Image registration by maximization of mutual information based on edge width matching using particle swarm optimization

Xuan Yang (杨 烜)[1] and Jihong Pei (裴继红)[2]

[1]*College of Information Engineering;* [2]*Modern Education Technology and Information Center, Shenzhen University, Shenzhen 518060*

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Mutual information (MI) based image registration has been found to be quite effective in many medical image applications. However, standard MI hampers the convergence of registration transformation parameters since it contains local maxima. In this paper, a novel registration method is proposed. At first, MI based on edge width matching is computed to avoid great change of joint probability distribution and get less local maxima. Particle swarm optimization (PSO), which combines local search methods with global ones balancing exploration and exploitation, is done to search the optimal registration parameter. PSO has less computational complexity as its complex behavior follows only a few simple rules. It could avoid local maxima and reach global optimal results. This method is applicable to a variety of multimodal images, and suitable to different interpolation methods. Theoretical analysis and experiments show that this method is effective and accurate to register multimodal medical images.

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Registration of multimodal medical images is the first important step in successful fusion of multimodal medical images, which produces different information. The mutual information (MI) between two images can be regarded as a statistical tool to measure the degree to which an image can be predicted from the other. It has been found to be an appropriate similarity measure for many multimodal images's registration problems[1,2]. However, the local maxima of MI make it difficult to register images, because the search algorithm will converge on the local maximum easily[3−5]. Ji *et al.*[3] and Tsao[4] analyzed both sampling and interpolation effects of MI. Likar and Pernus[5] proposed a combination of prior and floating information on the joint probability and random re-sampling of one image to improve the registration. Pluim *et al.*[6] combined both standard MI and gradient information to yield a better registration. However, these methods are limited by interpolation methods or not suitable to different multimodal images.

In the MI, the joint probability density function is approximated using joint intensity histogram in general. However, intensity of edges in an image would change greatly, which would reduce the statistical power of the joint intensity histogram and aggravate the local maxima problem.

In order to illustrate the local maxima of MI, we compute the MI of two discrete one-dimensional (1D) edges with different edge widths. Suppose the reference edge is $A(x)$ and the floating edge is $B(x)$. Let each sample length be 1 and the edge length be L, where the slope length of $A(x)$ and $B(x)$ are correspondingly M and N. Suppose two edges are central symmetry and $N < M$. When the center points of the two edges matched, two edges are matched.

When the edge $B(x)$ shifts horizontally rightwards (in the x orientation), $x \leq \frac{M-N}{2}$ is supposed. The joint intensity histogram is obtained by combining the intensity pairs of the overlapping parts of the reference edge and the floating edge[7]. Then

$$H(A) = -\frac{L-M-2x}{2L-2x}\log\frac{L-M-2x}{2L-2x} - \frac{L-M}{2L-2x}\log\frac{L-M}{2L-2x} - \frac{M}{L-x}\log\frac{1}{L-x}, \tag{1}$$

$$H(B) = -\frac{L-N-2x}{2L-2x}\log\frac{L-N-2x}{2L-2x} - \frac{L-N}{2L-2x}\log\frac{L-N}{2L-2x} - \frac{N}{L-x}\log\frac{1}{L-x}, \tag{2}$$

$$H(A,B) = \begin{cases} -\frac{L-M-2x}{2L-2x}\log\frac{L-M-2x}{2L-2x} - \frac{L-M}{2L-2x}\log\frac{L-M}{2L-2x} - \frac{M}{L-x}\log\frac{1}{L-x}, & N < M \\ -\frac{L-M-2x}{L-x}\log\frac{L-M-2x}{2L-2x} - \frac{M+2x}{L-x}\log\frac{1}{L-x}, & N = M \end{cases}, \tag{3}$$

$$\mathrm{MI}_x(A,B) = \begin{cases} -\frac{L-N-2x}{2L-2x}\log\frac{L-N-2x}{2L-2x} - \frac{L-N}{2L-2x}\log\frac{L-N}{2L-2x} - \frac{N}{L-x}\log\frac{1}{L-x}, & N < M \\ -\frac{L-M}{L-x}\log\frac{L-M}{2L-2x} - \frac{M-2x}{L-x}\log\frac{1}{L-x}, & N = M \end{cases}. \tag{4}$$

