Constructing a universal set of quantum gates via probabilistic teleportation

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We present a general method to construct a universal set of quantum gates using probabilistic teleportation as a basic primitive. The technique generalizes the teleportation method of gate construction to partially entangled quantum channels. Without recourse to local filtering or entanglement concentration, using local rotation and CNOT operations followed by measurements in the computational basis, one can construct many encoded quantum operations with unit fidelity but less than unit probability. The technique can also be applied to the construction of remote quantum gates that cannot be directly performed.

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Quantum teleportation[1,2] is an important subject in the area of quantum information and computation. It is not only a novel demonstration of the fundamental properties of quantum mechanical system, but also a very useful technique for quantum information processing. With the help of teleportation, many promising applications such as telecloning[3], distributed quantum computation[4], quantum network communication[5], entanglement manipulation[6], distributed state preparation[7], and quantum remote control[8] have been brought forward. Recently, some interesting works which correlate closely to teleportation have been presented by Chuang *et al.*[9]. They used quantum teleportation as basic primitive to enable construction of quantum operations that cannot be directly performed through unitary operations. The motivation behind their works is the possibility that many tasks of quantum information and computation based on these quantum logic gates could be implemented with the current technology such as single photon operations. However, this does not imply that their method is near to realization with current experimental technology. For example, the Bell measurement in the above scheme cannot be performed perfectly with linear element[10−12], and this difficulty can be seen in some teleportation experiments[2]. Furthermore, the detection of photons requires an ideal photon counter which is not available, and the entangled states which are used as quantum channels are sensitive to decoherence and dissipation etc.. Based on Ref. [9], Zhou *et al.*[13] have proposed a scheme for constructing quantum logic gate using "one-bit teleportation".

In this letter, we will provide an extension to the teleportation method of gates construction[9] with a similar but more practical primitive. We show that CNOT gates, single qubit rotations, computation basis measurements, and some partially entangled states are sufficient to construct a universal set of quantum gates. Our method offers possibilities for relaxing experimental constrains on realizing quantum information processing.

In standard teleportation protocol, Alice performs a Bell measurement on the unknown state and one-half of the maximally entangled pair, and depending on the measurement outcome Bob applies a local unitary operation to recover the unknown state. Considering that an entangled state may be partially entangled, Li *et al.*[14] have investigated a partially entangled quantum channel. In their report, Alice performs a Bell measurement on her side, while Bob introduces an auxiliary qubit, and operates a collective unitary transformation on his qubit and the auxiliary qubit to reestablish the initial state with certain probability. There have been other proposals to teleport an unknown state via a partially entangled quantum channel but using generalized measurements such as POVMs[15], non-maximally entangled measurements[16]. Also, there has been a qubit assisted conclusive teleportation process[17]. However, we will show that the same objective can be accomplished in a simpler manner if Alice performs a rotation about the Y axis on her teleported qubit and then measures the two qubits in her possession in computational basis respectively. The quantum circuit shown in Fig. 1 gives a precise description of the probabilistic teleportation. Suppose that Alice has an unknown quantum state $|\psi\rangle_A = \alpha|0\rangle_A + \beta|1\rangle_A$. She wants to teleport the unknown state $|\psi\rangle_A$ to Bob who is at a remote station. Alice and Bob are supposed to share a partially entangled state

$$|\varphi\rangle_{ab} = \cos\theta\,|00\rangle_{ab} + \sin\theta\,|11\rangle_{ab}\,, \tag{1}$$

the particles a and b belong to Alice and Bob, respectively. In the circuit, the input state is $|\psi\rangle_0 = |\psi\rangle_A|\varphi\rangle_{ab}$. Alice sends her qubits A and a through a local CNOT gate $U_{\text{CNOT}}(A;a)$ (with qubit A being control qubit), and then sends the qubit A through the $R_Y(\theta')$ gate

$$R_Y(\theta') = \begin{pmatrix} \cos\theta' & -\sin\theta' \\ \sin\theta' & \cos\theta' \end{pmatrix}, \tag{2}$$

