Bright and dark optical solitons in the nonlinear Schrödinger equation with fourth-order dispersion and cubic-quintic nonlinearity

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By the use of an auxiliary equation, we find bright and dark optical soliton and other soliton solutions for the higher-order nonlinear Schrödinger equation (NLSE) with fourth-order dispersion (FOD), cubic-quintic terms, self-steepening, and nonlinear dispersive terms. Moreover, we give the formation condition of the bright and dark solitons for this higher-order NLSE.

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The possibility of the propagation of envelop solitons in optical fibers was theoretically predicted by Hasegawa *et al.*[1], and then was experimentally demonstrated by Mollenauer *et al.*[2]. It is well known that the propagation of a picosecond optical pulse in a monomode optical fiber (not including optical fiber loss) is described by the nonlinear Schrödinger equation (NLSE)[1],

$$iE_z - \frac{\beta_2}{2}E_{tt} + \gamma_1|E|^2E = 0, \tag{1}$$

where E is a complex envelop amplitude, t represents the time (in the group-velocity frame), z represents the distance along the direction of propagation (the longitudinal coordinate), $\beta_2 = \frac{\partial^2 k}{\partial \omega^2}|_{\omega=\omega_0}$ describes group velocity dispersion (GVD), k is the axial wave number, ω_0 is the carrier wave frequency, $\gamma_1 = n_2\omega_0/(cA_{\rm eff})$ is the self phase modulation (SPM) parameter, where n_2 is the Kerr coefficient, c is the speed of light in vacuum, and $A_{\rm eff}$ is the effective core area of the fiber. Such an equation predicts bright soliton solutions in the anomalous dispersion region of the fiber and dark soliton solutions (the intensity profile contains a dip in a uniform background) in normal dispersion regime[3]. This equation is completely integrable by the inverse scattering transform[4].

Propagations of ultrashort light pulses in optical fibers are of particular interest and practical value because of their extensive applications to telecommunication and ultrafast signal-routing system. However, when short pulses are considered, e.g., to nearly 50 fs, even below 10 fs, the classic NLSE (Eq. (1)) fails in the physical description of the propagations of light pulses in fibers. In such a system, the third-order dispersion (TOD) and the fourth-order dispersion (FOD) must be taken into account[5]. Meanwhile, other phenomena like self-steepening (also called Kerr dispersion) are inevitable.

Moreover, when the optical field frequency approaches a resonant frequency of the optical fibers material, accounting for higher-order nonlinearities in optical fibers is related to stronger optical fields and the corresponding larger powers launched into them. Pushkarov *et al.*[6] modelled the refractive index nonlinearity, i.e., $n = n_0 + n_2|E|^2 + n_4|E|^4 + \cdots$ (E is the electric field amplitude, $n_2 = 3\chi^{(3)}/(8n_0)$, $n_4 = 5\chi^{(5)}/(16n_0)$, $\chi^{(3)}$ and $\chi^{(5)}$ being the components of the corresponding nonlinear dielectric tensors and n_0 being the linear refractive index coefficient). If the expansion is cut to fourth-order terms with respect to $|E|^4$, then the cubic-quintic nonlinearities are introduced into Eq. (1).

Generally, corrections to the linear dispersion should be taken into account together with nonlinear corrections to the cubic nonlinear term being the same order. With the effect of all these physical processes above mentioned, wave dynamics of optical fiber system is governed by the higher-order NLSE,

$$iE_z - \frac{\beta_2}{2}E_{tt} + \gamma_1|E|^2E = i\frac{\beta_3}{6}E_{ttt} + \frac{\beta_4}{24}E_{tttt} - \gamma_2|E|^4E + i\alpha_1(|E|^2E)_t + i\alpha_2E(|E|^2)_t, \tag{2}$$

where $\beta_3 = \frac{\partial^3 k}{\partial \omega^3}|_{\omega=\omega_0}$, $\beta_4 = \frac{\partial^4 k}{\partial \omega^4}|_{\omega=\omega_0}$ represent TOD and FOD, respectively, k is the axial wave number, ω_0 is the carrier wave frequency, and γ_1 and γ_2 are the cubic and quintic nonlinearities coefficients, respectively. The term proportional to α_1 results from the first derivative of the slowly varying part of the nonlinear polarization. It is responsible for self-steepening (SS) and shock formation at a pulse edge. The last term proportional to α_2 has its origin in the delayed Raman response, and, generally, α_2 should be complex. In many cases, however, ${\rm Im}\alpha_2 \ll {\rm Re}\alpha_2$[7]. So we consider only the real part of α_2 (here called nonlinear dispersion) in this letter. In our analysis, the parameters do not need to be small, so we are not limited to the perturbation regime.

