The phenomenon of stochastic resonance in a single-mode laser system with an input periodical signal

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Using the linear approximation method, we study a single-mode laser system driven by colored pump noise and quantum noise with coupling between the real and imaginary parts when the laser is operated well above threshold. The steady state mean intensity fluctuation $C(0)$ and signal-to-noise ratio (SNR) are calculated. It is found that there is a maximum in SNR when there is a minimum in the fluctuation of laser system if the coupling coefficient between real and imaginary parts of the quantum noise equals zero.

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The phenomenon of stochastic resonance (SR) was originally discovered by Benzi *et al.*[1,2] in 1981. Since then it has been proved experimentally in different scientific fields, and has attracted much attention[3−7]. In laser systems, SR was firstly detected in the two-mode ring laser[8]. It is shown that noise in the driving system and the different forms of cross-correlation between noises have much influence on SR. After inputting the signal into the laser system, SR appears due to the cooperative effect between the noise and the signal[9−11]. Therefore, the study of the phenomenon of SR in a single-mode laser system driven by colored pump noise and quantum noise with cross-correlation between the real and imaginary parts when there is an input additive periodical signal, and the analysis of the influence of noise, periodical signal, and net gain on the characteristic of SR, can provide a theoretic foundation for the optimization of designing laser system.

On the basis of the above-mentioned research works, we have adopted the linear approximation method[12] to calculate the signal-to-noise ratio (SNR) of the system when pump noise is colored noise and quantum noise is Gaussian white noise with cross-correlation between the real and imaginary parts. We have found that when self-correlation time of pump noise is very short ($\tau \ll 1$), there is a maximum in SNR. Moreover, we have also analyzed the subjects that influence SR such as the amplitude and frequency of input signal, the intensities of quantum noise and pump noise, the self-correlation time of pump noise, and the net gain.

The Langevin equation of amplitude r for a loss-noise model of a single-mode laser is given by[13]

$$\frac{\mathrm{d}r}{\mathrm{d}t'} = a_0 r - A r^3 + \frac{P}{2r}(1-|\lambda_\mathrm{q}|) + r p_\mathrm{R}(t') + \varepsilon_\mathrm{r}(t'). \quad (1)$$

Considering the influence of periodical signal $B\cos\Omega t'$ on laser system, the Langevin equation of intensity for a loss-noise model of a single-mode laser with an input signal is given by

$$\frac{\mathrm{d}I}{\mathrm{d}t'} = 2a_0 I - 2AI^2 + P(1-|\lambda_\mathrm{q}|) + 2I p_\mathrm{R}(t') + 2\sqrt{I}\varepsilon_\mathrm{r}(t') + B\cos\Omega t', \quad (2)$$

where $p_\mathrm{R}(t')$ and $\varepsilon_\mathrm{r}(t')$ are correlated as

$$\langle p_\mathrm{R}(t')\rangle = \langle \varepsilon_\mathrm{r}(t')\rangle = 0, \quad \langle p_\mathrm{R}(t') p_\mathrm{R}(s)\rangle = \frac{Q}{2\tau}\mathrm{e}^{-\frac{|t'-s|}{\tau}},$$
$$\langle \varepsilon_\mathrm{r}(t')\varepsilon_\mathrm{r}(s)\rangle = P(1+|\lambda_\mathrm{q}|)\delta(t'-s),$$
$$\langle p_\mathrm{R}(t')\varepsilon_\mathrm{r}(s)\rangle = 0. \quad (3)$$

Here a_0 and A represent the net gain coefficient and self-saturation coefficient, respectively; I is the intensity; B is the amplitude of input signal, Ω is the signal frequency; $p_\mathrm{R}(t')$ is the real part of the pump noise, and $\varepsilon_\mathrm{r}(t')$ is the quantum noise of phase locking after; P and Q are the intensities of the quantum noise and the pump noise, respectively; τ is the self-correlation time of pump noise; λ_q is the coefficient of cross-correlation between the real and imaginary parts of the quantum noise, and $-1 \le \lambda_\mathrm{q} \le 1$.

Let $I = I_0 + \delta(t')$, where $I_0 = \frac{a_0}{A}$ is the deterministic steady-state intensity, and $\delta(t')$ is the perturbational term. We linearize Eq. (2) around I_0, thus the linear equation of the laser intensity is obtained as

$$\frac{\mathrm{d}\delta(t)}{\mathrm{d}t} = -\gamma\delta(t) + 2I_0 p_\mathrm{R}(t) + 2\sqrt{I_0}\varepsilon_\mathrm{r}(t) + P(1-|\lambda_\mathrm{q}|) + B\cos\Omega t, \quad (4)$$

where $\gamma = 2a_0$.

