intensity; λ represents the noise correlation coefficient; Q and D are the intensities of the pump noise and quantum noise, respectively; τ_1 is the autocorrelation time for pump noise; τ_2 for quantum noise; $\beta = \widetilde{A}/\Gamma$, $\widetilde{A}$ and Γ represent the self-saturation and gain coefficients; K for loss coefficient; A is the amplitude of the periodical signal; Ω is the frequency; α indicates the bias signal. We use the linear approximation method discussed in Ref. [11] to process Eq. (1) and then get the laser intensity.

Linearize Eq. (1) around the deterministic steady-state intensity $I_0 = (\Gamma - K)/\beta K$, and let

$$I = I_0 + \varepsilon(t'),$$

where $\varepsilon(t')$ is the perturbation term substituted into Eq. (1), the linear equation of the laser intensity is found to be

$$\frac{\mathrm{d}\varepsilon(t')}{\mathrm{d}t} = -\gamma\varepsilon(t') + D + \frac{2I_0}{1+\beta I_0}\left[\alpha + \frac{1}{2}A\cos(\Omega t')\right]\xi(t') + 2\sqrt{I_0}\eta(t'), \quad (3)$$

where damping coefficient $\gamma = 2K(\Gamma - K)/\Gamma$, $\langle\varepsilon(t')\rangle = \frac{D}{\gamma}(1 - \mathrm{e}^{-\gamma t'})$.

According to the definition of mean laser intensity correlation function

$$C(t) = \lim_{t'\to\infty} \overline{\langle I(t'+t)I(t')\rangle}$$
$$= \lim_{t'\to\infty} \frac{\Omega}{2\pi}\int_{t'}^{t'+\frac{2\pi}{\Omega}} \langle I(t'+t)I(t')\rangle\,\mathrm{d}t', \quad (4)$$

the mean laser intensity correlation function is got as

$$C(t) = \left(I_0 + \frac{D}{\gamma}\right)^2 + \frac{A^2 I_0^2 Q \mathrm{e}^{-\tau_1^{-1}|t|}}{4\tau_1(1+\beta I_0)^2 k_+ k_-}$$
$$\times[(\gamma^2 - \tau_1^{-2})\cos(\Omega t) + (\gamma + \tau_1^{-1})\Omega\sin(\Omega t)]$$
$$+\frac{4\alpha I_0^{\frac{3}{2}}\lambda\sqrt{DQ}}{\gamma(1+\beta I_0)} + \left(\frac{\alpha I_0}{1+\beta I_0}\right)^2 \frac{2Q}{\tau_1}$$
$$\times\left[\frac{\mathrm{e}^{-\tau_1^{-1}|t|}}{\gamma^2 - \tau_1^{-2}} - \frac{\tau_1^{-1}\mathrm{e}^{-\gamma|t|}}{\gamma(\gamma^2 - \tau_1^{-2})}\right]$$
$$+\frac{2I_0 D}{\tau_2}\left[\frac{\mathrm{e}^{-\tau_2^{-1}|t|}}{\gamma^2 - \tau_2^{-2}} - \frac{\tau_2^{-1}\mathrm{e}^{-\gamma|t|}}{\gamma(\gamma^2 - \tau_2^{-2})}\right],$$

where $k_+ = (\gamma + \tau_1^{-1})^2 + \Omega^2$, $k_- = (\gamma - \tau_1^{-1})^2 + \Omega^2$.

Compared with other ways, the advantage of this method is no restriction to the signal and noise. However calculating the form of integral in mean square is much more complex.

Translating Eq. (4) by Fourier transform, the power spectrum $S(\omega)$ is $S(\omega) = S_1(\omega) + S_2(\omega)$, where $S_1(\omega)$ and $S_2(\omega)$ are the output power spectra of the signal and the noise, respectively. The overall power is defined by

$$P_{\mathrm{s}} = \int_0^\infty S_1(\omega)\mathrm{d}\omega. \quad (5)$$

SNR is defined as the ratio of the output power of signal P_{s} and the mean power of unit noise spectrum at $\omega = \Omega$ (only take its spectrum for positive frequency):

$$R = \frac{P_{\mathrm{s}}}{S_2(\omega = \Omega)}, \quad (6)$$

its concrete form is given by

$$P_{\mathrm{s}} = \frac{\pi A^2 I_0^2 Q[(\gamma^2 - \tau_1^{-2}) - (\gamma + \tau_1^{-1})\Omega]}{2\tau_1(1+\beta I_0)^2 k_+ k_-},$$

$$S_2(\omega = \Omega) = \frac{4I_0^2\alpha^2 Q}{(1+\beta I_0)^2\tau_1^2(\Omega^2+\gamma^2)(\Omega^2+\tau_1^{-2})}$$
$$+\frac{8\alpha I_0^{\frac{3}{2}}\lambda\sqrt{DQ}}{(1+\beta I_0)(\Omega^2+\gamma^2)} + \frac{4I_0 D}{\tau_2^2(\Omega^2+\gamma^2)(\Omega^2+\tau_2^{-2})}.$$

According to Eq. (5) the curve of P_{s} as a function of τ_1 is plotted in Fig. 1(a). From it, we can see that if the value of $\gamma - \tau_1^{-1} - \Omega$ is smaller than zero, P_{s} is negative, at the same time no signal power can be outputted. While under the condition of the value of $\gamma - \tau_1^{-1} - \Omega$ larger than zero, P_{s} is positive and one SR peak occurs, moreover, with the increase of γ, the position of the peak turns left and goes higher.