Suppose $x \ll L-M$, $\mathrm{MI}_x(A,B)$ is approximated to be

$$\mathrm{MI}_x(A,B) \approx \begin{cases} -\log\frac{L-N}{2L} + \frac{N}{L-x}\log\frac{L-N}{2}, & N < M \\ -\frac{L-M}{L-x}\log\frac{L-M}{2} + \frac{L}{L-x}\log\frac{1}{L} - 2\log\frac{1}{L}, & N = M \end{cases}. \tag{5}$$

It can be seen that, for $N < M$, $\mathrm{MI}_x(A,B)$ is a monotonic increase function when edge $B(x)$ shifts horizontally rightwards, which is similar to that of leftwards. That means a local minimum occurring at $x = 0$. We can also prove the same case for $M < N$. The conclusion is that local maxima of MI function would appear when the widths of two edges are different. For $N = M$, $\mathrm{MI}_x(A,B)$ would be approximated to monotonic descending when edge $B(x)$ shifts rightwards, which is similar to that of leftwards. That means a maximum occurring at $x = 0$. The conclusion is local maxima of MI would appear less when the widths of two edges are similar.

Based on the above analysis, we could make a conclusion that, the response of MI to transformation would be smoother when the edge widths of two images are similar. In order to match the edge widths of images, we use smooth function with compact support to blur the edge whose width is sharper.

Suppose the widths of the edges in two images are M and N alternatively. $N < M$ is supposed. The compact set of the two-dimensional (2D) smooth function is defined as $[-W,W]\times[-W,W]$, where $W = M - N + 1$. The smooth function with compact set is used to smooth the sharp edge. Then, the sharp edge would be expanded to match the wide edge. Any smooth function could be used as long as the size of its support set satisfies $W = M - N + 1$.

For most multimodal medical images, edge widths in an image would not change greatly. That means a majority of edge widths are quite similar. So it is not difficult to estimate the edge widths of two images in multimodal medical images. The MI is computed on two images whose edges are matched. Then the response of MI to transformation parameters is smoother than that of standard MI. It can be used to register multimodal medical images. So it is of great benefit to optimization algorithm searching transformation parameters.

Powell's method is a popular optimization method of MI. It repeatedly iterates the dimensions of the search space, performing 1D optimizations for each dimension, until convergence is reached. However, this method could run into local maxima and reach non-optimal results. In this paper, we applied particle swarm optimization (PSO) to the optimization of the MI based on edge width matching.

The PSO algorithm was first introduced by Eberhart and Kennedy[8], which is an extremely simple algorithm that seems to be effective for optimizing a wide range of functions. It began as a simulation of a simplified social environment. Agents were thought of as collision-proof birds, and the original intent was to graphically simulate the graceful but unpredictable choreography of a bird flock.

PSO is another form of evolutionary computation and is stochastic in nature much like genetic algorithms. In contrast to genetic algorithms and evolutionary strategies, which exploit the competitive characteristics of biological evolution, PSO exploits cooperative and social aspects. In PSO, the potential solutions, called particles, fly through the problem space by following the current optimum particles. Each particle keeps track of its coordinates in the problem space which are associated with the best solution (fitness) it has achieved so far. This value is called "pbest". Another "best" value that is tracked by the particle swarm optimizer is the best value, obtained so far by any particle in the neighbors of the particle. This location is called "lbest". When a particle takes all the population as its topological neighbors, the best value is a global best and is called "gbest".

The particle swarm optimization concept consists of, at each time step, changing the velocity of (accelerating) each particle toward its pbest and lbest locations (local version of PSO). Acceleration is weighted by a random term, with separate random numbers being generated for acceleration toward pbest and lbest locations.

