

# Spectral anomalies of polychromatic, spatially coherent light diffracted by an annular aperture in the far field

Xiandai Hu (胡现代) and Jixiong Pu (蒲继雄)

Department of Electronic Science and Technology, Huaqiao University, Quanzhou 362021

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The spectral behavior of polychromatic spatially fully coherent light diffracted by an annular aperture in the far field is studied. It is shown that the spectrum in the far field is generally different from that at the aperture, i.e., the spectrum in the far field is proportional to the spectrum at the aperture and a spectral modifier, which depends on the central obstruction ratio  $\varepsilon$  and diffraction angle  $\theta$ . Detailed numerical calculation results indicate that significant spectral changes take place in the vicinity of zeros of the Airy pattern. It is found that at the critical diffraction angle  $\theta_c$ , the spectrum is split into two lines, while at a diffraction angle a little smaller than  $\theta_c$ , the spectrum is red-shifted, and at a diffraction angle a little larger than  $\theta_c$ , the spectrum is blue-shifted. The influence of the central obstruction ratio on the spectral anomalies is presented.

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In recent years, a great deal of attention has been paid to the structure of wave fields in the neighborhood of points where the field amplitude has zero value<sup>[1-3]</sup>. At such points the phase of fields is singular. It has been known that in the neighborhood of the singular points, the wave fields exhibit a rather complex structure, for example, dislocations and optical vortices. Studies of phenomena associated with phase singularities have gradually developed into a new branch of physical optics, sometimes referred to as singular optics<sup>[1]</sup>.

Most of the publications concerned with singular optics dealt with monochromatic waves. More recently, singular optics has been extended to polychromatic cases by Wolf *et al.*<sup>[4,5]</sup>, they have shown that drastic spectral changes occur in the vicinity of intensity zeros in the focal region of polychromatic, spatially coherent, converging spherical waves, that is, the spectrum is red-shifted at some points, blue-shifted at some others, and split into two lines elsewhere. Similar phenomena of drastic spectral changes were also found in Young's double-slit interference experiments etc.<sup>[6-11]</sup>. These drastic spectral changes in the vicinity of singular points are shown to have close relation to spectral switches<sup>[12,13]</sup>, and the relation between the drastic spectral change in the vicinity of singular points and the spectral switches were studied in Ref. [14].

In this letter, we investigate Fraunhofer diffraction pattern of a polychromatic, spatially coherent wave diffracted by an annular aperture instead of the circular aperture before<sup>[6,10]</sup>. The spectral behaviors in the vicinity of the dark rings of Fraunhofer diffraction pattern are investigated. It is demonstrated that the spectral anomalies also take place in the vicinity of the dark rings. The effect of the parameters of annular aperture on the spectral changes is studied in detail.

Suppose that a spatially fully coherent polychromatic plane wave  $E(\mathbf{r}', \omega)$  which is written as  $E(\mathbf{r}', \omega) = A(\omega)$  is incident upon an annular aperture, with the inner radius  $b$  and the outer radius  $a$ , at the  $z = 0$  plane, as shown in Fig. 1. The spectral density function of the incident

polychromatic plane wave is

$$W^{(0)}(\mathbf{r}', z = 0, \omega) = E^*(\mathbf{r}', \omega)E(\mathbf{r}', \omega) = A^*(\omega)A(\omega) = S^{(0)}(\omega), \quad (1)$$

where  $\mathbf{r}'$  denotes the position vector of point  $P$  at the  $z = 0$  plane,  $S^{(0)}(\omega)$  is the original spectrum in the aperture,  $\omega$  is angular frequency, the asterisk denotes the complex conjugate.

According to the Collins formula<sup>[15]</sup>, we obtain the optical field at the point  $Q$

$$E(r, \varphi, z, \omega) = -\frac{ik}{2\pi z} \int_b^a \int_0^{2\pi} E(\mathbf{r}', \omega) \times \exp\left[\frac{ik}{2z}(r'^2 + r^2 - 2r' \cdot r \cos(\varphi' - \varphi))\right] dr' d\varphi', \quad (2)$$

here  $k = 2\pi/\lambda = \omega/c$  is the wave number associated with angular frequency  $\omega$ ,  $c$  is the velocity of light in vacuum.

