Effects of localized impurity of trap on characteristics of two Bose-Einstein condensates

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The characteristics of solitons with a localized impurity in Bose-Einstein condensates (BECs) are investigated with numerical simulations of the Gross-Pitaevskii (GP) equation, the effects of the impurity on BEC solitons are discussed, and the atom population transferring ratios between the two BECs as time goes on are analyzed. It is found that population transfer depends on the impurity strength and the parameters of the system of BECs.

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A convenient model to study the mean-field dynamics of Bose-Einstein condensates (BECs) is the Gross-Pitaevskii (GP) equation, which has the form of a three-dimensional (3D) nonlinear Schrödinger equation with an external trapping potential. In the case when the confinement for two of the three spatial dimensions is much stronger than for the third dimension, the GP equation can be reduced to an effective quasi-one-dimensional (1D) GP equation. This is in contrast to a truly 1D mean-field theory which requires transverse dimensions on the order of or less than the atomic interaction length. The recent trapping of a BEC in optical and magnetic traps demonstrates that a quasi-1D BEC is experimentally realizable. A variety of other experiments are also modeled by the 1D GP equation with an external potential. It is well known that the GP equation supports solitonic solutions. Dark solitons (with positive scattering length) of Bose condensed atoms were experimentally observed a few years ago, while bright solitons (with negative scattering length) have been detected only very recently with ^{7}Li using an optical red-detuned laser beam along the axial direction of the sample to impose a transverse (radial) confinement, and studied in 1D models[1−6].

Contrary to the 1D case, in two and three dimensions, stable bright solitons do not exist in a flat potential. However, if an external harmonic potential constrains the motion in two dimensions, such 3D soliton states exist but only below a critical number N_c of bosons. The value of N_c depends on the transverse confinement. More precisely it is proportional to the ratio of the transverse harmonic length to the scattering length[7].

The dynamics of a two-mode BEC of a mean-field dynamical instability are investigated recently. Using a density matrix formalism rather than the conventional wave-function methods, Anglin and Vardi derived an improved set of equations of motion for the mean-field plus the fluctuations, and showed that the leading quantum corrections appeared as decoherence of the reduced single-particle quantum state[8]. Salmond *et al.* considered a two-component BEC in two spatially localized modes of a double well potential with periodic modulation of the tunnel coupling, treated the driven quantum field using a two-mode expansion, and showed that the corresponding semiclassical mean-field dynamics can exhibit regions of regular and chaotic motion[9].

The mean field of a BEC with a delta function potential has been investigated, and the potential may be used to describe the interaction of a soliton with "artificial" impurities. Such an impurity could be induced with sharply focused laser beams intersecting with the condensate in experiments. It models the response of the condensate to an impurity of a length scale smaller than the healing length, which could be realized by a tightly focused laser beam, by another spin state of the same atom, or by any other object, such as another alkili atom, confined in an optical trap[10,11].

It is useful to transfer some of the techniques and insights from the fiber optics and the directional fiber coupler to the field of BEC dynamics[12−15]. In this letter, the characteristics of solitons with a localized impurity in BECs are investigated, and some novel results are obtained.

The basic equation governing the dynamics of BECs in confining potentials is given by the GP equation

$$j\frac{\partial u}{\partial t}+\frac{1}{2}\nabla^2 u+4\left|u\right|^2 u+Vu=0, \qquad (1)$$

where u is the condensate wave function, and $\rho^2 = x^2+y^2$. $V(x,y,z)$ is the normalized confining potential of the traps.

Two traps are separated from each other by a distance d. If the transverse confinement is much stronger than the confinement in the z direction and the transverse shape of the wave function does not change much in each trap, which implies that the confinement of atoms in two transverse (radial) directions is very strong and hence the transverse part of the order parameter is taken as being "frozen" to the ground-state wave function of the transverse confining potential, which has a Gaussian-type form for a two-dimensional (2D) harmonic oscillator potential. We will assume that the potential in z direction is weaker than the nonlinear interaction. Then the solution of Eq. (1) reads as[10]

