Effect of vacuum-induced coherence on lasing without inversion in an equispaced three-level ladder system

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The effects of vacuum-induced coherence (VIC) on the properties of the absorption and gain of the probe field in an equispaced three-level ladder atomic system are investigated. It is found that lasing without inversion (LWI) is remarkably enhanced due to the effect of VIC in the case of the small incoherent pump rate.

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There has been an increasing interest in the coherence arising from spontaneous emission, subject to the condition that the atomic dipole moments must be nonorthogonal. This kind of coherence is created by the interference of spontaneous emission of either two closely lying levels with a common level (V-type atom), or a single level with two closely lying levels (Λ-type atom). Zhou *et al.*[1] studied the absorption of a weak probe beam for a V-type atom and demonstrated that the vacuum-induced coherence (VIC) results in narrow resonance, transparency, and gain without inversion. Gong *et al.*[2] found an unexpected population inversion on one of the optical transition due to the effect of VIC. Furthermore, Zhu *et al.*[3−6] presented the quenching of spontaneous emission, and Paspalakis *et al.*[7−11] put forward some coherent phase-control schemes. Liu *et al.*[12] found that electromagnetically induced absorption (EIA) in a Λ-type system can occur due to the effect of VIC. Besides the coherence induced by spontaneous emission, the generation of coherence between the two lower levels via incoherent pumping also leads to some interesting phenomena. Hu *et al.*[13] predicted that for the Λ-type system with two degenerate lower levels, quantum interference of incoherent pump processes leads to coherent population trapping (CPT). Li *et al.*[14] found that in an open V-type atomic system VIC leads to lasing with and without inversion which is different from the corresponding closed V-type system.

In a three-level ladder atomic system, VIC can also be created with nearly equispaced levels[15−17]. Ficek *et al.*[16] discussed the effects of VIC on the steady-state intensity and squeezing properties of the fluorescence light. Ma *et al.*[17] showed that in the three-level ladder-type system with equispaced levels, the population inversion can be greatly enhanced on one of the optical transitions due to the VIC effect.

On the basis of the models in Refs [15—17], we first present the incoherently pumped equispaced three-level ladder system model. The simple analytical solutions and numerical results are given to show the effect of VIC on the gain of the weak probe field when the control field is resonant and non-resonant. We find that the gain profile sensitively depends on the effect of VIC, and lasing without inversion (LWI) can occur unexpectedly. In addition, LWI can be enhanced due to the effect of VIC with small incoherent pump rate.

A three-level ladder-type system with nearly equispaced levels is shown in Fig. 1. Two coherent fields (the control field and the probe field) of frequencies ω_c and ω_p with real-valued Rabi frequencies $G = \vec{\mu}_{23} \cdot \vec{\varepsilon}_c/\hbar$ and $g = \vec{\mu}_{12} \cdot \vec{\varepsilon}_p/\hbar$ drive the transitions $|2\rangle \leftrightarrow |3\rangle$ and $|1\rangle \leftrightarrow |2\rangle$, respectively. The excited level $|2\rangle$ ($|3\rangle$) is considered to decay spontaneously to $|1\rangle$ ($|2\rangle$) at the rate γ_1 (γ_2). The incoherent pump rate from $|1\rangle$ to $|3\rangle$ is R. In such a case, the set of differential equations for the density matrix (in a rotating frame) can be derived as

$$\mathrm{d}\rho_{11}/\mathrm{d}t = -2R\rho_{11} + 2\gamma\rho_{22} + ig(\rho_{21} - \rho_{12}), \quad (1)$$

$$\mathrm{d}\rho_{33}/\mathrm{d}t = 2R\rho_{11} - 2\gamma\rho_{33} + iG(\rho_{23} - \rho_{32}), \quad (2)$$

$$\mathrm{d}\rho_{23}/\mathrm{d}t = -(\gamma_1 + \gamma_2 + i\Delta_2)\rho_{23} + iG(\rho_{32} - \rho_{22}) + ig\rho_{13}, \quad (3)$$

$$\mathrm{d}\rho_{12}/\mathrm{d}t = -(R + \gamma_1 + i\Delta_1)\rho_{12} + ig(\rho_{22} - \rho_{11}) - iG\rho_{13} + 2p\sqrt{\gamma_1\gamma_2}\rho_{23}, \quad (4)$$

$$\mathrm{d}\rho_{13}/\mathrm{d}t = -\left[\gamma_1 + R + i(\Delta_1 + \Delta_2)\right]\rho_{13} - iG\rho_{12} + ig\rho_{23}, \quad (5)$$

with the closure relation $\rho_{11} + \rho_{22} + \rho_{33} = 1$ and $\rho_{mn} = \rho_{nm}^*$. Here $\Delta_1 = \omega_{21} - \omega_p$ and $\Delta_2 = \omega_{32} - \omega_c$ are the frequency detunings of two laser fields. The term $2p\sqrt{\gamma_1\gamma_2}$ represents the effect of VIC resulting from the cross-coupling between the transition $|2\rangle \to |1\rangle$ and $|3\rangle \to |2\rangle$.