The spectral density function  $S(r, \varphi, z, \omega)$  of the light at point  $Q$  can be given by

$$S(r, \varphi, z, \omega) = E^*(r, \varphi, z, \omega)E(r, \varphi, z, \omega). \quad (3)$$

In this paper, we only study the spectral anomalies in the Fraunhofer diffraction pattern. In this case, substituting Eq. (2) into Eq. (3) and after some algebras

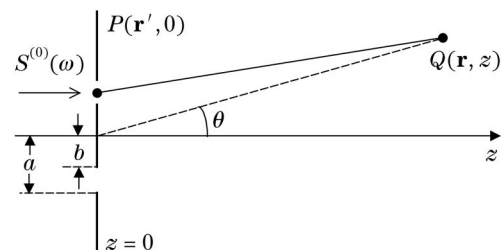


Fig. 1. Notation relating to the aperture geometry.

calculation, we obtain

$$S(\theta, z, \omega) = S^{(0)}(\omega)M(\theta, z, \omega), \quad (4)$$

with

$$M(\theta, z, \omega) = \left(\frac{a}{z\sin\theta}\right)^2 [J_1(ka\sin\theta) - \varepsilon J_1(ka\varepsilon\sin\theta)]^2, \quad (5)$$

where  $\varepsilon = \frac{b}{a}$  ( $1 > \varepsilon \geq 0$ ) is called the central obstruction ratio of the annular aperture,  $J_1(x)$  is the Bessel function of the first kind and of the first order.  $M(\theta, z, \omega)$  is called the spectral modifier which describes the modification of  $S^{(0)}(\omega)$  by the aperture diffraction. It follows from Eqs. (3)–(5) that the spectrum of diffracted, spatially fully coherent polychromatic plane beams in the far field depends on the original spectrum  $S^{(0)}(\omega)$  and spectral modifier  $M(\theta, z, \omega)$ , and the latter is related to the diffraction angle  $\theta$  and the central obstruction ratio  $\varepsilon$  of the annular aperture.

As we only discuss spectral behaviors of Fraunhofer diffraction, the observed point should be located in the far field, therefore the diffraction angle can be approximated as

$$\theta \approx \sin\theta = \frac{r}{z}, \quad (6)$$

here  $r$  is the distance of the observed point  $Q$  from the optical axis. Based on Eq. (6), Eq. (5) can be reduced to

$$M(\theta, z, \omega) = \left(\frac{a}{z\theta}\right)^2 [J_1(ka\theta) - \varepsilon J_1(ka\varepsilon\theta)]^2. \quad (7)$$

It is assumed that the original spectrum is a Gaussian distribution centered at frequency  $\omega_0$  and width  $\sigma_0$ , i.e.,

$$S^{(0)}(\omega) = S_0 \exp\left[-\frac{(\omega - \omega_0)^2}{2\sigma_0^2}\right], \quad (8)$$

where  $S_0$  is a constant. Substituting Eq. (8) into Eq. (4) yields

$$S(\theta, z, \omega) = S_0 \exp\left[-\frac{(\omega - \omega_0)^2}{2\sigma_0^2}\right] M(\theta, z, \omega). \quad (9)$$

The total intensity at the point  $Q(\theta, z)$  is expressed as

$$I(\theta, z) = \int_0^\infty S(\theta, z, \omega) d\omega. \quad (10)$$

Thus, the intensity distribution in the far field is obtained from Eqs. (9) and (10). Here a numerical example is shown in Fig. 2. It is seen that in the region  $0 \leq \theta \leq 3.50 \times 10^{-3}$ , there are three dark rings for  $\varepsilon = 0$ , at which the normalized intensity is zero, that is to say, these points are just the singular points. These three dark rings are located at  $\theta = 1.15 \times 10^{-3}$ ,  $2.105 \times 10^{-3}$ , and  $3.053 \times 10^{-3}$ , representing the first-, second-, and third-order dark Airy rings, respectively. With the increase of the central obstruction ratio  $\varepsilon$ , the radius for the corresponding order dark Airy ring decreases, and the normalized intensity of the second peak increases. For example, in the case of  $\varepsilon = 0$ , the normalized intensity of the second peak is about 0.017, but in the cases of  $\varepsilon = 0.6$  and  $0.9$ , the normalized intensities of the second

