Effect of third-order dispersion on breathing localized solutions in the quintic complex Ginzburg-Landau equation

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The effect of third-order dispersion on breathing localized solutions in the quintic complex Ginzburg-Landau (CGL) equation is investigated. It is found that even small third-order dispersion can cause dramatic changes in the behavior of the solutions, such as breathing solution asymmetrically and travelling slowly towards the right for the positive third-order dispersion. A little larger third-order dispersion causes the solution breathing only on one side and the other side keeping the soliton profile. For the negative dispersion, the same results can be obtained except for the change of the traveling direction. Otherwise, we analyzed the interaction of two breathing solitons and found a simple method to inhibit this interaction.

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The well-known complex Ginzburg-Landau (CGL) equation is a generic equation with both dissipation and dispersion, which describes the system near a subcritical bifurcation for travelling waves. The equation and its modifications describe a variety of phenomena[1−5], for example, open flow motions, travelling waves in binary fluid mixtures, and spatially extended nonequilibrium systems. More recently stable localized solutions have been found to occur in quintic CGL equation[4−7]. Although these solutions share some properties with solitons that occur in purely dispersive systems, such as a fixed shape for the modulus and interaction behavior in which shape and size are preserved during collisions, there are fundamental differences. For example, these solutions exhibit mutual annihilation during collisions[5,8], a property which does not occur for solitons. Furthermore, in contrast to solitons which require no energy input for their existence, the dissipation-dispersive localized solutions depend on a constant influx of energy in order to overcome the dissipation.

Recently, three novel solutions have been presented by Soto-Crespo *et al.*, which are pulsating, erupting, and creeping solitons[7,9]. Tian *et al.* have found that the nonlinear gradient terms can dramatically change the behavior of these solitons[5]. For the breathing solutions presented by Deissler *et al.*[10], the modulus breathes periodically, quasiperiodically and even chaotically and the effect of nonlinear gradient terms on the breathing localized solutions has also been analyzed[8]. They found that even small nonlinear gradient terms can cause dramatic changes in the behavior of the solution, such as causing opposite sides of an otherwise monoperiodic symmetrically breathing solution to breathe at different frequencies. But all of these reports have neglected third-order dispersion in the equation. In fact, the CGL equation with higher-order dispersions is worth further investigating because the third-order dispersion is an important factor in the passively mode-locked lasers[11−13] and the optical fiber transmission[14−17]. From these lasers, the very useful ultrashort pulses, whose widths are shorter than 100 fs, can be generated.

In this paper, we study the effect of third-order dispersion on the breathing solutions of the quintic CGL equation. We find that even small third-order dispersion makes the solution breathe asymmetrically and a little larger third-order dispersion causes the solution breathing only on one side and the other side keeping the soliton profile. Otherwise, the pulse envelope has a definite velocity towards the right under the effect of positive third-order dispersion.

The quintic CGL equation with third-order dispersion can be written as

$$U_z = A_0 U + A_1 U_{tt} + A_2 \left|U\right|^2 U + A_3 \left|U\right|^4 U + A_4 U_{ttt}, \tag{1}$$

where U is a slowly varying complex amplitude, z represents the distance along the propagation direction, t represents the time, and A_j $(j = 0, 1, \cdots, 4)$ is parameter of the equation, in which A_0 and A_4 are real, A_1, A_2, and A_3 are complex, i.e., of the form $A_j = A_{j\mathrm{r}} + iA_{j\mathrm{i}}$ $(j = 1, 2, 3)$. The physical meaning of each parameter depends on the particular problem.

Figure 1 shows the space-time plots of the modulus of U for various values of the third-order dispersion, keeping the other parameter values fixed as $A_0 = -0.1$, $A_{1\mathrm{r}} = 1.0$, $A_{1\mathrm{i}} = -1.1$, $A_{2\mathrm{r}} = 3.0$, $A_{2\mathrm{i}} = 1.0$, $A_{3\mathrm{r}} = -2.75$, and $A_{3\mathrm{i}} = 1.0$. The numerical simulations have been carried out using a split-step technique with up to 2048 points along the t direction and step size 0.1 along the z direction. For comparison purposes, the corresponding contour plots are shown in Fig. 2, respectively. Figures 1(a) and 2(a) show the solution with coefficient of the third-order-dispersion equals to zero, i.e. $A_4 = 0$. The waves breathe symmetrically down the left and right sides of the solution. Figures 1(b) and 2(b) show the solution with $A_4 = 0.03$. As can be seen, the solution breathes no longer symmetrically. The left side of the pulse breathes more weakly and the right side breathes more deeply as a result of the effect of third-order dispersion. We note that the solution is also travelling slowly towards the right. For larger third-order dispersion, the right side of the pulse breathes more deeply and the left even more weakly, as shown in Figs. 1(c) and 2(c) for $A_4 = 0.05$. When the third-order dispersion parameter $A_4 = 0.1$, the breathing on the left side of the pulse almost vanished and the pulse breathes only on the right side, as shown in Figs. 1(d) and 2(d). This result is in agreement with that described in Refs. [18—20], in which it has been pointed out that the effect of third-order dispersion will cause asymmetric pulse and lead to the appearance of oscillation on the trailing edge of the pulse for the positive third-order dispersion.