According to the steady-state mean intensity fluctuation (SSMICF) defined by

$$C(t) = \lim_{t'\to\infty}\frac{\overline{\langle I(t')I(t'+t)\rangle} - \overline{\langle I(t')\rangle}^2}{\overline{\langle I(t')\rangle}^2} = \lim_{t'\to\infty} \left(\frac{\frac{\Omega}{2\pi}\int_{t'}^{t'+\frac{2\pi}{\Omega}}\langle I(t')I(t'+t)\rangle \mathrm{d}t' - \frac{\Omega}{2\pi}\int_{t'}^{t'+\frac{2\pi}{\Omega}}\langle I(t')\rangle^2 \mathrm{d}t'}{\frac{\Omega}{2\pi}\int_{t}^{t'+\frac{2\pi}{\Omega}}\langle I(t')\rangle^2 \mathrm{d}t'}\right),$$

SSMICF is obtained by solving Eq. (4)

$$C(t) = \frac{A^2P^2\left(1-|\lambda_q|\right)^2}{4a_0^4} + \left(\frac{AP(1+|\lambda_q|)}{a_0^2} - \frac{Q}{a_0(4a_0^2\tau^2-1)}\right)e^{-2a_0|t|} + \frac{2Q\tau}{4a_0^2\tau^2-1}e^{-\frac{|t|}{\tau}} + \frac{A^2B^2\cos\Omega t}{2a_0^2(4a_0^2+\Omega^2)}, \quad (5)$$

where $\tau \neq \frac{1}{2a_0}$. Thus, translate Eq. (5) into the power spectrum $S(\omega)$ by Fourier transform

$$S(\omega) = S_1(\omega) + S_2(\omega), \quad (6)$$

where $S_1(\omega)$ and $S_2(\omega)$ are output power spectra of the signal and the noise, respectively.

$$S_1(\omega) = \frac{\pi B^2}{2(\gamma^2+\Omega^2)}(\delta(\omega-\Omega)+\delta(\omega+\Omega)), \quad (7)$$

$$S_2(\omega) = \frac{4I_0P(1+|\lambda_q|)}{\gamma^2+\omega^2} + \frac{4I_0^2Q\tau^2}{(\gamma^2\tau^2-1)(1+\omega^2\tau^2)} - \frac{4I_0^2Q}{(\gamma^2\tau^2-1)(\gamma^2+\omega^2)}. \quad (8)$$

The SNR is defined as the ratio of output power of the signal and the broadband noise output at $\omega = \Omega$ (only the spectrum of $\omega > 0$ is kept). We have

$$R = \frac{P_s}{S_2(\omega=\Omega)}. \quad (9)$$

According to Eq. (9) and

$$\begin{cases} P_s = \int_0^\infty S_1(\omega)d\omega = \frac{\pi B^2}{2(\gamma^2+\Omega^2)} \\ S_2(\omega=\Omega) = \frac{4I_0P(1+|\lambda_q|)}{\gamma^2+\Omega^2} + \frac{4I_0^2Q\tau^2}{(\gamma^2\tau^2-1)(1+\Omega^2\tau^2)} - \frac{4I_0^2Q}{(\gamma^2\tau^2-1)(\gamma^2+\Omega^2)} \\ I_0 = \frac{a_0}{A} \quad \gamma = 2a_0 \end{cases},$$

we can get the output SNR

$$R = \frac{\pi B^2}{8I_0P(1+|\lambda_q|) + \frac{8I_0^2Q}{\tau^2\Omega^2+1}}. \quad (10)$$

Because the linear approximation method is adopted, the validity of Eq. (10) is only appropriate in a small range of the steady state. Furthermore, the unified colored noise approximation is used in Eq. (1), so we only discuss SR in the case of $\tau \ll 1$ in this paper.

Figures 1—6 are SNR as a function of λ_q for $A = 1$ and different values of B, Ω, τ, Q, P, and a_0, respectively. From them, we can see that SNR appears resonance peaks at $\lambda_q = 0$, and the size of peak increases with increasing B, Ω, and τ increasing (see Figs. 1—3) or with decreasing Q, P, and a_0 (see Figs. 4—6), i.e., the resonance peak is higher and acuter.