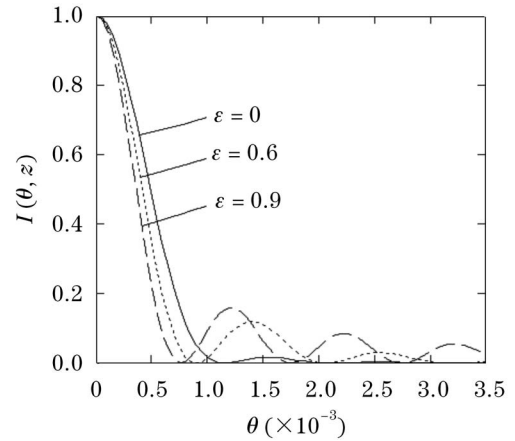


Fig. 2. Normalized intensity distribution in the far field versus diffraction angle  $\theta$  with different central obstruction ratios  $\varepsilon$ .  $\omega_0 = 1 \times 10^{15} \text{ s}^{-1}$ ,  $\sigma_0 = 0.01 \times 10^{15} \text{ s}^{-1}$ .

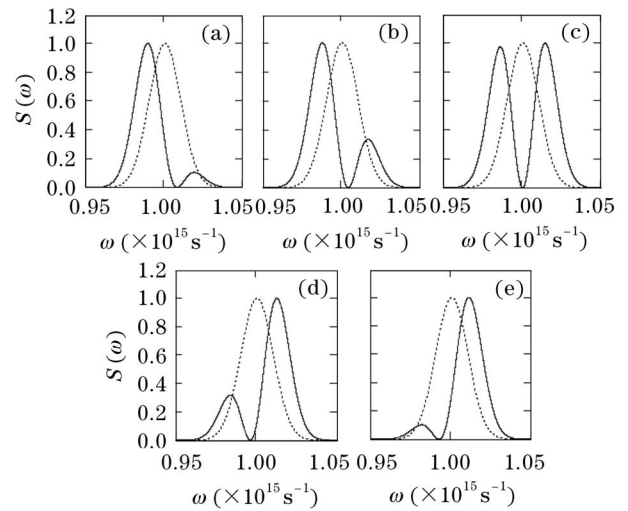


Fig. 3. Normalized spectra vary with diffraction angle  $\theta$ ,  $\omega_0 = 1 \times 10^{15} \text{ s}^{-1}$ ,  $\sigma_0 = 0.01 \times 10^{15} \text{ s}^{-1}$ , and  $\varepsilon = 0.9$ . (a)  $\theta = 0.7531 \times 10^{-3}$ ; (b)  $\theta = 0.7561 \times 10^{-3}$ ; (c)  $\theta = 0.7591 \times 10^{-3}$ ; (d)  $\theta = 0.7621 \times 10^{-3}$ ; (e)  $\theta = 0.7651 \times 10^{-3}$ . The dashed curves indicate the original spectra  $S^{(0)}(\omega)$ , and the solid curves indicate the spectra  $S(\omega)$  in the diffraction field.

peak are 0.12 and 0.16, respectively.

In Fig. 3, we give spectral changes in the vicinity of the first-order dark Airy ring ( $\theta = 0.7591 \times 10^{-3}$ ) for  $\varepsilon = 0.9$ ,  $\omega_0 = 1 \times 10^{15} \text{ s}^{-1}$ . As shown in Fig. 3(a), compared with the original spectrum, the observed spectrum at the diffraction angle  $\theta = 0.7531 \times 10^{-3}$  is split into two peaks, i.e., the maximum and the secondary peaks. To characterize the spectral changes, we define the relative spectral shift as<sup>[12]</sup>

$$\frac{\Delta\omega}{\omega_0} = \frac{\omega_m - \omega_0}{\omega_0}, \quad (11)$$

where  $\omega_m$  stands for the frequency at which the spectrum takes its maximum. When  $\Delta\omega/\omega_0 > 0$ , the spectrum is called blue-shifted, whereas  $\Delta\omega/\omega_0 < 0$  the spectrum is red-shifted. Based on this definition, the spectrum in Fig. 3(a) ( $\theta = 0.7531 \times 10^{-3}$ ) is red-shifted. When  $\theta = 0.7561 \times 10^{-3}$  (see Fig. 3(b)), the spectrum is also red-shifted. It is also shown that the spectrum is split

into two lines with nearly equal height at the critical diffraction angle  $\theta_c = 0.7591 \times 10^{-3}$  (Fig. 3(c)), sometimes this phenomenon is called a spectral switch<sup>[12,13]</sup>. This diffraction angle corresponds to the first-order dark Airy ring, at which the intensity for  $\omega_0$  component is zero, that is, the position of phase singularities for frequency  $\omega_0$  component. It is readily found that when  $\theta = 0.7621 \times 10^{-3}$  (Fig. 3(d)) or  $0.7651 \times 10^{-3}$  (Fig. 3(e)), the spectrum is blue-shifted.