When the terms on the right-hand side are negligible, the Eq. (2) becomes Eq. (1). Equation (2) with $\beta_3 = \beta_4 = \alpha_1 = \alpha_2 = 0$ is the cubic-quintic NLSE, whose exact travelling wave solutions were discussed in Ref. [8]. When $\beta_4 = \gamma_2 = \alpha_2 = 0$ in Eq. (2), the effects of high-order dispersion and nonlinearity on pulse shape and wavelength drift of femtosecond soliton pulse in single-mode fiber having normal dispersion were examined numerically by Xie *et al.*[9]. Recently, Karpman *et al.*[10] have investigated the resonant radiation and evolution of a soliton described by the higher-order NLSE with third and fourth derivatives ($\gamma_2 = \alpha_1 = \alpha_2 = 0$ in Eq. (2)). When $\beta_4 = \gamma_2 = \alpha_2 = 0$ of Eq. (2), Liu *et al.*[11] obtained the exact N-soliton solutions by Hirota's direct method. For the constrained case, i.e., $\alpha_1 = \alpha_2 = 0$ of Eq. (2), Li *et al.*[12] derived some analytic soliton solutions, and Ren *et al.*[13] discussed the modulational in-stability. In absence of the quintic nonlinear term and FOD term ($\beta_4 = \gamma_2 = 0$) of Eq. (2), it is shown that dark and bright soliton solutions exist[14,15]. Moreover, the stability of the bright optical solitary wave solution against some kinds of initial perturbations by employing numerical simulation was investigated[16]. However, the exact analytic solutions of Eq. (2) have not been previously obtained. It is always useful to construct exact analytical solutions and it is worthwhile to investigate the exact solutions (in particular soliton solutions).

In this paper, we derived analytic soliton solutions (bright and dark soliton and other soliton solutions) for Eq. (2) by using an algebraic method with an auxiliary equation. And the properties of the obtained solutions are shown graphically. It is difficult to solve a fourth-order derivative equation with quintic terms by a direct integration process used in Refs. [16, 17]. The virtue of this method in this letter is that, without much complicated calculations, we circumvent integration to directly get both bright and dark solitons in a uniform way (only difference in the parameters in the auxiliary equations (3)), which is simpler than the direct ansatz approach in Ref. [18]. Another feature of this method is that it is independent of the integrability of nonlinear equations. In fact, we can also derive the period sn, cn, and dn solutions with the auxiliary equations (3). Here we omit them.

To seek more travelling wave solutions to Eq. (2), we introduce the following auxiliary equation,

$$\begin{aligned} g_\xi^2 &= 2(c_0 + c_1 g + c_2 g^2 + c_3 g^3 + c_4 g^4), \\ g_{\xi\xi} &= c_1 g + 2c_2 g + 3c_3 g^2 + 4c_4 g^3, \end{aligned} \tag{3}$$

where $\xi = \xi(z,t)$ and c_i $(i = 0-4)$ are arbitrary constants. Equation (3) has the following solutions,

$$\begin{aligned} g &= \sqrt{-\frac{c_2}{c_4}}\operatorname{sech}(\sqrt{2c_2}\xi), \\ &\quad (c_0 = c_1 = c_3 = 0, c_2 > 0, c_4 < 0); \\ g &= \epsilon\sqrt{-\frac{c_2}{2c_4}}\tanh(\sqrt{-c_2}\xi), \\ &\quad (c_1 = c_3 = 0, c_0 = c_2^2/(2c_4), c_2 < 0, c_4 > 0, \epsilon^2 = 1); \\ g &= \frac{c_2\operatorname{sech}^2(\sqrt{2c_2}\xi/2)}{2\epsilon\sqrt{c_2c_4}\tanh(\sqrt{2c_2}\xi/2) - c_3}, \\ &\quad (c_0 = c_1 = 0, c_2, c_4 > 0, \epsilon^2 = 1); \\ g &= \frac{2c_2\operatorname{sech}^2(\sqrt{2c_2}\xi)}{\sqrt{c_3^2 - 4c_2c_4} - c_3\operatorname{sech}(\sqrt{2c_2}\xi)}, \\ &\quad (c_0 = c_1 = 0, c_2 > 0, c_3^2 - 4c_2c_4 > 0); \\ g &= \frac{c_2c_3\operatorname{sech}^2(\epsilon\sqrt{2c_2}\xi/2)}{c_2c_4(1 - \tanh(\epsilon\sqrt{2c_2}\xi/2))^2 - c_3^2}, \\ &\quad (c_0 = c_1 = 0, c_2 > 0, \epsilon^2 = 1). \end{aligned} \tag{4}$$