$$u(\rho,z,t)=f_-(\rho+\frac{d}{2})u_1+f_+(\rho-\frac{d}{2})u_2, \qquad (2)$$

where $\rho=0$ occurs in the middle of the two traps. f_- and f_+ satisfy the eigenvalue problem of the 2D isotropic harmonic oscillator, $-\frac{1}{2}\nabla_\perp^2 f+\frac{1}{2}\rho^2 f=v_\rho f$. Its ground-state solution is $f_0(\rho)\sim \mathrm{e}^{-\rho^2/2}$. Multiplying Eq. (1) by

f^* and integrating to eliminate the ρ result in the following coupled 1D equations:

$$j\frac{\partial u_1}{\partial t}+\frac{1}{2}\frac{\partial^2 u_1}{\partial z^2}+2|u_1|^2u_1 +V'(z)u_1+E_0u_1+Ku_2=0,$$
$$j\frac{\partial u_2}{\partial t}+\frac{1}{2}\frac{\partial^2 u_2}{\partial z^2}+2|u_2|^2u_2 +V'(z)u_2+E_0u_2+Ku_1=0, \quad (3)$$

where E_0 is the ground-state energy and K is the linear coupling coefficient arising from overlaps of the transverse parts of the wave functions. $V'(z)$ is the normalized confining potential of the traps in the longitudinal direction (z direction). $N_1=\int_{-\infty}^{\infty}|u_1|^2\mathrm{d}z$ and $N_2=\int_{-\infty}^{\infty}|u_2|^2\mathrm{d}z$ are the respective numbers of atoms in each trap, and $N=N_1+N_2$ is the total number of atoms in two traps (a conserved quantity). The E_0 term can be eliminated by the transformation $u_i\to u_i\exp(-jE_0t)$ ($i=1,2$). The coupled BEC system can be described by the following two coupled nonlinear Schrödinger equations:

$$j\frac{\partial u_1}{\partial t}+\frac{1}{2}\frac{\partial^2 u_1}{\partial z^2}+2|u_1|^2u_1+V'(z)u_1+Ku_2=0,$$
$$j\frac{\partial u_2}{\partial t}+\frac{1}{2}\frac{\partial^2 u_2}{\partial z^2}+2|u_2|^2u_2+V'(z)u_2+Ku_1=0. \quad (4)$$

In order to study the characteristics of BEC solitons with a localized impurity in the framework of Eq. (4), it is convenient to decompose the external potential $V'(z)$ as[11]

$$V'(z)=U_{\mathrm{con}}+b\delta(z), \quad (5)$$

where U_{con} is the conventional time independent trapping potential, which is assumed to be smooth and slowly varying on the soliton scale, and the additional sharp potential $b\delta(z)$ accounts for an impurity localized in space at the point $z=0$, described by a Dirac $\delta(z)$ function. The parameter b in Eq. (5) which measures the impurity strength is assumed to be small and may take either positive or negative values for repulsive or attractive impurities, respectively.

In order to treat analytically the effect of the impurity on BEC solitons, the conventional trapping potential is assumed to be the external potential:

$$U_{\mathrm{con}}=\frac{1}{2}v^2z^2+V_0sn^2(x,k), \quad (6)$$

where the two terms represent the external magnetic trapping (MT) and the optical lattice (OL) potential respectively. MT is cylindrically symmetric with the harmonic frequency in the radial direction, and its geometry is cigar-shaped. v is the coefficient of the trap in the longitudinal direction (z direction). The OL trap created by interference patterns from multiple laser beams allows for a systematic study of the dynamics of coherent structures in the presence of periodic potentials. $sn(x,k)$ is Jacobi's elliptic sine (we always assume $k\to 0$ for simplicity in this letter), V_0 is the amplitude of potential.

We have used a split-step Fourier method to integrate the GP equation (4). In Figs. 1—4, we plot evolution of atom density (namely, $|u_1|^2$ and $|u_2|^2$) of each trap versus time by direct numerical simulation. The initial input pulses are $u_1(t=0)=\sqrt{N/2}\cos\alpha\sec h(z)$ and $u_2(t=0)=\sqrt{N/2}\sin\alpha\sec h(z)$ ($\alpha=\pi/6$ for simplicity). Except the especial indications in figure captions, the simulation parameters are selected as the coupling coefficient $K=1$, the amplitude of potential 0.1, the coefficient of trap $v=0.1$, the total number $N=6$, and the impurity strength $b=0.01$. In these figures, we find that the impurity strength plays an important role in the switching and self-trapping characteristics of the solitons. The impurity potential deforms evolution of the condensate wave function, and change characteristics of the system. For example, the large impurity may lead to the splitting of solitons and appearance of multi-peak pulse. Switching alternation in the periodic manner disappears and becomes disorderly as time goes on when the impurity strength becomes large.

