MAP-based infrared image expansion

Nan Zhang (张 楠), Weiqi Jin (金伟其), Binghua Su (苏秉华), Yangyang Liu (刘扬阳), and Hua Chen (陈 华)

Department of Optical Engineering, School of Information Science and Technology, Beijing Institute of Technology, Beijing 100081

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Image expansion plays a very important role in image analysis. Common methods of image expansion, such as the zero-order hold method, may generate a visual mosaic to the expanded image, linear and cubic spline interpolation may blur the image data at peripheral regions. Since infrared images have the characteristics of low contrast and low signal-to-noise ratio (SNR), the expanded images derived from common methods are not satisfactory. As shown in the analysis of the course from images with low resolution to those with high resolution, the expansion of image is found to be an ill-posed inverse problem. An image interpolation algorithm based on MAP estimation under Bayesian framework is proposed in this paper, which can effectively preserve the discontinuities in the original image. Experimental results demonstrate that the expanded images by this method are visually and quantitatively (analyzed by using the criteria of mean squared error (MSE) and mean absolute error (MAE)) superior to the images expanded by common methods of linear interpolation. Even in expansion of infrared images, this method can also give good results. An analysis about choosing regularization parameter α in this algorithm is given.

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Image expansion, as we know, is one of image processing algorithms that acquire new pixels from the original image by a certain method, so as to expand the size (pixel numbers) of the image. Image expansion is applicable in many aspects of image processing. To generate precise maps, cartographers must expand small regions of satellite image data. In medical imaging, computerized tomography and X-rays need to be zoomed locally to find out foci of anomalies. Reconnaissance photographs must be locally expanded to show accurately hidden details of military targets. However, owing to limitations of image resolution, especially current wide application of digital images, common methods of image expansion tend to smooth out details of such original images, and may even generate a visual mosaic, thus deteriorate the quality of the observed images. Hence the researches on effective processing for image expansion are widely concerned.

The methods of image expansion commonly used are zero-order hold method, linear and cubic spline interpolation[1,2]. The zero-order hold method is the most basic interpolation method, in which relevant pixels in neighborhood are produced by replicating single pixels, and a visual mosaic effect might easily be generated in the expanded image. The smoothing effect of linear and cubic spline interpolation methods may effectively overcome the visual mosaic, but the discontinuous regions in the image data will also be smoothed out, thus blurring the expanded images. Since the infrared images have the characteristics of low signal-to-noise ratio (SNR) and low contrast, expansions made by the above methods are not satisfactory at all.

Image expansion is an ill-posed inverse problem because there may exist virtually infinite sets of expanded images from the original date. Bayesian statistical framework is an effective method in solving the ill-posed problem. An image interpolation expansion algorithm based on MAP estimation is presented in this paper, which can preserve the original data at edges and suppress noises simultaneously.

In order to understand how the inverse problem is solved, it is necessary to examine first the process of low-resolution image acquisition[3]. Let the function $z(x,y,t)$ represent the continuous image of sceneries projected onto a sensor plane of detector. Figure 1 shows that the low-resolution image sensor plane is comprised of $N_1 \times N_2$ sensor elements with each element dimension of $\omega \times \omega$. The output of each sensor element, or the pixel value at each point, is directly proportional to the incident light intensity on the charge coupled device (CCD) sensor. Let $\{y_{ij}\}$ $(i = 0, \cdots, N_1 - 1; j = 0, \cdots, N_2 - 1)$ be the set of low-resolution sensor output values from the image, where y_{ij} is the summation of light intensity received by the (i,j)th sensor element during each integral period, represented as

$$y_{ij} = \int_0^1 \int_{\omega i}^{\omega(i+1)} \int_{\omega j}^{\omega(j+1)} z(x,y,t) \mathrm{d}x \mathrm{d}y \mathrm{d}t. \quad (1)$$

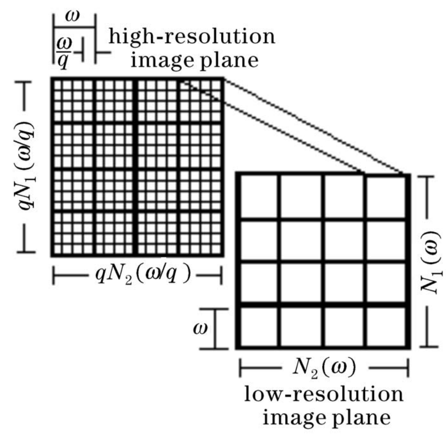

Fig. 1. Image model of high-resolution to low-resolution.