Meanwhile, in order to derive some exact solutions of Eq. (2), we make the transformation

$$\begin{aligned} E(z,t) &= A(\xi)\exp(i\theta), \\ \xi &= pz - t, \quad \theta = kz - ct, \end{aligned} \tag{5}$$

where the amplitude $A(\xi)$ is real. The inverse pulse width p, the phase shift k, and the frequency shift c are all constants. Substituting Eq. (5) into Eq. (2) and separating the real and imaginary parts lead to

$$\begin{aligned} &(\beta_3 - \beta_4 c)\,A''' + \left(6p - 6\beta_2 c - 3\beta_3 c^2 + \beta_4 c^3\right) A' \\ &\quad + (18\alpha_1 + 12\alpha_2)A^2A' = 0, \end{aligned} \tag{6}$$

$$\begin{aligned} &\beta_4 A^{(4)} + \left(12\beta_2 + 12\beta_3 c - 6\beta_4 c^2\right) A'' \\ &\quad + \left(24k - 12\beta_2 c^2 - 4\beta_3 c^3 + \beta_4 c^4\right) A \\ &\quad - 24(\gamma_1 - \alpha_1 c)A^3 - 24\gamma_2 A^5 = 0. \end{aligned} \tag{7}$$

Inserting the differential of Eq. (6) in Eq. (7) have

$$\begin{aligned} &\left[\beta_4\left(6p - 6\beta_2 c - 3\,\beta_3 c^2 + \beta_4 c^3\right)\right. \\ &\quad \left. - (\beta_3 - \beta_4 c)\left(12\beta_2 + 12\beta_3 c - 6\beta_4 c^2\right)\right] A'' \\ &+ \beta_4(18\alpha_1 + 12\alpha_2)(2AA'^2 + A^2A'') \\ &- (\beta_3 - \beta_4 c)\left[\left(24k - 12\beta_2 c^2 - 4\beta_3 c^3 + \beta_4 c^4\right) A\right. \\ &\quad \left. - 24(\gamma_1 - \alpha_1 c)A^3 - 24\gamma_2 A^5\right] = 0. \end{aligned} \tag{8}$$

Assuming

$$A(\xi) = a_0 + \sum_{j=1}^{l} a_j[g(\xi)]^j, \tag{9}$$

where $g(\xi)$ satisfies Eq. (3). By balancing the highest-order derivative A^2A'' term with the nonlinear A^5 term in Eq. (8), we get $l = 1$ in Eq. (9). Substituting Eq. (9) with $l = 1$ into Eq. (8) and setting the coefficients of individual term to zero. To avoid the tediousness, we omit the over-determined system. With the aid of Maple, we have