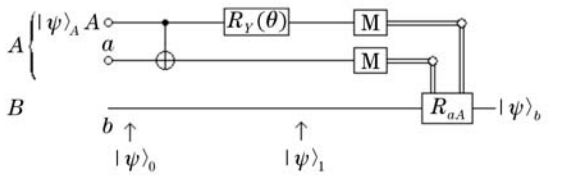

Fig. 1. Quantum circuit for probabilistic teleporting a qubit. The double lines carry classical bits, and single lines, qubits. The boxes "M" represent measurements in computation basis. R_{aA} is either the identity if $aA = 01$, or a single qubit Pauli operation $R_{10} = -iY$, $R_{00} = Z$, $R_{11} = X$.

where $R_Y(\theta')$ corresponds to the rotation about the Y-axis. After this, the state will be

$$\begin{aligned}|\psi\rangle_1 = |00\rangle_{aA}\, Z_b(\alpha\cos\theta\cos\theta'\,|0\rangle + \beta\sin\theta\sin\theta'\,|1\rangle)_b \\ + |01\rangle_{aA}\,(\alpha\cos\theta\sin\theta'\,|0\rangle + \beta\sin\theta\cos\theta'\,|1\rangle)_b \\ - |10\rangle_{aA}\, iY_b(\alpha\sin\theta\cos\theta'\,|0\rangle + \beta\cos\theta\sin\theta'\,|1\rangle)_b \\ + |11\rangle_{aA}\, X_b(\alpha\sin\theta\sin\theta'\,|0\rangle + \beta\cos\theta\cos\theta'\,|1\rangle)_b. \quad (3)\end{aligned}$$

Here, and in what follows, we will write X, Y, and Z instead of σ_x, σ_y, and σ_z; X_b stands for the operator X acting on the qubit b, and so on. Then Alice performs the computation basis measurements on her qubits a and A, respectively. From Eq. (3), we can read off Bob's post-measurement state, given the results of Alice's measurement — which is two classical bits (two c-bits) of information. We now wish to have faithful transportation with nonzero probability. Let us consider several scenarios involving various choices of the parameter θ', given the value of θ. Choice is at the disposal of Alice. 1) Standard teleportation protocol: Let us choose $\theta' = \theta = \pi/4$, then faithful teleportation is possible with unit fidelity and unit probability. 2) Probabilistic teleportation protocol: If we make the choice $\theta' = \theta \neq \pi/4$, then for any of these choices, reliable teleportation is possible for only two out of four possible results of the measurement. The total probability of this successful teleportation is given by

$$P = 2\left|\sin^2\theta\cos^2\theta\right|. \quad (4)$$

This is one of the main results of our letter, which will be used to construct a universal set of quantum gates. In this protocol, single qubit state needs only two particles to teleport in comparison with Li's three particles[14]. The price to be paid, however, is that our protocol succeeds with $2\sin^2\theta\cdot\cos^2\theta$ probability. In Li's protocol, the probability of successful teleportation is $2\sin^2\theta$ ($|\sin\theta| < |\cos\theta|$). 3) No teleportation: If the value of θ' is not related with that of θ, the teleportation is not possible with unit fidelity.

We now consider a general method for constructing universal quantum gates using the probabilistic teleportation protocol as a basic primitive. The basic idea is following. To apply the m-qubit unitary operation U to an m-qubit arbitrary unknown state $|\psi\rangle$, we firstly construct a $2m$ qubits input entangled state

$$|\chi\rangle = (I_m \otimes U)\,|\varphi^m\rangle, \quad (5)$$

where $|\varphi^m\rangle = \prod_{i=1}^{m}(\cos\theta_i\,|00\rangle + \sin\theta_i\,|11\rangle)$ is the $2m$-qubit partially entangled state, rearranged so that the first m qubits represent qubits from half of each partially entangled pair, and the last m the other halves. $I_m \otimes U$ means "do nothing to the first m qubits, and apply U to the last m qubits". Then, utilizing the partially entangled state $|\chi\rangle$ of Eq. (5) as quantum channels, the probabilistic teleportation procedure described above leaves the receiver with the state $U\,|\psi\rangle$ with a certain probability. That is, the output is a transformed version of the input, where the transformation is determined by the entangled state $|\chi\rangle$ used by the procedure.