For acquiring a higher resolution image, a finer grid is used by dividing each of the original low-resolution sensors into q^2 fine sensor elements. This high-resolution image will consist of $qN_1 \times qN_2$ pixel elements. Let this set be represented as $\{z_{kl}\}$ $(k = 0, \cdots, qN_1 - 1; l = 0, \cdots, qN_2 - 1)$. The value of each pixel element is given by

$$z_{kl} = \int_0^{q^2} \int_{\frac{\omega}{q}k}^{\frac{\omega}{q}(k+1)} \int_{\frac{\omega}{q}l}^{\frac{\omega}{q}(l+1)} z(x, y, t) \mathrm{d}x \mathrm{d}y \mathrm{d}t. \quad (2)$$

The integration limits over time have been changed from $[0, 1]$ to $[0, q^2]$ to maintain the same relative intensity value received for each of the image pixels.

The problem addressed in this paper is to estimate the high-resolution image $\{z_{kl}\}$, from the low-resolution image $\{y_{ij}\}$. The forward process of obtaining $\{y_{ij}\}$ from $\{z_{kl}\}$ is given by

$$y_{ij} = \frac{1}{q^2} \sum_{k=qi}^{q(i+1)-1} \sum_{l=qj}^{q(j+1)-1} z_{kl}. \quad (3)$$

Thus, from Eq. (3), the low-resolution pixel value y_{ij} is the average value of the high-resolution image pixels located within neighborhood N_{ij}.

Define y as the $N_1N_2 \times 1$ lexicographically ordered vector that contains pixel values from low-resolution image. Let z be the $q^2N_1N_2 \times 1$ vector containing pixel values expanded from the high-resolution image. The order of arrangement of pixels in z will be derived lexicographically and sequentially from pixel elements in each $q \times q$ neighborhood corresponding to y. Equation (3) may be written in matrix form as

$$y = Dz + n, \quad (4)$$

where n represents the white Gaussian noise of zero average value, and D is a constant $N_1N_2 \times q^2N_1N_2$ matrix with the form

$$D = \frac{1}{q^2} \begin{bmatrix} \overbrace{11\cdots1}^{q^2} & & & 0 \\ & \overbrace{11\cdots1}^{q^2} & & \\ & & \ddots & \\ 0 & & & \overbrace{11\cdots1}^{q^2} \end{bmatrix}. \quad (5)$$

This matrix contains q^2, $1/q^2$ in each row, corresponding to the number of pixels in each fine neighborhood.

To estimate high-resolution image z from low-resolution image y, a MAP technique is proposed. According to Bayesian analysis theory, given the low-resolution image y, the probability of high-resolution image z may be written as[4]

$$P(z/y) = P(y/z)P(z)/P(y). \quad (6)$$

MAP estimation of the high-resolution image z is done by maximizing the conditional probability density function $P(z/y)$, shown as

$$\hat{z} = \arg \max_z [P(z/y)]. \quad (7)$$

Both sides of Eq. (6) are taken logarithm operations. Since $P(y)$ is not dependent on z, it can be eliminated. Also considering the Markov property in the original image (i.e. every pixel merely has relations with its neighborhood pixels), we assume that the high-resolution image follows Markov distribution. So the optimization in Eq. (7) can be computed by

$$\hat{z} = \arg \max_z [\ln P(y/z) + \ln P(z) - \alpha U(z)], \quad (8)$$

where $U(z)$ is the energy function in the Gibbs distribution[4], and α is the regularization parameter which balances the weights of $P(z/y)$ and the energy function $U(z)$.

