

# A novel polarization modulator for a moiré system with grating imaging

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A polarization modulator based on splitting with a Savart plate and rotation of an analyzer for a moiré system with grating imaging is presented, and its modulation principle is analyzed. The polarization modulator is simple and achromatic. It is composed of a polarizer, a Savart plate, and an analyzer. The polarizer and the Savart plate are placed in front of the index grating to split the image of the scale grating in the moiré system. The analyzer is placed behind the grating and rotated to realize the modulation of the moiré signal. The analyzer can be rotated either continually with high speed or step by step with low speed to form different modulation modes. The polarization modulator makes the moiré system insensitive to the change of initial intensity. In experiments, we verified the usefulness of the polarization modulator.

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Displacement measurements and position sensing have been playing an important role in many fields such as fabrication, biophysics, and autocontrol. Interferometry, laser triangulation, optical fiber sensing and moiré technique are common optical measuring methods<sup>[1-3]</sup>. Among these methods, moiré technique holds the most interest due to its high accuracy, large range, low cost, and other advantages. When a long working distance is required, or when space is not at a premium, a moiré system with grating imaging is used with prior consideration in which a scale grating is imaged on an index grating to form moiré fringes. The first merit of the moiré system with grating imaging is the sinusoidal feature of the moiré signal. The moiré signal with sinusoidal variation is easily obtained by setting a diaphragm in the imaging system<sup>[4]</sup>. Its second merit is that multiple imaging of the scale grating can be used to enlarge the application range of moiré technique. For example, it can be used for measurement of the displacement or position of a reflective object using reflection of the grating image<sup>[5]</sup>. When the moiré system needs to fit for scale gratings with different transmissivity or reflectivity (for transmission grating or reflection grating)<sup>[4]</sup>, or when multiple imaging of the scale grating and reflection of grating image are used to measure reflecting objects with different reflectivity<sup>[5]</sup>, the replacement of the scale grating or reflecting object will change the initial intensity forming the moiré signal. In Refs. [4] and [5], a polarization modulation system including an electro-optic modulator and two quarter-wave plates is applied to make the measurement independent of the initial intensity in moiré systems with grating imaging. The polarization modulation system has been adopted to high-accuracy measurement, but it is a complex system with high cost and needs complicated calibration due to the electro-optic modulator's dispersion. In this paper, a simple and achromatic polarization modulator is proposed. It makes the moiré systems with grating imaging independent of its initial intensity.

A simplified moiré system with grating imaging is shown in Fig. 1. A scale grating illuminated by colli-

ated beam is imaged on an index grating through a lens  $L_1$ . If the scale grating is a reflection grating, the illumination beam is at the back of the scale grating. The image of the scale grating superimposes with the index grating to produce moiré fringes. In fact, the diffraction occurs on the scale grating and the image of scale grating is formed by the diffraction rays. The light intensity of the image which is transmitted by the index grating, namely the moiré signal, is detected by a detector through lens  $L_2$ . In the moiré system, a diaphragm is placed to stop the high-order and zero-order or just high-order diffraction rays to obtain sinusoidal moiré signal. The index grating has the same period as the image of scale grating. The grooves of the scale grating and index grating are parallel to each other. In the system, the moiré signal is given by

$$I = I_{in}S_{dc} + I_{in}S_{ac} \cos\left(\frac{x \cdot 2\pi}{p}\right), \quad (1)$$

where  $I_{in}$  is the initial intensity forming moiré fringes,  $x$  is the distance that the image of scale grating is moved relative to the index grating in the direction perpendicular to the groove of gratings,  $p$  is the period of the index grating. The moiré signal in Eq. (1) is composed of two parts. The first part is called mean component, it keeps constant in course of the movement of scale grating. The second part is called variation component that varies cosinoidally with the distance. Accordingly,  $S_{dc}$  and  $S_{ac}$  are the coefficients of the mean component and variation component, respectively. Both the mean component and the variation component are proportional to the initial intensity. When the scale grating is moved and the moiré signal is detected, the distance can be obtained

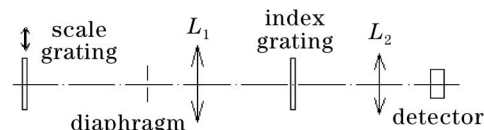


Fig. 1. Schematic diagram of the moiré system with grating imaging.

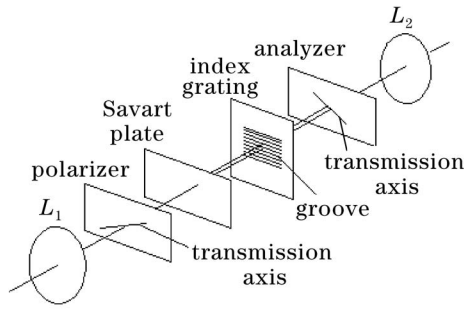


Fig. 2. Schematic diagram of the polarization modulator for a moiré system with grating imaging.

with Eq. (1). If the initial intensity is changed, the measurement accuracy of the distance is severely influenced. So we adopt a polarization modulator based on splitting with a Savart plate and rotation of an analyzer to eliminate the influence of the initial intensity variation in the system.

