Fidelity of quantum state for interacting system of light field and atomic Bose-Einstein condensate

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The evolution characteristics of quantum state fidelity in an interacting system of single-mode light field and atomic Bose-Einstein condensate have been studied and the influence of the initial light field intensity and the interaction among atoms of Bose-Einstein condensate on the quantum state fidelity respectively have been discussed.

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The realization of atomic Bose-Einstein condensation and the obtainment of coherent atomic beam coupling-output from atomic Bose-Einstein condensate (BEC), namely “the atomic laser”, were important breakthroughs in the field of physics towards the end of 20th century and they opened the new research field of coherent atoms. In July 1995, a group headed by Wieman and Cornell of American National Institute of Standard and Technology (NIST) and Joint Institute of Laboratory Astrophysics (JILA) of Colorado University carried out BEC in the vapor of metallic rubidium[1] and later, in November of this year, another group headed by Ketterle, Massachusetts Institute of Technology, reported the BEC results from natrium atoms[2]. On October 9, 2001 the above mentioned three scientists were awarded the 2001 Nobel Prize in physics for the outstanding contribution to the experimental research of BEC. In recent years, a large of experimental and theoretical researches have focused on the generation of BEC, its unique features and the interaction between atomic BEC and light field have been carried out with great interest, and a series of important results have been achieved[3−14]. As is well known, the interaction between light field and atoms plays an important role in atomic cooling and the preparation and detection of BEC. Thus a further exploration of BEC’s optical properties is not only favorable to research BEC’s intrinsic characteristics but also provides effective methods for its preparation and detection.

Fidelity is an important concept in the field of quantum optics and information science, which expresses the degree of closeness to the original state of information in the transmission. It has been adopted broadly as an important physical parameter in quantum communication and quantum calculation theories in recent years[15−18]. The carrier of information in quantum communication, quantum computer, and quantum cryptography is quantum state. Because the transmission of quantum state is involved in quantum computer, the fidelity must be taken into account in the transmission of quantum state. The quantum information is not wholly reflected by its entropy for the quantum system and the density operator interval is better than the entropy in quantum theory to provide more related information about quantum state[19]. Besides that, the density operator interval is equivalent to the fidelity in describing the connection and the entanglement of information. Based on the Ref. [9], this paper is dedicated to the evolution characteristics of the interacting system consisting of single-mode light field and two-level atomic BEC and the influences of the light field intensity and the interaction among BEC’ atoms on the fidelity of quantum state for the system. The light field and the atomic BEC are respectively discussed.

Taking into account the interaction among atoms, the Hamiltonian of the interaction system of light field and atomic BEC is[9]

$$H = \omega_0 b_1^+ b_1 + \omega a^+ a + \varepsilon \left(a^+ b_0^+ b_1 + a b_0 b_1^+ \right) + \Omega \left(b_0^+ b_0^+ b_0 b_0 + b_0^+ b_1^+ b_0 b_1 + b_1^+ b_0^+ b_1 b_0 + b_1^+ b_1^+ b_1 b_1 \right), \tag{1}$$

where a^+ and a denote the creation and annihilation operators of the photons of light field respectively, b_0^+ and b_0 as well as b_1^+ and b_1 denote the creation and annihilation operators for the atoms in ground state as well as in excited state respectively, ω and ω_0 denote the circular frequencies of the light field and the atomic transition respectively, ε and Ω denote the interaction intensity between light field and atoms and among BEC’s atoms respectively.

Here we only discuss the case of weak light field. In order to facilitate the solution of the dynamic equation for the system, the well known Bogoliubov’s approximation is adopted[20]. Namely, suppose that the atom number of BEC is so large at initial moment that the slow change of atom number in ground state can be neglected in process of the interaction with light field, replace b_0 and b_0^+ by $\sqrt{N_0}\mathrm{e}^{-i\theta}$ and $\sqrt{N_0}\mathrm{e}^{i\theta}$. Omit the terms containing $b_1^+ b_1^+ b_1 b_1$ and let $b = b_1$, $b^+ = b_1^+$, simplify system’s Hamiltonian as

$$H = \omega_0 b^+ b + \omega a^+ a + \varepsilon \sqrt{N_0} \left(a^+ b \mathrm{e}^{i\theta} + a b^+ \mathrm{e}^{-i\theta} \right) + \Omega \left(N_0^2 + 2 N_0 b^+ b \right). \tag{2}$$