It is proved that any unitary operation on multiple qubits may be implemented exactly by composing single qubit and CNOT gates[18]. Unfortunately, no straightforward method is known to implement all these gates in a fashion which is resistant to errors. Fortunately, the second construction — two discrete sets of gates have been proved[18] that the Hadamard gate H ($H = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$), phase gate S ($S = \begin{pmatrix}1 & 0\\ 0 & i\end{pmatrix}$), CNOT gate, and $\pi/8$ gate T ($T = \begin{pmatrix}1 & 0\\ 0 & e^{i\pi/4}\end{pmatrix}$) are universal for quantum information processing, in the sense that an arbitrary unitary operation on multiple qubits can be approximated to an arbitrary accuracy using a quantum circuit composed of only these gates. Replacing the $\pi/8$ gate in this list with the Toffoli (control-control-NOT) gate also gives a universal family. Moreover, the fault-tolerant constructions of these gates are known. Now we turn to construct these important essential quantum gates using the method described above.

Firstly, let U denote the single qubit gate, such as the Hadamard gate H, phase gate S, and $\pi/8$ gate T.

We pre-prepare the entangled state

$$\begin{aligned}|\chi\rangle_{ab} &= (I_a \otimes U_b)\,|\varphi\rangle_{ab} \\ &= \cos\theta\,|0\rangle_a U_b\,|0\rangle_b + \sin\theta\,|1\rangle_a U_b\,|1\rangle_b, \quad (6)\end{aligned}$$

where the particles a and b belong to Alice and Bob, respectively. The quantum circuit derived here is the same as the one shown in Fig. 1. Instead of $|\varphi\rangle_{ab}$, we use $|\chi\rangle_{ab}$ as quantum channel, and the probabilistic teleportation procedure shown in Fig. 1 leaves the receiver with the state $U\,|\psi\rangle_b$, which is the desired transfer. The probability of successful transformation is $P = 1$ (when $\theta = \pi/4$) or$P = 2\sin^2\theta\cdot\cos^2\theta$ (when $\theta \neq \pi/4$), independent of the initial state $|\psi\rangle_A$ or the quantum gate U.

Then, let U denote the non-local CNOT gate.

Suppose Alice and Bob have been in their possession quantum states $|\psi\rangle_A$ and $|\psi\rangle_B$, respectively. How can they perform a non-local CNOT gate on the state $|\psi\rangle_{BA}$, here

$$\begin{aligned}|\psi\rangle_{BA} &= |\psi\rangle_B\,|\psi\rangle_A \\ &= (\lambda_1\,|00\rangle + \lambda_2\,|10\rangle + \lambda_3\,|01\rangle + \lambda_4\,|11\rangle)_{BA}, \quad (7)\end{aligned}$$

i.e. transform it to

$$\begin{aligned}&U_{\mathrm{CNOT}}(A;B)\,|\psi\rangle_{BA} \\ &= (\lambda_1\,|00\rangle + \lambda_2\,|10\rangle + \lambda_3\,|11\rangle + \lambda_4\,|01\rangle)_{BA}, \quad (8)\end{aligned}$$

without communicating any quantum information between them, but perhaps with the aid of some initially shared partially entangled state.