In PSO, the ith particle x_i $(i = 1, \cdots, N)$ moves by addition of a velocity vector v_i, which is a function of the best position found by the particle and of the best position found so far among all particles.

$$v_i(k+1) = \phi(k)v_i(k) + \alpha_1\{\gamma_{1i}[p_i - x_i(k)]\} + \alpha_2\{\gamma_{2i}[G - x_i(k)]\}, \tag{6}$$

$$x_i(k+1) = x_i(k) + v_i(k+1), \tag{7}$$

where i is particle index, k is iteration index, $\phi(k)$ is the inertial weight, which is monotonically decreasing function of the iterations index. $\phi(k+1) = \phi(k) + (\phi_{\min} - \phi_{\max})/K$. K is the iteration number. v_i is the velocity of ith particle, x_i is the position of ith particle, p_i is the best position found by ith particle, G is the best position found by swarm, $\gamma_{1,2}$ is the random numbers on the interval [0,1] applied to ith particle, $\alpha_{1,2}$ is acceleration constants.

In order to maximize MI based on edge width matching, we set $\phi_{\max} = 0.9$ and $\phi_{\min} = 0.4$. The iteration number is up to 200. The population size is 20 and acceleration constants are $\alpha_1 = \alpha_2 = 2$. Our search space is three dimensions, one rotation and two translations. $x_i = (t_x, t_y, \theta)$, where t_x and t_y are translation parameter, θ is rotation parameter.

We compare our method with the method based on standard MI, using multimodal medical images, by transforming these images with translation (unit is pixel) and rotation (unit is degree). MR-CBF and MRI-CT image pairs, which are registered already, are experimented with. Original sizes of them are 256×256. MR-CBF is down-sampled as 128×128 and MRI-CT is down-sampled as 64×64. In order to compare the registration accuracy of our method and standard MI method, PSO is used to search the maximum of standard MI. We transformed one image round the known transformation parameters. The registration results and registration error of two methods are shown in Tables 1 and 2. The registration error is defined as $\mathrm{RE} = |t_x - t_x^*| + |t_y - t_y^*| + |\theta - \theta^*|$, where (t_x, t_y, θ) are the registration result parameters corresponding to horizon translation, vertical translation and rotation, (t_x^*, t_y^*, θ^*) are the transformation parameter. Our registration accuracy is one pixel for translation and one degree for rotation. The accuracy of our method is shown to be more robust and more accurate than that of standard MI method because of the pre-process of edge width matching.

Table 1. Registration Results of MR-CBF Using Our Method and Maximization of Standard MI

Transformation Parameters	Our Method	MI	Error of Our Method	Error of MI
(0, 4, 7)	(0, 6, 7)	(0, 9, 11)	2	9
(−2, 0, 5)	(−2, 2, 5)	(−2, 5, 8)	2	8
(5, 5, 0)	(5, 6, 0)	(2, 9, 6)	1	13
(5, −3, −5)	(5, 0, −5)	(3, 2, −1)	4	12

Table 2. Registration Results of MRI-CT Using Our Method and Maximization of Standard MI

Transformation Parameters	Our Method	MI	Error of Our Method	Error of MI
(0, 4, 7)	(1, 3, 5)	(0, 2, 17)	4	12
(−2, 0, 5)	(−2, 0, 8)	(−3, 2, 20*)	3	18
(5, 5, 0)	(4, 4, 1)	(1, 5, 6)	3	10
(5, −3, −6)	(4, −3, −6)	(3, −1, 20*)	1	30

* The registration result is out of the search range.

Moreover, great deals of multimodal medical images have been used to test our method. Experiments show that in rigid registration, where traditional MI method does well, our method works well too. And the registration accuracy of our method is similar to that of standard MI method. While standard MI method performs worse, our method still works well and the registration accuracy is much better than that of standard MI method.

MI has developed into an accurate measure for multimodal medical image registration. But local extrema impede the registration optimization process and rule out the subpixel accuracy. We proposed a novel registration method based on edge width matching to improve the registration accuracy of multimodal medical image. After preprocessed the images by a smooth function to blur the image whose edge widths are sharper, the MI is computed on two images whose edge widths are similar. PSO is used to search the maximum of the MI based on edge width matching. Theoretical analysis and experiments show that the registration accuracy of our method is better than that of standard MI method. PSO is feasible to search the optimal transformation parameters of multimodal medical images.

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