In order to have a clear description of the influence of the interaction among BEC’s atoms on the energy of the system, rewrite system's Hamiltonian as

$$H = (\omega_0 + 2N_0\Omega)\, b^+ b + \omega a^+ a + \varepsilon\sqrt{N_0}\left(a^+ b e^{i\theta} + a b^+ e^{-i\theta}\right) + N_0^2\Omega. \quad (3)$$

Seen from Eq. (3), the interaction among BEC's atoms makes the original atomic energy gap between ground and excited states ω_0 increase to $\omega_0 + 2N_0\Omega$ and the increment $\Delta = 2N_0\Omega$ is proportional to the atomic number N_0 and the interaction intensity Ω.

Under the condition of resonance ($\omega = \omega_0$), solve system's Heisenberg equation

$$i\dot{a} = [a, H] = \omega a + \varepsilon\sqrt{N_0} b e^{i\theta}, \quad (4)$$

$$i\dot{b} = [b, H] = \varepsilon\sqrt{N_0} a e^{-i\theta} + (\omega_0 + 2N_0\Omega)\, b, \quad (5)$$

we get

$$a(t) = \alpha_a(t)\, a(0) + \beta_a(t)\, b(0), \quad (6)$$

$$b(t) = \alpha_b(t)\, a(0) + \beta_b(t)\, b(0), \quad (7)$$

where

$$\alpha_a(t) = \frac{e^{-i(\omega+N_0\Omega)t}}{\gamma}\left[\gamma\cos(\gamma t) + iN_0\Omega\sin(\gamma t)\right], \quad (8)$$

$$\beta_a(t) = -i\frac{\sqrt{N_0}\varepsilon}{\gamma} e^{-i(\omega+N_0\Omega)t}\sin(\gamma t)\, e^{i\theta}, \quad (9)$$

$$\alpha_b(t) = -i\frac{\sqrt{N_0}\varepsilon}{\gamma} e^{-i(\omega+N_0\Omega)t}\sin(\gamma t)\, e^{-i\theta}, \quad (10)$$

$$\beta_b(t) = \frac{e^{-i(\omega+N_0\Omega)t}}{\gamma}\left[\gamma\cos(\gamma t) - iN_0\Omega\sin(\gamma t)\right], \quad (11)$$

where $\gamma = \sqrt{N_0(\varepsilon^2 + N_0\Omega^2)}$.

Assuming that all atoms occurring Bose-Einstein condensation in ground state can be described by the coherent state $|\beta\rangle_g$, the atom's excited state is the vacuum state $|0\rangle_e$, and the light field in the coherent state $|\alpha\rangle_f$ at time $t = 0$. The initial state vector of the system is

$$|\psi(0)\rangle = |\beta\rangle_g \otimes |0\rangle_e \otimes |\alpha\rangle_f \equiv |\beta, 0, \alpha\rangle, \quad (12)$$

where $|\alpha\rangle_f = D(\alpha)|0\rangle_f$, $D(\alpha) = \exp(\alpha a^+ - \alpha^* a)$, $\alpha = \sqrt{n_0}e^{i\eta}$, n_0 being the average photon number of initial light field. Under the Bogoliubov's approximation, the BEC's atoms can be regarded as always in the coherent state $|\beta\rangle_g$ in the process of system evolution.

At any moment the state vector of system evolves into

$$|\psi(t)\rangle = U(t)|\psi(0)\rangle, \quad (13)$$

where $U(t) = \exp(-iHt)$, H is expressed as Eq. (3).

From Eqs. (12) and (13) the density operators $\rho(0)$ and $\rho(t)$ of system are obtained respectively as

$$\rho(0) = |\psi(0)\rangle\langle\psi(0)| = \exp\left(-|\alpha|^2\right)\sum_{m,n}\frac{\alpha^n(\alpha^*)^m}{\sqrt{m!n!}}|\beta,0,n\rangle\langle\beta,0,m|, \quad (14)$$