The dependence of the normalized spectrum  $S(\omega) = S(\theta, z, \omega)/S_{\max}(\theta, z, \omega)$  ( $S_{\max}(\theta, z, \omega)$  is the maximum spectrum) on both diffraction angle  $\theta$  and frequency  $\omega$  is given in Fig. 4 with  $\theta$  in the region from  $0.7471 \times 10^{-3}$  to  $0.7711 \times 10^{-3}$ . From Fig. 4, we can find that when  $\theta = 0.7471 \times 10^{-3}$ , the spectrum is red-shifted, and the second peak is very small. With the increment of  $\theta$ , the second peak increases, and at  $\theta_c = 0.7591 \times 10^{-3}$ , two peaks are nearly the same. When  $\theta$  continues to increase, the second peak turns into the maximum peak, i.e., in this case, the spectrum is changed into blue-shift.

The relative spectral shift  $\Delta\omega/\omega_0$  versus diffraction angle  $\theta$  in the region of  $0 \leq \theta \leq 3.5 \times 10^{-3}$  for  $\varepsilon = 0.9, 0.6,$  and  $0$  is given in Fig. 5. It is shown that, for  $\varepsilon = 0.9$  the drastic changes of the spectral shift occur at three diffraction angles  $\theta = 0.7591 \times 10^{-3}, 1.742 \times 10^{-3},$  and  $2.731 \times 10^{-3}$ , respectively. They just indicate the diffraction angles at which the first-, second-, and third-order dark Airy rings occur. These diffraction angles are also the angles at which spectral switches take place. With the decrease of  $\varepsilon$ , the corresponding angle at which spectral anomalies take place increases. To characterize the spectral anomalies further, we introduce the mean frequency, which is defined as

$$\bar{\omega}(\theta, z) = \frac{\int \omega S(\theta, z, \omega) d\omega}{\int S(\theta, z, \omega) d\omega}. \tag{12}$$

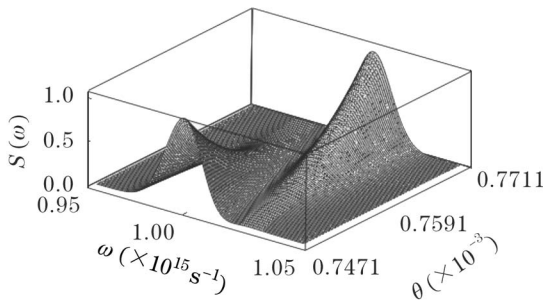


Fig. 4. Normalized spectrum  $S(\omega)$  as a function of  $\omega$  and  $\theta$  in the region of  $0.7471 \times 10^{-3} \leq \theta \leq 0.7711 \times 10^{-3}$ . The other calculation parameters are the same as those in Fig. 3.

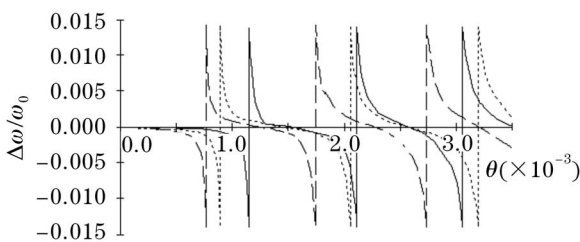


Fig. 5. Normalized spectral shift  $\Delta\omega/\omega_0$  as a function of the diffraction angle  $\theta$  for three different values of  $\varepsilon$ :  $\varepsilon = 0$  (solid curves),  $0.6$  (dotted curves), and  $0.9$  (dashed curves).