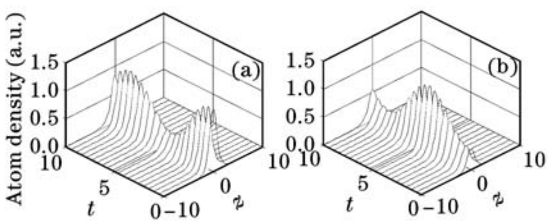

Fig. 1. Evolution of atom density of each trap versus time. The impurity strength $b=0$. (a) $|u_1|^2$; (b) $|u_2|^2$.

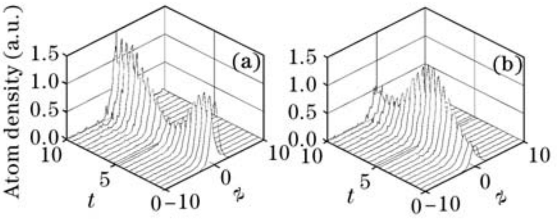

Fig. 2. Evolution of atom density of each trap versus time. The impurity strength $b=0.005$. (a) $|u_1|^2$; (b) $|u_2|^2$.

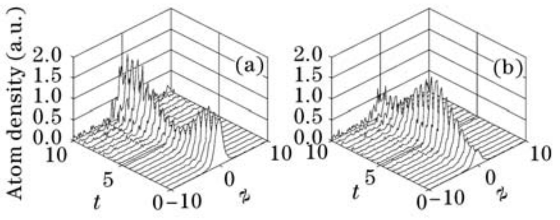

Fig. 3. Evolution of atom density of each trap versus time. The impurity strength $b=0.01$. (a) $|u_1|^2$; (b) $|u_2|^2$.

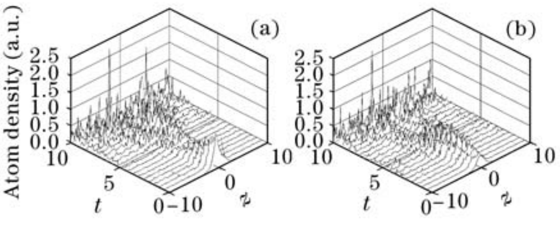

Fig. 4. Evolution of atom density of each trap versus time. The impurity strength $b=0.1$. (a) $|u_1|^2$; (b) $|u_2|^2$.

In order to further demonstrate the effect of the impurity, we introduce the population transferring ratios

$$R(t) = \frac{N_1 - N_2}{N}. \quad (7)$$

We plot the changes in population transferring ratios versus time in Figs. 5—8. The initial input pulses and the simulation parameters have been selected except the especial indications in captions. We can see the effects of the parameters on characteristics of the solitons. For example, it is found that the behavior of the system would be sensitive to the change in the OL potential and linear coupling coefficient in presence of impurity potential, but insensitive to the change in the external magnetic trapping.

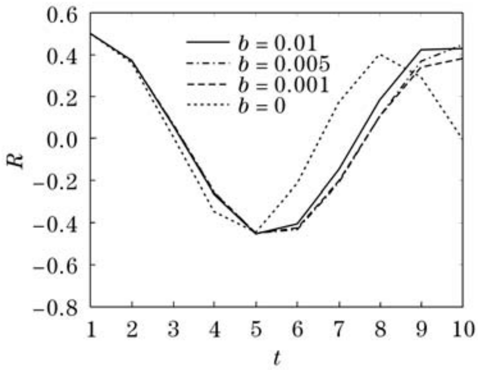

Fig. 5. Changes in the population transferring ratios versus time with different impurity strengths.