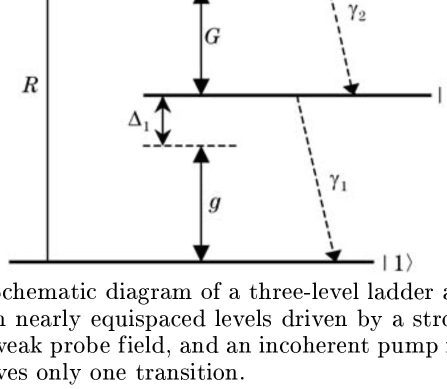

Fig. 1. Schematic diagram of a three-level ladder atomic system with nearly equispaced levels driven by a strong control field, a weak probe field, and an incoherent pump field. Each field drives only one transition.

It should be noted that nearly equispaced levels, the effects arising from VIC must be taken into account. The parameter p denotes the alignment of the two matrix elements which is nonparallel as well as nonorthogonal, and is defined as $p = \vec{\mu}_{12} \cdot \vec{\mu}_{23} / |\vec{\mu}_{12} \cdot \vec{\mu}_{23}| = \cos\theta$, where θ is the angle between the two induced dipole moments $\vec{\mu}_{23}$ and $\vec{\mu}_{12}$. We can see that the Rabi frequencies are connected to the parameter p by the relation $G = G_0\sqrt{1-p^2} = G_0 \sin\theta$ and $g = g_0\sqrt{1-p^2} = g_0 \sin\theta$, G_0 and g_0 are the Rabi frequencies at the situation of no VIC.

It is convenient to check the analytical solutions when the control field is resonant, i.e., $\Delta_2 = 0$. Under the steady-state condition, in the limit of a weak probe, the solutions for ρ_{nn} to the zero order of g, but to all the orders of G, are given as

$$\rho_{11}^{(0)} = \frac{\gamma_1}{D}\left[\gamma_2(\gamma_1+\gamma_2)+G^2\right], \quad (6)$$

$$\rho_{22}^{(0)} = \frac{R}{D}\left[\gamma_2(\gamma_1+\gamma_2)+G^2\right], \quad (7)$$

$$\rho_{33}^{(0)} = \frac{R}{D}\left[\gamma_1(\gamma_1+\gamma_2)+G^2\right], \quad (8)$$

$$\frac{\rho_{12}^{(1)}}{g} = \frac{i\left[(\gamma_2+R+i\Delta_1)(\rho_{22}^{(0)}-\rho_{11}^{(0)})+\frac{G^2}{\gamma_1+\gamma_2}(\rho_{33}^{(0)}-\rho_{22}^{(0)})\right]}{G^2+(\gamma_1+R+i\Delta_1)(\gamma_2+R+i\Delta_1)}, \quad (9)$$

$$D = (\gamma_1+2R)G^2+(\gamma_1+\gamma_2)(\gamma_1\gamma_2+R\gamma_1+R\gamma_2). \quad (10)$$

The gain (or absorption) of the probe field is proportional to the imaginary part of the ρ_{12}/g. Clearly, as shown in Eq. (9), the gain depends on the population inversion of two optical transitions. Ma *et al.*[17] have shown that the population inversion can be greatly enhanced on one of the transitions due to the effect of VIC. When $\rho_{22}^{(0)} - \rho_{11}^{(0)} < 0$, the large gain is obtained because VIC and incoherent pump are helpful to produce enough population inversion $\rho_{33}^{(0)} - \rho_{22}^{(0)}$, i.e., LWI is achieved. Zhu[18] has shown that pump mechanism plays a very important role and given the necessary condition of the realization of LWI in this three-level system (but he did not consider the effect of VIC). However, we confirm that the existence of VIC has a crucial influence on the properties of this system.

The steady-state solutions can be found by setting the Eqs. (1)—(5) to be zero. In numerical calculations, we use computational package Maple (all the parameters are scaled by γ_1 and $\gamma_1 = 1$).