$$\begin{aligned} p = {} & (20\gamma_2\beta_4{}^3c^4c_4 - 80\gamma_2\beta_4{}^2c^3\beta_3c_4 - 24\gamma_2\beta_4{}^2c^2\beta_2c_4 \\ & - 96\alpha_2c_4\beta_4c^2\alpha_1 - 144\alpha_1{}^2c_4\beta_4c^2 + 108\gamma_2\beta_4c^2\beta_3{}^2c_4 \\ & + 144\alpha_1c_4\beta_4c\gamma_1 + 72\gamma_2\beta_4c\beta_3\beta_2c_4 + 96\alpha_2c_4\beta_4c\gamma_1 \\ & - 48\gamma_2\beta_3{}^3cc_4 + 96\alpha_2c_4\beta_3\alpha_1c + 144\alpha_1{}^2c_4\beta_3c \\ & + 243c_3{}^2\alpha_1{}^2 + 108c_3{}^2\alpha_2{}^2 - 144\alpha_1c_4\beta_3\gamma_1 + 324c_3{}^2\alpha_1\alpha_2 \\ & - 288\alpha_2{}^2c_4c_2 - 48\gamma_2\beta_3{}^2\beta_2c_4 - 96\alpha_2c_4\beta_3\gamma_1 \\ & - 864\alpha_2c_4c_2\alpha_1 - 648\alpha_1{}^2c_4c_2) \\ & /(24\gamma_2\beta_4c_4\left(\beta_4c - \beta_3\right)), \\ k = {} & -(3456\alpha_1{}^2c_4{}^3c_0 + 1536\alpha_2{}^2c_4{}^3c_0 + 8\gamma_2\beta_4{}^3c^6c_4{}^2 \\ & - 96\gamma_2\beta_3{}^2\beta_2c^2c_4{}^2 + 4608\alpha_2c_4{}^3\alpha_1c_0 - 32\gamma_2\beta_3{}^3c^3c_4{}^2 \\ & + 192\gamma_2\beta_4c^3\beta_2\beta_3c_4{}^2 - 96\gamma_2\beta_4{}^2c^4\beta_2c_4{}^2 - 48\gamma_2\beta_4{}^2c^5\beta_3c_4{}^2 \\ & + 72\gamma_2\beta_4c^4\beta_3{}^2c_4{}^2 + 144c_4c_3{}^2\beta_3\gamma_1\alpha_2 + 216c_4c_3{}^2\beta_3\gamma_1\alpha_1 \\ & - 132c_3{}^4\alpha_2{}^2 - 297c_3{}^4\alpha_1{}^2 - 396c_3{}^4\alpha_1\alpha_2 \\ & - 216c\beta_4c_4c_3{}^2\gamma_1\alpha_1 - 144c\beta_4c_4c_3{}^2\gamma_1\alpha_2 - 144cc_4c_3{}^2\beta_3\alpha_1\alpha_2 \\ & - 216cc_4c_3{}^2\beta_3\alpha_1{}^2 + 144c^2\beta_4c_4c_3{}^2\alpha_1\alpha_2 + 216c^2\beta_4c_4c_3{}^2\alpha_1{}^2 \\ & + 2304c_4c_3{}^2\alpha_1c_2\alpha_2 + 1728c_4c_3{}^2\alpha_1{}^2c_2 + 768c_4c_3{}^2\alpha_2{}^2c_2 \\ & - 2592c_2{}^2c_4{}^2\alpha_1{}^2 - 3456c_2{}^2c_4{}^2\alpha_2\alpha_1 - 384c_2c_4{}^2\alpha_2\beta_4c^2\alpha_1 \\ & - 1152c_2{}^2c_4{}^2\alpha_2{}^2 + 576c_2c_4{}^2\alpha_1\beta_4c\gamma_1 + 384c_2c_4{}^2\alpha_2\beta_4c\gamma_1 \\ & + 384c_2c_4{}^2\alpha_2\beta_3\alpha_1c + 576c_2c_4{}^2\alpha_1{}^2\beta_3c - 576c_2c_4{}^2\alpha_1\beta_3\gamma_1 \\ & - 384c_2c_4{}^2\alpha_2\beta_3\gamma_1 - 576c_2c_4{}^2\alpha_1{}^2\beta_4c^2) \\ & /\left(192\gamma_2c_4{}^2\left(\beta_4{}^2c^2 - 2\beta_4c\beta_3 + \beta_3{}^2\right)\right), \end{aligned}$$

$$c_1 = \frac{(4c_2c_4 - c_3^2)c_3}{8c_4^2},$$
$$a_0 = \pm\frac{c_3}{4c_4}\sqrt{\frac{4\alpha_2c_4 + 6\alpha_1c_4}{\gamma_2\beta_4c - \gamma_2\beta_3}},$$
$$a_1 = \pm\sqrt{\frac{4\alpha_2c_4 + 6\alpha_1c_4}{\gamma_2\beta_4c - \gamma_2\beta_3}}, \qquad (10)$$

where c_0, c_2, c_3, c_4, and c are arbitrary constants. Therefore from Eqs. (4), (5), (9), and (10), we can obtain bright and dark soliton solutions and other soliton solutions to Eq. (2).

Bright soliton

$$E_1 = a_1\sqrt{-\frac{c_2}{c_4}}\text{sech}[\sqrt{2c_2}(pz - t)]\exp[i(kz - ct)], \quad (11)$$

where p, k, a_1 are given by Eq. (10), c is an arbitrary constant, and $c_2 > 0$, $c_4 < 0$. In contrast with dark soliton, for fixed frequency and intensity, just one bright solution is possible, and bright solitons have a constant phase. As shown in Fig. 1(a), if setting $\alpha_1 = \alpha_2 = 0.1$, $\beta_2 = -1$, $\beta_3 = 0.9$, $\beta_4 = 0.5$, $\gamma_1 = \gamma_2 = 1$, $c = 1$, $c_2 = 0.5$, $c_4 = -1$, $c_0 = c_3 = 0$, $a_1 = 1.58$, at $z = 0.1$ in Eq. (11), the intensity $|E|^2$ is bright soliton, and the corresponding spatio-temporal plot is depicted in Fig. 1(b).