The polarization modulator is illustrated in Fig. 2. A polarizer and a Savart plate are placed in front of the index grating. An analyzer is located behind the index grating. The polarizer, the Savart plate and the analyzer constitute the polarization modulator for the moiré system with grating imaging. The polarizer is used to linearly polarize the rays which forming the image of scale grating, so the incident rays on the Savart plate are linearly polarized. The Savart plate comprises two beam splitting plates with orthogonal optic axes and equal thicknesses. The Savart plate splits the beam forming the image of the scale grating into two orthogonally polarized and mutually sheared beams, in other words, ordinary and extraordinary images of the scale grating are formed on the index grating. When the analyzer is rotated around the optical axis of the lens  $L_2$ , polarization modulation of the moiré signal is realized.

The rays forming the image of scale grating are polarized after they pass the polarizer. The angle between the polarizer's transmission axis and the groove direction of index grating is  $45^\circ$ . The shearing directions of Savart plate's two beam splitting plates are parallel and perpendicular to the groove direction of index grating, respectively, so the Savart plate splits the beam into two mutually sheared beams with equal light intensity. For the Savart plate, only one splitting plate, whose beam displacement is in the direction perpendicular to the groove direction of index grating, functions in the change of the moiré signal. The other splitting plate, whose beam displacement is in the direction parallel to the groove direction of index grating, compensates the optical path difference generated by the above-mentioned splitting plate. In the polarization modulator, the beam displacement of one beam splitting plate of the Savart plate is a half of the index grating's period. So two moiré signals produced by the index grating's superimposition with the ordinary and extraordinary images of the scale grating are written as

$$I_o = I_{in}k \left[ S_{dc} + S_{ac} \cos \left( \frac{x \cdot 2\pi}{p} \right) \right], \quad (2)$$

and

$$I_e = I_{in}k \left[ S_{dc} - S_{ac} \cos \left( \frac{x \cdot 2\pi}{p} \right) \right], \quad (3)$$

respectively, where  $k$  is a coefficient less than 0.5 that is generated due to insertion of the polarizer and the Savart plate's beam splitting. When the analyzer is rotated, the intensities of the ordinary and extraordinary images of scale grating through the analyzer are written as

$$I'_o = I_o \cos^2 \alpha = \frac{I_o}{2} (1 + \cos 2\alpha), \quad (4)$$

and

$$I'_e = I_e \cos^2 (90^\circ - \alpha) = \frac{I_e}{2} (1 - \cos 2\alpha), \quad (5)$$

respectively, where  $\alpha$ , termed azimuth angle, is the angle between the analyzer's transmission axis and the groove direction of index grating. Because there is no optical path difference in the two orthogonally polarized beams exiting from the Savart plate, the interference will not occur and the intensities of the two beams are independent of each other when they pass the analyzer. The detector detects just the light flux through the analyzer and the detected intensity is given by

$$I_{out} = I'_o + I'_e = I_{in}kS_{dc} + I_{in}kS_{ac} \cos \left( \frac{x \cdot 2\pi}{p} \right) \cos 2\alpha. \quad (6)$$

It is obvious that the moiré signal in Eq. (1) is modulated when the analyzer is rotated to change its azimuth angle and only the moiré signal's variation component is modulated by the polarization modulator.

When the analyzer is rotated continuously with high speed, the azimuth angle of the analyzer is expressed as

$$\alpha = \omega t, \quad (7)$$

where  $\omega$  is the angular frequency. The detected intensity is converted into electric signal, which includes a direct current (dc) component and an alternating current (ac) component. The electric signal is amplified by an amplifier and its dc and ac components are separated by a filter circuit. Using the electric signal, the mean component and variation component of the moiré signal are obtained. Equation (6) can be rewritten as

$$I_{out} = DC + AC \cos 2\alpha, \quad (8)$$

where  $DC$  and  $AC$  are proportional to the mean component and variation component of the moiré signal. So the amplitudes of the dc and ac components of the electric signal are  $DC$  and  $AC$ , respectively. According to Eqs. (6) and (8), the distance is given by

$$x = \frac{p}{2\pi} \arccos \left( \frac{S_{dc}}{S_{ac}} \cdot \frac{AC}{DC} \right). \quad (9)$$

In Eq. (9),  $AC$  and  $DC$  have common factors  $I_{in}$  and  $k$ , so the distance is independent of the initial intensity.