$$\rho(t) = |\psi(t)\rangle\langle\psi(t)| = \exp\left(-|\alpha\beta_a^*(-t)|^2 - |\alpha\alpha_a^*(-t)|^2\right) \sum_{m',n',M',N'}\frac{[\alpha\alpha_a^*(-t)]^{n'}[\alpha^*\alpha_a(-t)]^{m}[\alpha\beta_a^*(-t)]^{N'}[\alpha^*\beta_a(-t)]^{M'}}{\sqrt{m'!n'!}\sqrt{M'!N'!}} \times |\beta, N', n'\rangle\langle\beta, M', m'|, \quad (15)$$

and the reduced density operators of field and BEC, ρ_f and ρ_b are gained respectively as

$$\rho_f = \mathrm{Tr}_b\rho, \quad (16)$$

$$\rho_b = \mathrm{Tr}_f\rho. \quad (17)$$

We introduce the fidelities of quantum state for the interaction system, light field, and atomic BEC, denoted by F_s, F_f and F_b, respectively, so as to depict the deviation of output state (end state) from input state (initial state) of the system, light field, and atomic BEC. They are defined as[16]

$$F_s = \mathrm{Tr}\{\rho(0)\rho(t)\}, \quad (18)$$

$$F_f = \mathrm{Tr}\{\rho_f(0)\rho_f(t)\}, \quad (19)$$

$$F_b = \mathrm{Tr}\{\rho_b(0)\rho_b(t)\}. \quad (20)$$

Obviously we have $0 \le F \le 1$. If initial state and end state are orthogonal each other, then $F = 0$ and this means the quantum state being totally distorted. If initial state and end state are coincided, then $F = 1$ and this means the quantum state being wholly undisturbed, therefore an ideal transmitting process taking place. In general, we have $0 < F < 1$, it means certain distortion exists in the transmission process of information. Using Eqs. (14) and (15), we obtain the expressions of F_s, F_f, and F_b respectively as

$$F_s = \exp\left\{-n_0\left[1 + |\beta_a(-t)|^2 + |\alpha_a(-t)|^2 - 2\mathrm{Re}(\alpha_a(-t))\right]\right\}, \quad (21)$$

$$F_f = \exp\left\{-n_0\left[1 + |\alpha_a(-t)|^2 - 2\mathrm{Re}(\alpha_a(-t))\right]\right\}, \quad (22)$$

$$F_b = \exp\left\{-n_0|\beta_a(-t)|^2\right\}, \quad (23)$$

$\alpha_a(-t)$ as well as $\beta_a(-t)$ can be got from Eqs. (8) and (9) as

$$\alpha_a(-t) = \frac{e^{i(\omega+N_0\Omega)t}}{\gamma}\left[\gamma\cos(\gamma t) - iN_0\Omega\sin(\gamma t)\right], \quad (24)$$

$$\beta_a(-t) = i\frac{\sqrt{N_0}\varepsilon}{\gamma}e^{i(\omega+N_0\Omega)t}\sin(\gamma t)\, e^{i\theta}, \quad (25)$$

where $\gamma = \sqrt{N_0(\varepsilon^2 + N_0\Omega^2)}$. We see that the fidelity F_s of quantum state for the interaction system is exactly the product of the fidelities F_f and F_b of quantum state for light field and atomic BEC and it is not only dependent on the average atomic number N_0 of atomic BEC and the average photon number n_0 of initial light field but also related to the coupling intensity ε of atoms and light field and to the dipole interaction constant Ω and the resonance frequency ω.

The time-dependent evolutions of the fidelity of quantum state for the interaction system of light field and atomic BEC are numerically shown in Figs. 1 and 2 (for simplicity, let $N_0 = 2000$, $\varepsilon = 1$, and $\omega = 1$). They describe the influences of the average photon number n_0 of initial light field and the dipole interaction constant Ω among atoms on the fidelity of quantum state of the system, respectively.

Seen from Fig. 1, in case of weak initial light field, the quantum state fidelity of the system very approximates to one with the least distortion as shown in Fig. 1(a). With increasing the intensity of initial light field, the quantum state fidelity is distinctly decreased because of enhancing interaction between light field and atomic BEC. In case of stronger initial light field, the distortion of the system's quantum state is very much as shown in Fig. 1(d), indicating that the Bogoliubov's approximation is not suitable. In addition, Fig. 1 indicates clearly that the changing intensity does not vary the Rabi oscillation frequency and the modulation frequency of fidelity of the quantum information of the system.