Dark soliton

$$E_2 = \epsilon a_1\sqrt{-\frac{c_2}{2c_4}}\tanh[\sqrt{-c_2}(pz - t)]\exp[i(kz - ct)], \quad (12)$$

where p, k, a_1 are given by Eq. (10), c is an arbitrary constant, and $c_2 < 0, c_4 > 0, \epsilon^2 = 1$. Dark optical solitons have been researched theoretically and experimentally since the early work of Zakharov and Shabat *et al.*[19]. One main reason for such a pursuit could be the possible application of dark solitons for long-distance communications taking advantage of its stability under the influence of the material losses, a lower Gordon-Haus jitter[20]. As shown in Fig. 2(a), if setting $\alpha_1 = \alpha_2 = 0.1$, $\beta_2 = -1$, $\beta_3 = 0.9$, $\beta_4 = 0.5$, $\gamma_1 = \gamma_2 = 1$, $c = 2$, $c_2 = -0.5$, $c_4 = 1$, $c_0 = c_3 = 0$, $a_1 = 3.16$, at $z = 0.1$ in Eq. (12), the intensity $|E|^2$ is dark soliton, and the corresponding spatio-temporal plot is depicted in Fig. 2(b).

From the expression a_1 in Eq. (10), we have the condition $(4\alpha_2c_4 + 6\alpha_1c_4)(\gamma_2\beta_4c - \gamma_2\beta_3) > 0$. From the condition we can see the formation of the soliton are decided by α, β_3, β_4, γ_2, γ_4, namely, the right terms in Eq. (2). When the SS and the nonlinear dispersion (ND) are both neglected ($\alpha_1 \to 0$, $\alpha_2 \to 0$), both the bright soliton and the dark soliton disappear, but either the SS or the ND exists, the bright soliton and the dark soliton do not disappear. Neither the quintic nonlinearity nor both TOD and FOD are considered ($\gamma_2 \to 0$ and $\beta_3, \beta_4 \to 0$), both the bright soliton and the dark soliton are destroyed, but either TOD or FOD is not neglected (when $\gamma_2 \neq 0$) the bright soliton and the dark soliton exist. Moreover, opposite values of c_4 in Eqs. (11) and (12) hint that the formation condition of the bright and dark solitons is different, i.e., the formation condition for the bright soliton

$$(4\alpha_2 + 6\alpha_1)(\gamma_2\beta_4c - \gamma_2\beta_3) < 0, \qquad (13)$$

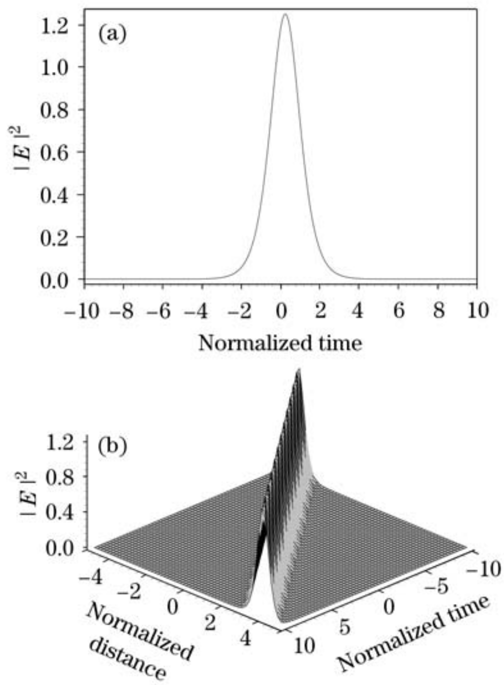

Fig. 1. (a) The intensity of bright soliton at $z = 0.1$ for $\alpha_1 = \alpha_2 = 0.1$, $\beta_2 = -1$, $\beta_3 = 0.9$, $\beta_4 = 0.5$, $\gamma_1 = \gamma_2 = 1$, $c = 1$, $c_2 = 0.5$, $c_4 = -1$, $c_0 = c_3 = 0$, $a_1 = 1.58$ in Eq. (11), (b) the corresponding spatio-temporal plot with the same parameters as (a).