A quantum circuit to accomplish the task is given in Fig. 2. We pre-prepare the entangled state $|\chi\rangle_{b_1,a_1,b_2,a_2}$ by simply performing a CNOT gate on two partially entangled pairs $|\varphi\rangle_{b_1,a_1} = (\cos\theta_1\,|00\rangle + \sin\theta_1\,|11\rangle)_{b_1,a_1}$ and $|\varphi\rangle_{b_2,a_2} = (\cos\theta_2\,|00\rangle + \sin\theta_2\,|11\rangle)_{b_2,a_2}$,

$$|\chi\rangle_{b_1,a_1,b_2,a_2} = U_{\mathrm{CNOT}}(b_2;b_1)\,|\varphi\rangle_{b_1,a_1}\,|\varphi\rangle_{b_2,a_2}. \quad (9)$$

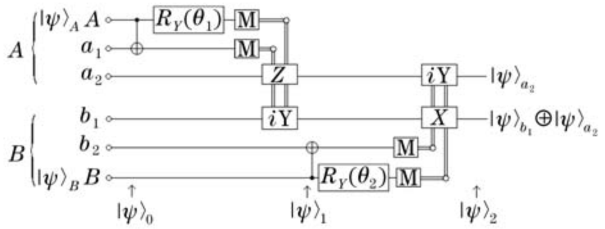

Fig. 2. Quantum circuit for constructing a non-local CNOT gate.

We give particles a_1, a_2 to Alice and particles b_1, b_2 to Bob. The state of the whole system at this moment is $|\psi\rangle_0 = |\chi\rangle_{b_1,a_1,b_2,a_2} |\psi\rangle_{BA}$.

Alice sends her qubits A and a_1 through a local CNOT gate $U_{\rm CNOT}(A;a_1)$, and then sends the qubit A through the $R_Y(\theta_1) = \begin{pmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{pmatrix}$ gate to give

$$
\begin{aligned}
&R_Y(\theta_1)U_{\rm CNOT}(A;a_1)\,|\psi\rangle_0 \\
&= |00\rangle_{a_1,A} \left[\cos^2\theta_1 \cos\theta_2(\lambda_1 |0000\rangle + \lambda_3 |0001\rangle)\right. \\
&\quad + \cos^2\theta_1 \sin\theta_2(\lambda_1 |1110\rangle + \lambda_3 |1111\rangle) \\
&\quad - \sin^2\theta_1 \cos\theta_2(\lambda_2 |1000\rangle + \lambda_4 |1001\rangle) \\
&\quad \left. - \sin^2\theta_1 \sin\theta_2(\lambda_2 |0110\rangle + \lambda_4 |0111\rangle)\right]_{b_1,b_2,a_2,B} \\
&\quad + |01\rangle_{a_1,A} [\cos\theta_1 \sin\theta_1 \cos\theta_2(\lambda_1 |0000\rangle + \lambda_3 |0001\rangle) \\
&\quad + \cos\theta_1 \sin\theta_1 \sin\theta_2(\lambda_1 |1110\rangle + \lambda_3 |1111\rangle) \\
&\quad + \cos\theta_1 \sin\theta_1 \cos\theta_2(\lambda_2 |1000\rangle + \lambda_4 |0001\rangle) \\
&\quad + \cos\theta_1 \sin\theta_1 \sin\theta_2(\lambda_2 |0110\rangle + \lambda_4 |0111\rangle)]_{b_1,b_2,a_2,B} \\
&\quad + |10\rangle_{a_1,A} [-\cos\theta_1 \sin\theta_1 \cos\theta_2(\lambda_2 |0000\rangle + \lambda_4 |0001\rangle) \\
&\quad - \cos\theta_1 \sin\theta_1 \sin\theta_2(\lambda_2 |1110\rangle + \lambda_4 |1111\rangle) \\
&\quad + \cos\theta_1 \sin\theta_1 \cos\theta_2(\lambda_1 |1000\rangle + \lambda_3 |1001\rangle) \\
&\quad + \cos\theta_1 \sin\theta_1 \sin\theta_2(\lambda_1 |0110\rangle + \lambda_3 |0111\rangle)]_{b_1.b_2,a_2,B} \\
&\quad + |11\rangle_{a_1,A} \left[\cos^2\theta_1 \cos\theta_2(\lambda_2 |0000\rangle + \lambda_4 |0001\rangle)\right. \\
&\quad + \cos^2\theta_1 \sin\theta_2(\lambda_2 |1110\rangle + \lambda_4 |1111\rangle) \\
&\quad + \sin^2\theta_1 \cos\theta_2(\lambda_1 |1000\rangle + \lambda_3 |1001\rangle) \\
&\quad \left. + \sin^2\theta_1 \sin\theta_2(\lambda_1 |0110\rangle + \lambda_3 |0111\rangle)\right]_{b_1,b_2,a_2,B}. \qquad (10)
\end{aligned}
$$