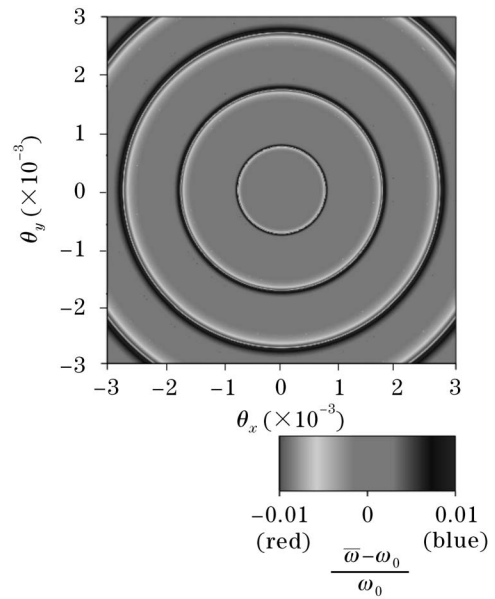


Fig. 6. Colour-coded plot of the mean frequency  $\bar{\omega}$  of the spectrum in the far field for  $\varepsilon = 0.9, \omega_0 = 1 \times 10^{15} \text{ s}^{-1},$  and  $\sigma_0 = 0.01 \times 10^{15} \text{ s}^{-1}$ .

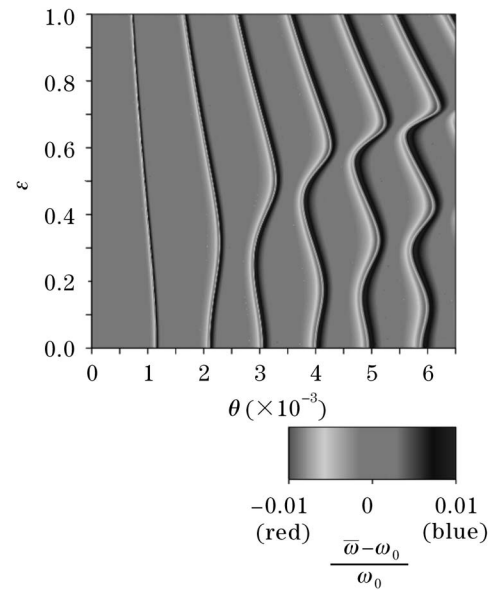


Fig. 7. Colour-coded plot of the mean frequency  $\bar{\omega}$  of the spectrum in the far field as a function of the diffraction angle  $\theta$  and the central obstruction ratio  $\varepsilon$  of the annular aperture.

In Fig. 6, the mean frequency of the spectrum for  $\varepsilon = 0.9, \omega_0 = 1 \times 10^{15} \text{ s}^{-1},$  and  $\sigma_0 = 0.01 \times 10^{15} \text{ s}^{-1}$  in the far field is plotted as a function of transverse coordinates  $\theta_x, \theta_y$ . The ‘colour’ in Fig. 6 is more ‘red’ and more ‘blue’ as the spectrum is more red-shifted and more blue-shifted, respectively. It is seen that when the diffraction angle  $\theta = (\theta_x^2 + \theta_y^2)^{1/2}$  approaches  $0.75911 \times 10^{-3},$  or  $1.742 \times 10^{-3},$  or  $2.731 \times 10^{-3}$ , there exist spectral anomalies, that is, the spectral shift is changed suddenly from red-shift to blue-shift. It is also found that at the first-order dark Airy ring, the spectral anomaly occurs in the very narrow zone, and the higher order of dark Airy ring is, the larger zone at which the spectral anomaly occurs.

Figure 7 shows the mean frequency of the spectrum at the diffraction field related to the central obstruction ratio  $\varepsilon$  and the diffraction angle  $\theta$ . It is shown that, for the first-order dark Airy ring, namely, the position of the critical diffraction angle  $\theta_c$ , the increase of  $\varepsilon$  results in the monotonous decrease of the critical diffraction angle  $\theta_c$  at which the spectrum is anomalous. However, for the second order and higher order dark Airy rings, the relation between the critical diffraction angle  $\theta_c$  and  $\varepsilon$  is not monotonous any more.

In conclusion, we have studied the spectral behavior of spatially fully coherent, polychromatic plane waves diffracted by an annular aperture in the far field. It has been shown that spectral anomalies also take place in the vicinity of zeros of the Airy pattern (the corresponding diffraction angle is  $\theta_c$ ), i.e., the spectral shift exhibits a rapid transition at the critical diffraction angle  $\theta_c$ . This phenomenon is called a spectral switch. It has been shown that the critical diffraction angle  $\theta_c$  at which the spectral anomalies occur is dependent on the central obstruction ratio  $\varepsilon$ . The spectral switch has potential applications in optical interlink and optical communication<sup>[16,17]</sup>.

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