In this paper, we are interested in the polarization induced in the $|2\rangle \leftrightarrow |1\rangle$ transition. The non-diagonal element of density matrix ρ_{12} can be expanded in powers of the $|2\rangle \leftrightarrow |1\rangle$ probe field Rabi frequency g as [19,20]

$$\rho_{12} = A(G,\gamma_1,\gamma_2,p,\Delta_1,\Delta_2)+B(G,\gamma_1,\gamma_2,p,\Delta_1,\Delta_2)g, \quad (11)$$

where A is the lowest order nonlinear susceptibility. The dispersion and gain of the medium correspond to the real and imaginary parts of $\rho_{12} - A = \sigma_{12}$.

Firstly, we investigate the gain $\mathrm{Im}\sigma_{12}/g$ when the control field is resonant on the $|2\rangle \leftrightarrow |3\rangle$ transition ($\Delta_2 = 0$). The gain will be obtained if $\mathrm{Im}\sigma_{12}/g > 0$. The absence of population inversion on the $|2\rangle \leftrightarrow |1\rangle$ transition will hold for Figs. 2 and 3 in the subsection.

In Fig. 2, we present the gain as a function of the probe field frequency detuning Δ_1. As shown in Fig. 2(a), when VIC ($p = 0$) is absent, along with incoherent pump rate R increasing, the absorption of the probe field decreases at the Autler-Townes components. The Autler-Townes absorption doublets can be related to the dressed states which is a mixture of states $|2\rangle$ and $|3\rangle$, and the corresponding eigenvalues is $\frac{1}{2}(\Delta_2 \pm \sqrt{\Delta_2^2+4G^2})$. In the region around $\Delta_1 = 0$, only negligible gain occurs and it is hard to obtain large gain without inversion only with small incoherent pump rate. This is agreement with Ref. [18]. In Fig. 2(b), we calculate $\mathrm{Im}\sigma_{12}/g$ versus the probe detuning Δ_1 but for the case of $p = 0.99$ (VIC is considered). It is clear that we obtain remarkable gain without inversion around the two-photon resonance $\Delta_1 = 0$. For example, the values of $\mathrm{Im}\sigma_{12}/g$ (when $R = 0.25$, $\Delta_1 = 0$) corresponding to $p = 0.99$ and $p = 0$ are 1.36×10^{-2} and 3.48×10^{-4}. Clearly, the gain in the case of $p = 0.99$ is about 40 times lager than $p = 0$ with the same incoherent pump rate. And when $R = 0.75$, we find gain at zero is much larger than $R = 0.1$ and 0.25. Zhu[18] analyzed the origin of the steady gain and showed that the large gain depends on sufficient large incoherent pumping. But as we have shown above, the gain without inversion may be obtained and enhanced effectively with the help of VIC. The gain with inversion can be also improved due to the effect of VIC, but large incoherent pumping is necessary. Another interesting phenomenon is that the absorption peaks get closer to the center and the linewidth of the absorption profile becomes very narrower than previously.

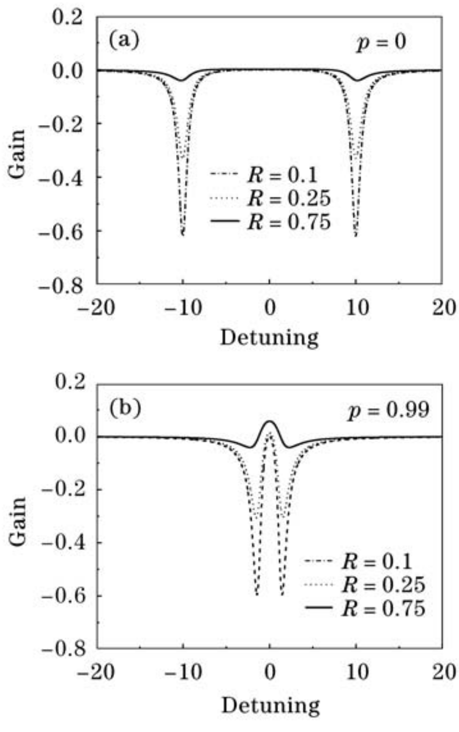

Fig. 2. The gain of the probe field (σ_{12}/g) versus the probe detuning Δ_1. $G_0 = 10$, $g_0 = 0.1$, $\gamma_1 = 1$, $\gamma_2 = 0.01$, and $\Delta_2 = 0$.