Since the photon detection and conversion on image plane is an independent Poisson random process, we suppose that the low-resolution image y follows Poisson distribution, and its probability density function may be given by

$$P(y/z) = \prod_{k=1}^{N} \frac{(\bar{y}_k)^{y_k} \exp(-\bar{y}_k)}{y_k!}, \quad (9)$$

where $N = N_1N_2$, $\bar{y}_k = \sum_{i=1}^{q^2N} D_{ki} z_i$.

Similarly, the high-resolution image z follows Poisson distribution too; its probability density function may be given by

$$P(z) = \prod_{s=1}^{q^2N} \frac{(\bar{z}_s)^{z_s} \exp(-\bar{z}_s)}{z_s!}. \quad (10)$$

Using Eqs. (9) and (10) we get

$$\ln P(y/z) + \ln P(z) = \sum_{k=1}^{N} [y_k \ln \bar{y}_k - \bar{y}_k - \ln(y_k!)] + \sum_{s=1}^{q^2N} [z_s \ln \bar{z}_s - \bar{z}_s - \ln(z_s!)]. \quad (11)$$

The first and second terms in the right side of Eq. (8) are replaced with the result of Eq. (11). Since $\ln(z_s!) \approx z_s \ln z_s - z_s$, $\bar{z}_s$ and $\ln(y_k!)$ are constants relative to z, we now differentiate Eq. (8) with respect to z_i. We have

$$\sum_{k=1}^{N} \left\{ y_k D_{ki} \Big/ \sum_{j=1}^{q^2N} D_{kj} z_j - D_{ki} \right\} + \ln \bar{z}_i - \ln z_i - \alpha \frac{\partial U(z)}{\partial z_i}. \quad (12)$$

Let expression (12) equal to zero, we get

$$z_i = \bar{z}_i \exp \left\{ \sum_{k=1}^{N} \left[y_k \Big/ \sum_{j=1}^{q^2N} D_{kj} z_j - 1 \right] D_{ki} - \alpha \frac{\partial U(z)}{\partial z_i} \right\}, \quad (13)$$

where $i = 1, \cdots, q^2N$, and z is the MAP estimate of the image. Since Eq. (13) is difficult to be directly calculated, we usually solve it in an iterative scheme. The iterative expression of Eq. (13) is

$$z_i^{n+1} = z_i^n \times \exp\left\{\sum_{k=1}^{N}\left[y_k \Big/ \sum_{j=1}^{q^2N} D_{kj} z_j^n - 1\right] D_{ki} - \alpha \frac{\partial U(z^n)}{\partial z_i}\right\}, \quad (14)$$

where n is the iteration number. The initial estimate of z, z^0, is the zero-order hold expansion of the low-resolution image y.

The mean squared error (MSE) and the mean absolute error (MAE) are used to compare the various methods of expansion. The MSE and the MAE used in this paper are defined as

$$\text{MSE} = \frac{\sum_{k=0}^{qN_1-1}\sum_{l=0}^{qN_2-1}(\hat{z}_{kl} - z_{kl})^2}{\sum_{k=0}^{qN_1-1}\sum_{l=0}^{qN_2-1}(z_{kl})^2},$$

$$\text{MAE} = \frac{\sum_{k=0}^{qN_1-1}\sum_{l=0}^{qN_2-1}|\hat{z}_{kl} - z_{kl}|}{\sum_{k=0}^{qN_1-1}\sum_{l=0}^{qN_2-1}|z_{kl}|}. \quad (15)$$

The lower the values of MSE and MAE are, the closer to the original image the expanded image will be.