Obviously, Figs. 1 and 2 show that there is a great influence on characteristic of SR by periodical signal. When B and Ω increase, the SR increases, therefore, changing input signal can control the SR effectively. Figure 6 shows that the nearer the threshold, the stronger the SR of laser system.

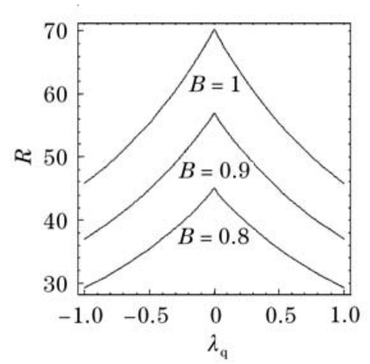

Fig. 1. SNR as versus λ_q for different values of B with the parameters $P = 0.003$, $Q = 0.003$, $a_0 = 1$, $\Omega = 20$, $\tau = 0.02$.

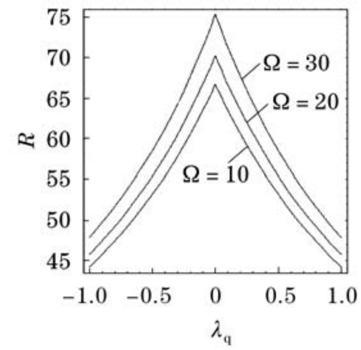

Fig. 2. SNR versus λ_q for different values of Ω with the parameters $P = 0.003$, $Q = 0.003$, $a_0 = 1$, $B = 1$, $\tau = 0.02$.

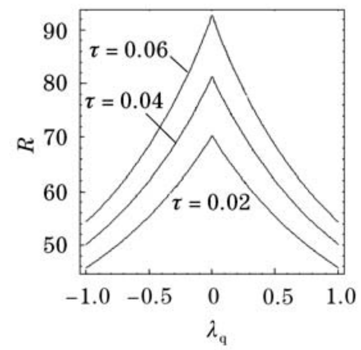

Fig. 3. SNR versus λ_q for different values of τ with the parameters $P = 0.003$, $Q = 0.003$, $a_0 = 1$, $B = 1$, $\Omega = 20$.

Let $t = 0$ in Eq. (5), the SSMICF $C(0)$ is obtained as

$$C(0) = \frac{A^2P^2\left(1-|\lambda_q|\right)^2}{4a_0^4} + \frac{AP(1+|\lambda_q|)}{a_0^2} + \frac{Q}{4a_0^2\tau^2-1}\left(2\tau - \frac{1}{a_0}\right) + \frac{A^2B^2}{2a_0^2(4a_0^2+\Omega^2)}. \quad (11)$$

Because of the adoption of the linear approximation method in solving Eq. (2), it is necessary to analyze whether the valid ranges of the parameters satisfy the condition of this method. The valid range of linear approximation method is $C(0) \ll 1$, which well meets within all parameter ranges. $C(0)$ versus λ_q is plotted in Fig. 7.

In Fig. 7, we can see: 1) within the parameter range, the SSMICF $C(0)$ is less than 1.3×10^{-2}, which meets the condition of $C(0) \ll 1$; 2) $C(0)$ decreases with decreasing

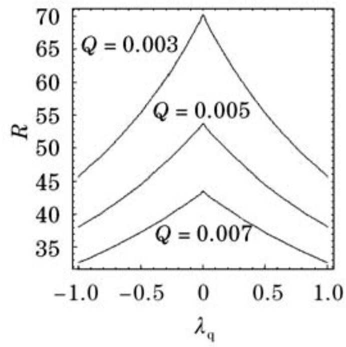

Fig. 4. SNR versus λ_q for different values of Q with the parameters $P = 0.003$, $a_0 = 1$, $B = 1$, $\Omega = 20$, $\tau = 0.02$.

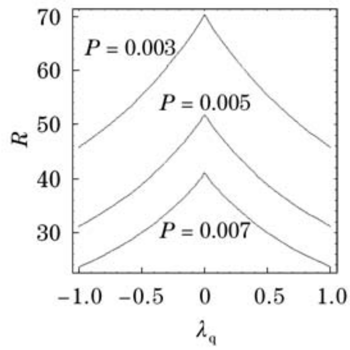

Fig. 5. SNR versus λ_q for different values of P with the parameters $Q = 0.003$, $a_0 = 1$, $B = 1$, $\Omega = 20$, $\tau = 0.02$.