This is then followed by computation basis measurements on Alice's qubits a_1 and A, respectively. Let $\theta_1, \theta_2 \neq \pi/4$. If the result is $|00\rangle_{a_1,A}$ or $|11\rangle_{a_1,A}$, the implementation fails. Otherwise the result is $|01\rangle_{a_1,A}$ or $|10\rangle_{a_1,A}$. According to $|01\rangle_{a_1,A}$ and $|10\rangle_{a_1,A}$, Alice and Bob perform the corresponding transformations $I_{b_1} \otimes I_{a_2}$ and $Y_{b_1} \otimes Z_{a_2}$ on the qubits b_1 and a_2, respectively, so that the qubits b_1, b_2, a_2, and B can be transformed to the common form

$$
\begin{aligned}
&|\psi\rangle_1 = \sin\theta_1 \cos\theta_1 [\cos\theta_2(\lambda_1 |0000\rangle + \lambda_3 |0001\rangle) \\
&+ \sin\theta_2(\lambda_1 |1110\rangle + \lambda_3 |1111\rangle) + \cos\theta_2(\lambda_2 |1000\rangle + \lambda_4 |0001\rangle) \\
&+ \sin\theta_2(\lambda_2 |0110\rangle + \lambda_4 |0111\rangle)]_{b_1,b_2,a_2,B}. \qquad (11)
\end{aligned}
$$

Now Bob performs a CNOT on qubits B and b_2, then a rotation operation $R_Y(\theta_2) = \begin{pmatrix} \cos\theta_2 & -\sin\theta_2 \\ \sin\theta_2 & \cos\theta_2 \end{pmatrix}$ on B. After this, Eq. (11) will be

$$
\begin{aligned}
&R_Y(\theta_2)U_{\rm CNOT}(B;b_2)\,|\psi\rangle_1 \\
&= |00\rangle_{b_2,B} \left[\cos\theta_1 \cos^2\theta_2 \sin\theta_1(\lambda_1 |00\rangle + \lambda_2 |10\rangle)\right. \\
&\quad \left. - \cos\theta_1 \sin\theta_1 \sin^2\theta_2(\lambda_3 |11\rangle + \lambda_4 |01\rangle)\right]_{b_1,a_2} \\
&\quad + |11\rangle_{b_2,B} \left[\cos\theta_1 \cos\theta_1 \sin^2\theta_2(\lambda_1 |11\rangle + \lambda_2 |01\rangle)\right. \\
&\quad \left. + \cos\theta_1 \cos^2\theta_2 \sin\theta_1(\lambda_3 |00\rangle + \lambda_4 |10\rangle)\right]_{b_1,a_2} \\
&\quad + (|01\rangle_{b_2,B} + |10\rangle_{b_2,B}\, iY_{a_2} X_{b_1}) \\
&\quad \times [\cos\theta_1 \cos\theta_2 \sin\theta_1 \sin\theta_2 U_{\rm CNOT}(a_2;b_1) \\
&\quad (\lambda_1 |00\rangle + \lambda_2 |10\rangle + \lambda_3 |01\rangle + \lambda_4 |11\rangle)]_{b_1,a_2}. \qquad (12)
\end{aligned}
$$

Now Bob measures qubits b_2 and B in the computation basis. If the result is $|00\rangle_{b_2,B}$ or $|11\rangle_{b_2,B}$, the implementation fails. Otherwise the result is $|01\rangle_{b_2,B}$ or $|10\rangle_{b_2,B}$. By performing corresponding transformations $I_{a_2} \otimes I_{b_1}$ or $Y_{a_2} \otimes X_{b_1}$, Alice and Bob complete the non-local CNOT with probability $P = 4\,|\sin\theta_1 \sin\theta_2 \cos\theta_1 \cos\theta_2|^2$ (4 kinds all). For the maximally entangled channels, $\theta_1 = \theta_2 = \pi/4$, $P = 1$ (8 kinds all).