The original image (128 × 128 pixels) in Fig. 2(a) was reduced in size by a factor of $q = 4$ by Eq. (3) (i.e., obtaining the average per $4 \times 4 = 16$ pixels). Then three expansion methods, including zero-order hold, bilinear interpolation, and the proposed MAP estimation method in this paper, were used to expand the reduced image back to its initial size so that the expansions could be compared to the original image. Figures 2(b), (c) and (d) shows the images that resulted from the three expansion methods. The comparison of these methods is given in Table 1. It is easy to see that our method could give a more accurate result.

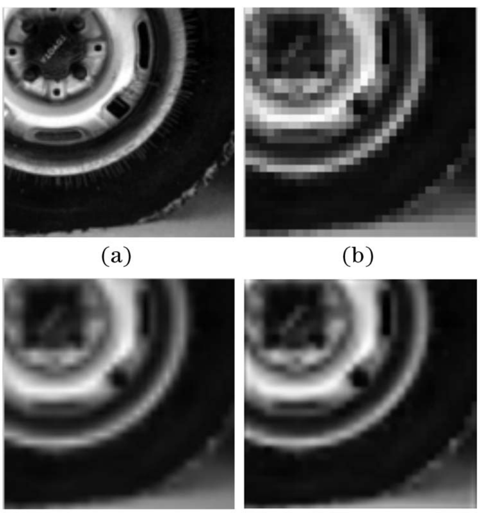

Fig. 2. Original image (a) and expanded image of zero-order hold method (b), bilinear interpolation (c), and our method (d).

Table 1. Comparison of Expansion Methods

Method of Expansion	MSE	MAE
Zero-Order Hold	0.0509	0.1870
Bilinear	0.0447	0.1886
Proposed Method	0.0367	0.1566

Fig. 3. Original infrared image.

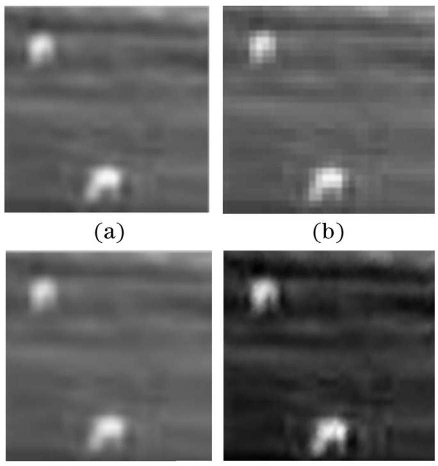

Fig. 4. Expanded images of one original infrared image. (a) Photoshop; (b) zero-order hold; (c) bilinear interpolation; (d) our method.

Figure 3 shows an original infrared image. The proposed MAP estimation method was compared with other common expansion methods, including expansion by Photoshop software, zero-order hold, and bilinear interpolation (see in Fig. 4). Each of the methods was used to expand the broken-line region (32×32) in the infrared image to four times of its size (128 × 128). The MAP method was found to obtain the best visual evaluation.

The problem of image expansion has been shown to be an ill-posed inverse problem. A MAP-based image expansion method is proposed in this paper to resolve the problem, which is a new nonlinear interpolation algorithm for image expansion. Experimental results indicate that the proposed algorithm outperforms conventional algorithms (such as zero-order hold, bilinear interpolation, and so on) both in visual and quantitative analysis. When infrared images are expanded by common methods, the blurred object edges that originally present in infrared images will become further blurred. While in our method, a satisfactory result is obtained. Not only the image data on the edges are preserved, but also the noises are efficiently controlled.

Experiments demonstrate that regularization parameter α is a factor affecting the quality of expanded images most. If α is set too large, the image appears rather blurry due to over smoothing. While, if α is set too small, the image estimation procedure tends to remain in the state of the zero-order hold initialization, resulting in a mosaic effect in the images. The value α is dependent on the quantity and magnitude of the edges within the constrained image. In this paper, α value is chosen through experiments. Experiments are run over a wide range of α values, and the best expanded image is determined by the lowest MSE and MAE. To simplify the process of estimating the regularization parameter α and achieve optimality, next focus of our research will be to find an auto-adaptive algorithm for choosing α.

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