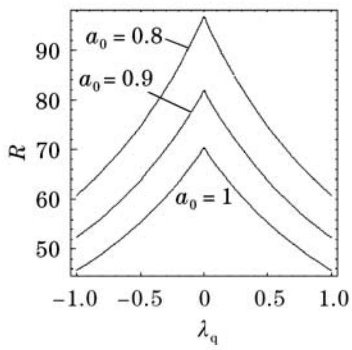

Fig. 6. SNR versus λ_q for different values of a_0 with the parameters $P = 0.003$, $Q = 0.003$, $B = 1$, $\Omega = 20$, $\tau = 0.02$.

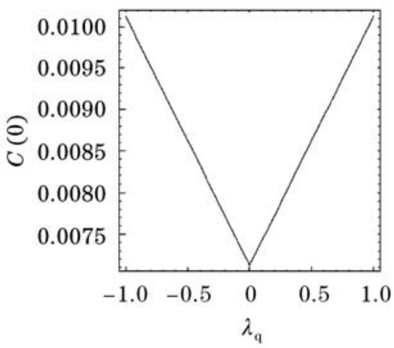

Fig. 7. SSMICF $C(0)$ versus λ_q with the parameters $A = 1$, $B = 1$, $P = 0.003$, $Q = 0.003$, $a_0 = 1$, $\tau = 0.02$, $\Omega = 20$.

$|\lambda_q|$, and there is a minimum at $\lambda_q = 0$, which shows that the fluctuation becomes smaller when the cross-correlation coefficient between real and imaginary parts of the quantum noise is weaker. There is a minimum in the case of no correlation ($\lambda_q = 0$); 3) Both the maximum of SNR and the minimum of SSMICF correspond to $\lambda_q = 0$, which shows that there is an optimum matching between output SNR and stochastic parameters, that is, there is a maximum in the output SNR while there is a minimum in the fluctuation of laser system at $\lambda_q = 0$.

In conclusion, 1) the phenomenon of SR appears by virtue of the curves of SNR versus λ_q, and there is an optimum matching between output SNR and stochastic parameters if the coupling coefficient between the real and imaginary parts of quantum noise $\lambda_q = 0$. It is shown that the output signal is of high quality and laser system is relatively stable. 2) When the amplitude of signal, signal frequency, and pump noise self-correlation time are augmented, the SR enhances. However, when the noise intensity increases and the laser system is far away from threshold, the SR weakens. 3) Using the linear approximation method in solving process of $C(0)$ and SNR, we have analyzed the valid ranges of all parameters, which satisfy the demand of the linear approximation method. 4) Under the condition that noises of quantum and pump are all white, we previously studied the phenomenon of SR in a signal-mode laser system driven by pump noise and quantum noise with cross-correlation between the real and imaginary parts, and found that SNR is independent of input signal frequency[11]. However, the SNR is related to input signal frequency and the self-correlation time of colored pump noise. Because of the great influence of the signal frequency on the SR of laser system, changing input signal can control the SR effectively. This provides a new method and theoretical basis for the application of SR to design and optimize optical communication.

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References

1. R. Benzi, A. Sutera, and A. Vulpiani, J. Phys. A **14,** 453 (1981).
2. C. Nicolis and G. Nicolis, Tellus **33,** 225 (1982).
3. X.-Q. Luo and S.-Q. Zhu, Chin. Phys. **13,** 1201 (2004).
4. R. Rozenfeld, J. A. Freund, A. Neiman, and L. Schimansky-Geier, Phys. Rev. E **64,** 051107 (2001).
5. X. Luo and S. Zhu, Phys. Rev. E **67,** 021104 (2003).
6. C. J. Tessone, H. S. Wio, and P. Hänggi, Phys. Rev. E **62,** 4623 (2000).
7. X. Luo, S. Zhu, and X. Chen, Phys. Lett. A **287,** 111 (2001).
8. L. Gammaitoni, P. Hänggi, P. Jung, and F. Marchesoni, Rev. Mod. Phys. **70,** 223 (1998).
9. J. Wang, L. Cao, and D.-J. Wu, Chin. Phys. Lett. **19,** 1416 (2002).
10. Q. Cheng, L. Cao, D. Xu, and D. Wu, Chin. Opt. Lett. **2,** 331 (2004).
11. Q.-H. Cheng, L. Cao, J. Wang, and D.-J. Wu, Commun. Theor. Phys. **42,** 553 (2004).
12. E. Hernández-Gorcia, R. Toral, and M. S. Miguel, Phys. Rev. A **42,** 6823 (1990).
13. S.-Z. Ke, L. Cao, D.-J. Wu, and K.-L. Yao, Chin. Phys. Lett. **18,** 370 (2001).