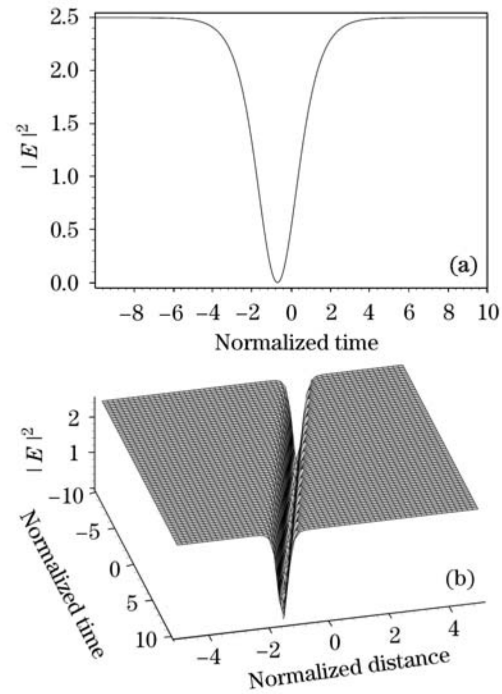

Fig. 2. (a) The intensity of dark soliton at $z = 0.1$ for $\alpha_1 = \alpha_2 = 0.1$, $\beta_2 = -1$, $\beta_3 = 0.9$, $\beta_4 = 0.5$, $\gamma_1 = \gamma_2 = 1$, $c = 2$, $c_2 = -0.5$, $c_4 = 1$, $c_0 = c_3 = 0$, $a_1 = 3.16$ in Eq. (12), (b) the corresponding spatio-temporal plot with the same parameters as (a).

and for the dark soliton

$$(4\alpha_2 + 6\alpha_1)(\gamma_2\beta_4c - \gamma_2\beta_3) > 0. \qquad (14)$$

From Eqs. (13) and (14), the bright soliton exists both for the anomalous-dispersion fiber and the normal-dispersion fiber if satisfying Eq. (13), and the dark soliton also exists both for the anomalous-dispersion fiber and the normal-dispersion fiber if satisfying Eq. (14), which is different from the existence of the bright soliton in anomalous-dispersion fiber ($\beta_2 < 0$) and dark soliton in normal dispersion fiber ($\beta_2 > 0$) for standard NLSE (Eq. (1)). It is interesting that the bright and dark solitons in the higher-order NLSE (Eq. (2)) exist even if there is not effect of the GVD and SPM ($\beta_2 \to 0$, $\gamma_1 \to 0$).

Other soliton solutions

$$E_3 = [a_0 + a_1 \frac{c_2 \mathrm{sech}^2[\sqrt{c_2/2}(pz-t)]}{2\epsilon\sqrt{c_2 c_4}\tanh[\sqrt{c_2/2}(pz-t)] - c_3}] \exp[i(kz - ct)], \quad (15)$$

where p, k, a_0, a_1 are given by Eq. (10), c is an arbitrary constant, and $c_2 > 0$, $c_4 > 0$, $4c_2c_4 = c_3^2$, $\epsilon^2 = 1$. And

$$E_4 = [a_0 + a_1 \frac{c_2 c_3 \mathrm{sech}^2[\sqrt{c_2/2}(pz-t)]}{c_2 c_4[1 - \tanh(\epsilon\sqrt{c_2/2}(pz-t))]^2 - c_3^2}] \exp[i(kz - ct)], \quad (16)$$

where p, k, a_0, a_1 are given by Eq. (10), c is an arbitrary constant, and $c_2 > 0$, $4c_2c_4 = c_3^2$, $\epsilon^2 = 1$.

In conclusion, by utilizing an auxiliary equation, we find bright and dark optical soliton and other soliton solutions for the higher-order nonlinear Schrödinger equation with FOD, the cubic-quintic nonlinear terms, self-steepening and nonlinear dispersive terms. Moreover, we discuss the formation conditions of the bright and dark solitons in Eqs. (13) and (14), respectively. Although these solutions represent only a small subset of the large variety of possible solutions admitted by this higher-order NLSE (Eq. (2)), those presented here are the first examples of exact analytic solutions found so far. The properties of the soliton solutions for Eq. (2) are shown in Figs. 1 and 2, respectively. The propagation of ultrashort light pulses in optical fibers is of particular interest because of the common expectation that solitary waves may be of extensive use in telecommunication, the potential application of these solutions (particularly dark and bright solitons) for this higher-order NLSE (Eq. (2)) is an important problem worth exploring.

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