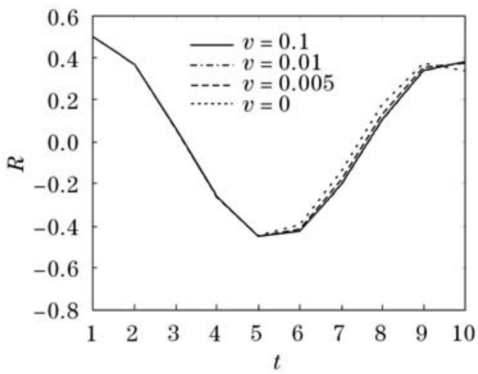

Fig. 6. Changes in the population transferring ratios versus time with different coefficients of trap.

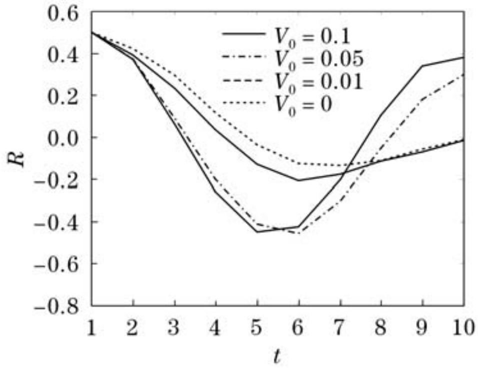

Fig. 7. Changes in the population transferring ratios versus time with different amplitudes of the potential.

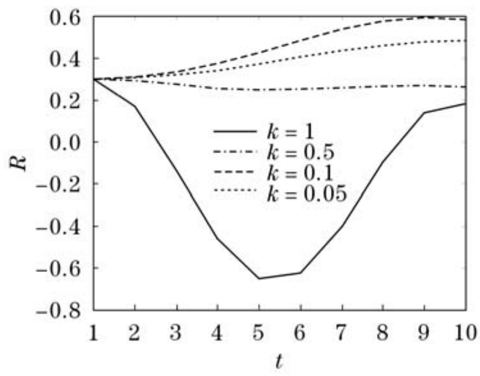

Fig. 8. Changes in the population transferring ratios versus time with different linear coupling coefficients.

In summary, the switching and self-trapping characteristics of bright solitons with a localized impurity in BECs are investigated, and the evolution of atom density of each trap as time goes on is discussed. It is found that the impurity strength plays an important role in the switching and self-trapping characteristics of the solitons. The impurity potential deforms evolution of the condensate wave function, and changes characteristics of the system. The large impurity may lead to the splitting of solitons and appearance of multi-peak pulse. The population transfer between two BECs depends on the impurity strength and the parameters of the system.

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References

1. M. H. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman, and E. A. Cornell, Science **269,** 198 (1995).
2. Y. Wu, X. Yang, and Y. Xiao, Phys. Rev. Lett. **86,** 2200 (2001).
3. G.-S. Chong, W.-H. Hai, and Q.-T. Xie, Chin. Phys. Lett. **20,** 2098 (2003).
4. G.-X. Huang, M. G. Velarde, and V. A. Makarov, Phys. Rev. A **64,** 013617 (2001).
5. T. Hong, Y. Z. Wang, and Y. S. Huo, Phys. Rev. A **58,** 3128 (1998).
6. G.-X. Huang, V. A. Makarov, and M. G. Velarde, Phys. Rev. A **67,** 023604 (2003).
7. L. Salasnich, A. Parola, and L. Reatto, Phys. Rev. A **66,** 043603 (2002).
8. J. R. Anglin and A. Vardi, Phys. Rev. A **64,** 013605 (2001).
9. G. L. Salmond, C. A. Holmes, and G. J. Milburn, Phys. Rev. A **65,** 033623 (2002).
10. S. Raghavan and G. P. Agrawal, J. Mod. Opt. **47,** 1155 (2000).
11. D. J. Frantzeskakis, G. Theocharis, F. K. Diakonos, P. Schmelcher, and Y. S. Kivshar, Phys. Rev. A **66,** 053608 (2002).
12. H. Li and D.-X. Huang, Chin. Phys. Lett. **20,** 417 (2003).
13. H. Li, D.-X. Huang, and D. N. Wang, Microwave and Opt. Technol. Lett. **35,** 502 (2002).
14. H. Li and D.-N. Wang, Opt. Commun. **191,** 405 (2001).
15. H. Li, D.-X. Huang, and D.-N. Wang, Chin. Phys. **12,** 415 (2003).