When the analyzer is rotated step by step with low speed, the azimuth angle of the analyzer is expressed as

$$\alpha = mA, \quad (10)$$

where  $A$  is the angle increment and  $m$  is the step number. In the process of the rotation of the analyzer, a series of detected intensities are obtained with different azimuth angles. The distance can be obtained from these intensities and azimuth angles. First,  $AC$  and  $DC$  in Eq. (8) are calculated using least-squares method. The detected intensity and the azimuth angle are written as  $\alpha_i$  and  $I_i$  at the  $i$ th step, respectively, and the total step number is  $n$ . For the calculation of  $AC$  and  $DC$ , the normal equations are

$$\begin{cases} nDC + \sum_{i=1}^n \cos(\alpha_i)AC = \sum_{i=1}^n I_i \\ \sum_{i=1}^n \cos(\alpha_i)DC + \sum_{i=1}^n \cos^2(\alpha_i)AC = \sum_{i=1}^n \cos(\alpha_i)I_i \end{cases} \quad (11)$$

Then the distance is calculated using  $AC$  and  $DC$  with Eq. (9). Thus, the distance is independent of the initial intensity, too.

Two rotation modes, continuous rotation with high speed and stepping rotation with low speed, of the analyzer form two modulation modes for the polarization modulator. Under the two modulation modes, the moiré system with grating imaging is independent of the initial intensity. In the polarization modulator, only the analyzer is rotated in course of polarization modulation, so the polarization modulator is simple with low cost and the modulation is easy to be realized. Common polarizer and analyzer, for example polarizing prisms, are used in the spectral range from near ultraviolet to infrared. An achromatic Savart plate can be designed easily. Thus the polarization modulator is achromatic and can be applied in moiré systems with grating imaging using white light illumination.

In our experiment, a scale grating was illuminated by a collimated beam emitted from a 785-nm laser diode. The scale grating was imaged on an index grating with  $-1^{\times}$  magnification through a  $4f$  system. The scale grating and index grating were etched on optical flats with the period of 0.1 mm and line to space ratio of 1:1. At the frequency plane of the  $4f$  system, a diaphragm was placed to stop the high-order diffraction rays of the scale grating. The scale grating and index grating together with the  $4f$  system formed a moiré system with grating imaging. The scale grating was located on a linear stage supplying up/down movement and moved to change the moiré signal. In the moiré system with grating imaging, a polarization modulator shown in Fig. 2 was arranged. The polarizer and the analyzer were the Glan-Taylor prisms with an extinction ratio greater than 100000:1. The Savart plate was made of two quartz prisms whose beam displacements were both 0.05 mm. The analyzer was placed in a rotation mount providing continuous  $360^{\circ}$  rotation. The light intensity passing the analyzer was detected by a photodiode through a lens and transmitted into an electrical signal. The electrical signal was amplified by an amplifier and transferred to a computer through a signal capturer to record its magnitude of voltage.

The light path of the moiré system was adjusted using an internally focusing telescope and the azimuth angles of the polarization elements were adjusted like Fig. 2 with the light extinction method. At a measuring point, at first, the analyzer was rotated step and step with  $6^{\circ}$

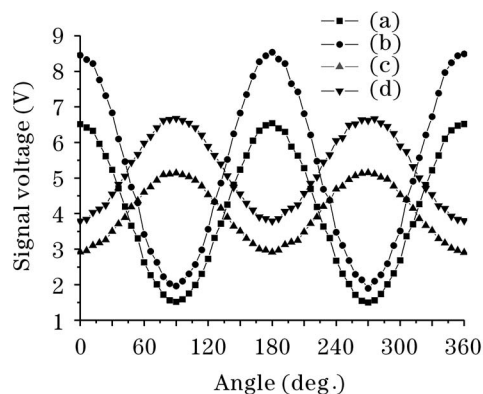


Fig. 3. Experiment results.

angle increment when the injection current of the laser diode was kept 36 mA. In the angle range of  $360^{\circ}$ , the variation of signal voltages is given as curve (a) in Fig. 3. Then the injection current of the laser diode was changed to 40 mA to vary the initial intensity of the grating system. The process of rotating the analyzer and recording the signal voltages was repeated. The variation of signal voltage is shown as curve (b) in Fig. 3. The scale grating was moved 0.0175 mm to change the moiré signal. Similarly, before and after the injection current of the laser diode was changed, the signal voltages in the course of rotation of the analyzer are given as curves (c) and (d) in Fig. 3. In Fig. 3, the signals vary sinusoidally with the rotation angles of the analyzer. For every signal curve, an ac component is superimposed on a dc component. So the moiré signal is effectively modulated by the polarization modulator. For the two groups of curves (a) and (b), (c) and (d) in Fig. 3, the changes of ratios of the ac component to dc component are less than 0.9%, while the change of initial intensity is larger than 28% before and after the change of injection current. So the results accord with Eqs. (6) and (9) and the polarization modulation makes the moiré system with grating imaging insensitive to the change of the initial intensity.

In conclusion, we proposed a polarization modulator for a moiré system with grating imaging and analyzed its two modulation modes. The polarization modulator is simple and achromatic with low cost, and it makes the moiré system insensitive to the change of initial intensity. In experiments, the usefulness of the polarization modulator was verified.

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