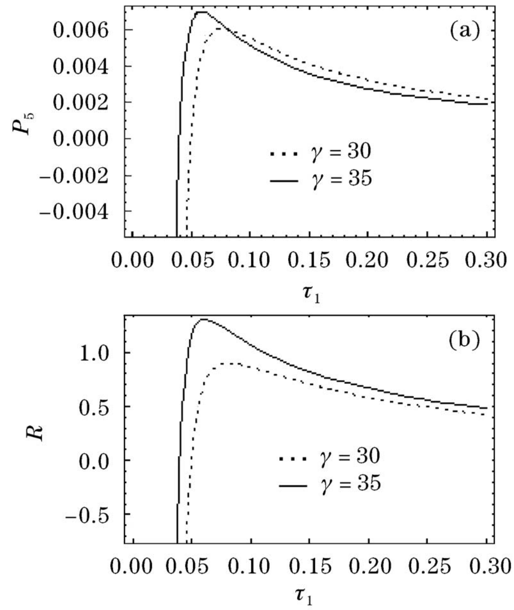

Fig. 1. (a) Signal power P_{s} as a function of the pump noise correlation time τ_1; (b) SNR as a function of the pump noise correlation time τ_1. $\beta = 1$, $A = 1$, $I_0 = 1$, $\Omega = 10$, $\lambda = 0.6$, $\tau_2 = 0.01$, $\alpha = 1$, $Q = 3$, $D = 0.5$.

According to Eq. (6) and taking γ as the parameter, Fig. 1(b) can be got. When the value of $\gamma - \tau_1^{-1} - \Omega$ is smaller than zero, SNR is always negative, also no signal power is outputted. Under the condition of $\gamma - \tau_1^{-1} - \Omega > 0$, as γ increases the same tendency appears as the corresponding part of Fig. 1(a).

For the effect of the bias signal α on SNR, we choose α as the parameter. Figure 2(a) depicts the dependence of

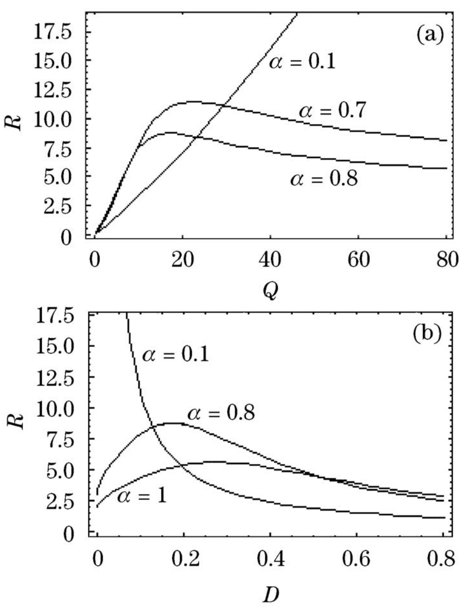

Fig. 2. (a) SNR as a function of the pump noise Q, $D = 0.5$; (b) SNR as a function of the quantum noise D, $Q = 3$. $\beta = 1$, $A = 1$, $I_0 = 1$, $\gamma = 20$, $\Omega = 3$, $\lambda = -0.6$, $\tau_2 = 0.01$, $\tau_1 = 0.3$.

SNR on Q. When α is very small, there is a monotone increasing curve; with the increase of α, SR occurs. Further increasing α leads the position of peak to turn left and the maximum value to go down. Figure 2(b) is the dependence of SNR on D. When α is very small, there is a monotone increasing curve, the increase of α also leads to the occurrence of SR. However further increasing α can move the position of peak right and decrease the maximum.

Adopting the addition form of input signal and noise[10], from both R-Q and R-D curves (see Fig. 2), we can see the occurrence of SR, while R-τ_1 does not. Adopting the multiplying form between periodical signal and noise, from R-τ_1 curve (see Fig. 1(b)), we can find the existance of SR but not from R-Q and R-D curves. In this letter, combining the bias signal and the modulated pump noise, in all the curves of R-Q, R-D, and R-τ_1, SR phenomena can be observed. Here bias signal α plays a role of additive term. When α is very small, the additive term is too weak to produce SR, as α increasing, SR occurs (see Fig. 2).

In conclusion, in the model of bias signal combining modulated pump noise, only when τ_1 is greater than a certain value can the signal power be outputted. It all depends on the values of damping coefficient γ and frequency Ω. SR occurs not only in the SNR upon the change of pump noise autocorrelation time τ_1, but also in the output power P_s upon the change of τ_1. Bias signal α acts as an additive term with the noise. When α is very small, changing the intensities of pump noise and quantum noise, SR does not appear. With increasing α, SR occurs.

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