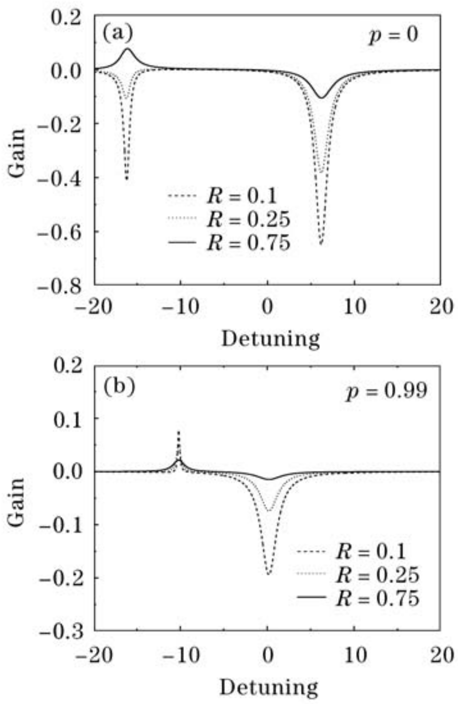

Fig. 3. The gain of the probe field (σ_{12}/g) versus the probe detuning Δ_1. $G_0 = 10$, $g_0 = 0.1$, $\gamma_1 = 1$, $\gamma_2 = 0.01$, and $\Delta_2 = 10$.

In Fig. 3, we show the effect of VIC on LWI as a function of the probe field detuning Δ_1 in the case of the control field detuning $\Delta_2 = 10$. As shown in Fig. 3(a) (no VIC considered, i.e., $p = 0$), when $R = 0.1$ and 0.25, the Autler-Townes absorption doublets become asymmetrical and larger incoherent pump accompanies smaller absorption. In Fig. 3(b), due to effects of VIC, even if the incoherent pump is very small, LWI can be realized at the two-photon resonance. For example, we show that when $R = 0.1$ and 0.25, the transition from absorption to gain for the probe field results from the effect of VIC ($p = 0.99$).

In conclusion, interferences via different decay paths have been proved to have substantial effects on the properties of the gain and absorption of the probe field in ladder-type atomic system with nearly equispaced levels. In particular, LWI can be significantly enhanced due to the effect of VIC with the small incoherent pump rate.

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References

1. P. Zhou and S. Swain, Phys. Rev. Lett. **78,** 832 (1997).
2. S.-Q. Gong, E. Paspalakis, and P. L. Knight, J. Mod. Opt. **45,** 2433 (1998).
3. S. Y. Zhu and M. O. Scully, Phys. Rev. Lett. **76,** 388 (1996).
4. H. Huang, S.-Y. Zhu, and M. S. Zubairy, Phys. Rev. A **55,** 744 (1997).
5. H. Lee, P. Polynkin, M. O. Scully, and S.-Y. Zhu, Phys. Rev. A **55,** 4454 (1997).
6. H.-R. Xia, C.-Y. Ye, and S.-Y. Zhu, Phys. Rev. Lett. **77,** 1032 (1996).
7. E. Paspalakis and P. L. Knight, Phys. Rev. Lett. **81,** 293 (1998).
8. E. Paspalakis, C. H. Keitel, and P. L. Knight, Phys. Rev A **58,** 4868 (1998).
9. F. Ghafoor, S.-Y. Zhu, and M. S. Zubairy, Phys. Rev. A **62,** 013811 (2000).
10. M. Macovei, J. Evers, and C. H. Keitel, Phys. Rev. Lett. **91,** 233601 (2003).
11. J.-H. Wu and J.-Y. Gao, Phys. Rev. A **65,** 063807 (2002).
12. C. Liu, S. Gong, X. Fan, and Z. Xu, Opt. Commun. **231,** 289 (2004).
13. X. Hu and J. Zhang, J. Phys. B **37,** 345 (2004).
14. J. Li, C. Liu, N. Cui, H. Li, and X. Jun, Chin. Opt. Lett. **2,** 725 (2004).
15. R. M. Whitley and C. R. Stroud Jr., Phys. Rev. A **14,** 1498 (1976).
16. Z. Ficek, B. J. Dalton, and P. L. Knight, Phys. Rev. A **51,** 4062 (1995).
17. H. Ma, S. Gong, C. Liu, Z. Sun, and Z. Xu, Opt. Commun. **223,** 97 (2003).
18. Y. Zhu, Opt. Commun. **105,** 253 (1994).
19. E. Paspalakis, S.-Q. Gong, and P. L. Knight, Opt. Commun. **152,** 293 (1998).
20. J. C. Petch, C. H. Keitel, P. L. Knight, and J. P. Marangos, Phys. Rev. A **53,** 543 (1996).