Otherwise, we have analyzed the interaction between the adjacent two localized solutions. Figures 3(a) and (b) show the space-time and contour plots of two pulses when the distance of them is 42. The initial condition is given as $U(t,0) = \mathrm{sech}(t-q_0) + \mathrm{sech}(t+q_0)$ and $q_0 = 21$. From Fig. 3, we can see that there is strong interplay between the adjacent pulses when the distance of them is equal or shorter than 42. As mentioned in Refs. [5, 8], mutual annihilation is exhibited during the collision happens between them, as shown in Fig. 3. But the interplay is not a favorable factor for the transmission of the pulses. It will cause the deformation of the pulse and the missing of the information. Therefore it is important to search for methods to maintain the soliton separation. For this intention, three methods have been suggested[21]: 1) phase alteration between neighbouring solitons,

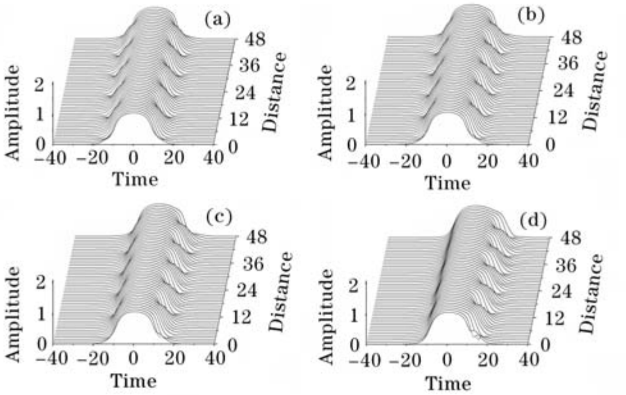

Fig. 1. Space-time plots of the breathing localized solutions for various values of third-order dispersion. (a) $A_4 = 0$, (b) $A_4 = 0.03$, (c) $A_4 = 0.05$, (d) $A_4 = 0.1$.

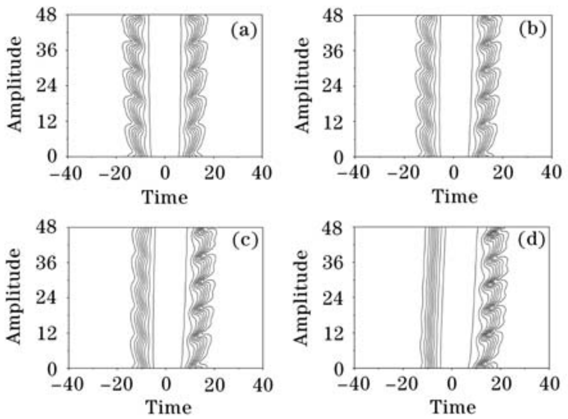

Fig. 2. Contour plots of Fig. 1. (a) $A_4 = 0$, (b) $A_4 = 0.03$, (c) $A_4 = 0.05$, (d) $A_4 = 0.1$.

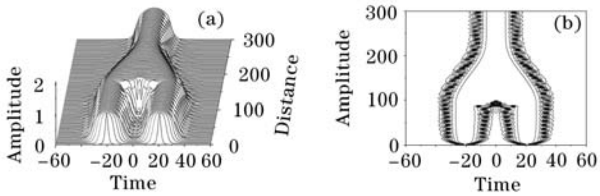

Fig. 3. (a) Space-time plots and (b) corresponding contour plots of two adjacent pulses for $q_0 = 21$.

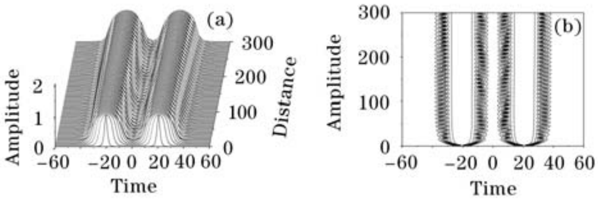

Fig. 4. (a) Space-time plots and (b) corresponding contour plots of two adjacent pulses for $q_0 = 21$ and initial phase $\Omega = 0.18$.

2) unequal amplitudes between neighbouring solitons, and 3) inclusion of third-order dispersion. Among these, the second method seems to offer more flexibility and is easier to implement than the other two. But it can not work here. So we choose the first method, named the initial condition as $U(t,0) = \mathrm{sech}(t-q_0)\exp(i\Omega t) + \mathrm{sech}(t+q_0)$, in which $q_0 = 21$ and $\Omega = 0.18$. The numerical results are shown in Figs. 4(a) and (b), the interactions between neighbouring solitons are suppressed effectively.

In general, we have studied the effect of third-order dispersion on the breathing localized solutions in the quintic CGL equation. This is important since the third-order dispersion is an important factor in many systems, such as passively mode-locking lasers. We find that even small third-order dispersion can dramatically change the behavior of the solution, causing one side of the pulse breathe more weakly and the other more deeply. Otherwise, the interplay between two adjacent pulses is analyzed and the interaction has been suppressed effectively by using the method of phase alteration between neighbouring solitons.

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