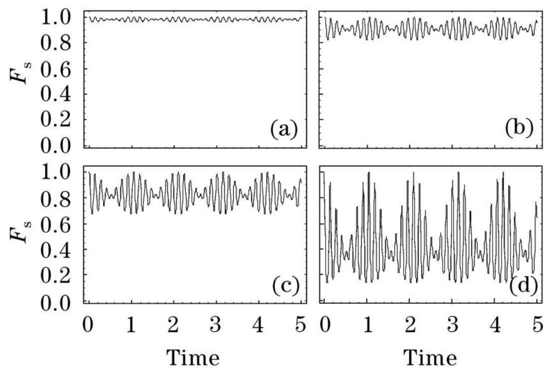

Fig. 1. The influences of n_0 on fidelity F_s with $\Omega = 0.001$. (a) $n_0 = 0.01$; (b) $n_0 = 0.05$; (c) $n_0 = 0.1$; (d) $n_0 = 0.5$.

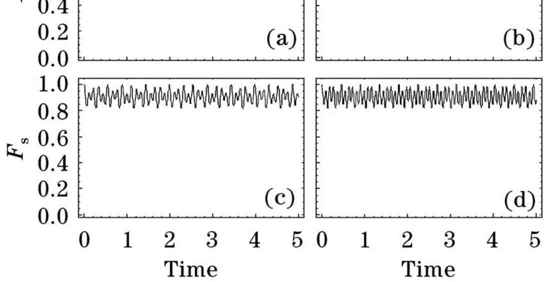

Fig. 2. The influences of Ω on fidelity F_s with $n_0 = 0.05$. (a) $\Omega = 0.0001$; (b) $\Omega = 0.001$; (c) $\Omega = 0.005$; (d) $\Omega = 0.01$.

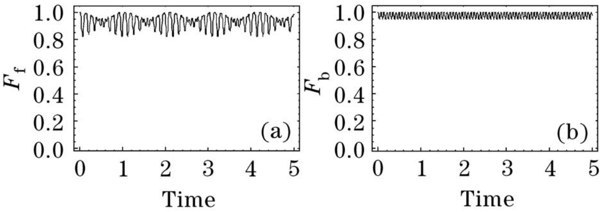

Fig. 3. The evolutions of F_f (a) and F_b (b) with $n_0 = 0.05$ and $\Omega = 0.001$.

The evolution of the fidelity of quantum state of the system with different interaction constants Ω among atoms is shown in Fig. 2. Being different from the case with varying initial light field intensity, the change of the interaction among atoms almost does not vary the average value of fidelity F_s and influences distinctly the oscillation frequency of modulation for the fidelity F_s. In the case of weaker interaction among atoms (smaller Ω), the change of Ω mainly varies the modulation frequency of F_s as shown in Figs. 2(a) and (b). In the case of stronger interaction (larger Ω), the change of Ω varies both the modulation frequency and the Rabi oscillation frequency of F_s as shown in Figs. 2(c) and (d).

As above mentioned, F_s being the product of F_f and F_b, the fidelity of quantum state of the interaction system consequently should be the combined expression of the fidelities of quantum state of the light field and the atomic BEC. Numerical results show that there is great difference in the evolution of F_f and F_b. The evolution of F_f and F_b under the same condition as in Fig. 2(b) is shown in Fig. 3. Comparing Figs. 3(a) and (b) with Fig. 2(b), it is obvious that the evolution of the fidelity F_f is very similar to that of the fidelity F_s and the fidelity F_b displays periodic oscillation with equal amplitude. This behavior has been verified by many numerical results of F_f and F_b. It indicates clearly that the light field is the key factor for influencing the fidelity of quantum information of system.

From the above theoretical analyses and numerical results, we have the following conclusion on the fidelity of quantum state of the interaction system of light field and atomic BEC.

1) The light field is the key factor influencing the fidelity of quantum information of system. The stronger the initial light field, the lower the fidelity of quantum information of system. The change of initial light field intensity does not vary the Rabi oscillation frequency and the modulation frequency for the fidelity of quantum information of system.

2) The strength of the interaction among BEC's atoms influences mainly the oscillation frequency of the fidelity of quantum state of system.

3) The evolution of the fidelity of quantum state of light field is very similar to that of the interaction system and the evolution of the fidelity of quantum state of atomic BEC displays periodic oscillation with equal amplitude.

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