Finally, let U denote the Toffoli gate.

Suppose Alice has an unknown arbitrary three-particle state $|\psi\rangle_A = |xyz\rangle_{A_1,A_2,A_3}$. We first prepare the entangled state $|\chi\rangle_{a_1,b_1,a_2,b_2,a_3,b_3}$ by performing a Toffoli gate on three partially independent entangled pairs $\prod_{i=1}^{3} (\cos\theta_i |00\rangle + \sin\theta_i |11\rangle)_{a_i,b_i}$,

$$
\begin{aligned}
&|\chi\rangle_{a_1,b_1,a_2,b_2,a_3,b_3} \\
&= U_{\rm Toffoli}(b_3,b_2;b_1) \prod_{i=1}^{3} (\cos\theta_i |00\rangle + \sin\theta_i |11\rangle)_{a_i,b_i}. \qquad (13)
\end{aligned}
$$

We give particles a_1, a_2, and a_3 to Alice and particles b_1, b_2, and b_3 to Bob. The state of the whole system at this moment is $|\chi\rangle_{a_1,b_1,a_2,b_2,a_3,b_3} |xyz\rangle_{A_1,A_2,A_3}$. The goal is Bob ends up with the processed state

$$
|\psi\rangle_B = U_{\rm Toffoli}(b_3,b_2;b_1)\, |xyz\rangle_{b_1,b_2,b_3}. \qquad (14)
$$

This can be verified by direct computation, but it is easier to understand by using the quantum circuit shown in Fig. 3. We recall that multiple-qubit state can be teleported simply by replicating instances of the single-qubit procedure. Normally, for three qubits, the receiver would obtain the state $|xyz\rangle_{b_1,b_2,b_3}$ with certain probability P, but we have replaced the usual partially entangled pairs $\prod_{i=1}^{3} (\cos\theta_i |00\rangle + \sin\theta_i |11\rangle)_{a_i,b_i}$ by special partially entangled state $|\chi\rangle_{a_1,b_1,a_2,b_2,a_3,b_3}$, the receiver instead obtains $U_{\rm Toffoli}(b_3,b_2;b_1)\,|xyz\rangle_{b_1,b_2,b_3}$ with probability P. In Fig. 3, the double lines represent the two-bit classical outcome and control all the operations in the corresponding boxes. For the maximally entangled channels, $\theta_1 = \theta_2 = \theta_3 = \pi/4$, and $P = 1$. If $\theta_1, \theta_2, \theta_3 \neq \pi/4$, $P = 2^3 \prod_{i=1}^{3} |\sin\theta_i \cos\theta_i|^2$.

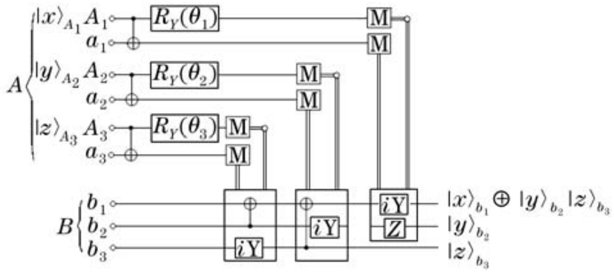

Fig. 3. Quantum circuit for constructing a Toffoli gate.

This method can be generalized to construct m-qubit Toffoli gate ($m \geq 3$).

In summary, we have presented a systematic technique to construct a universal set of quantum gates, by using a probabilistic teleportation as a basic primitive. Our probabilistic teleportation protocol gains an advantage over the one shown in Ref. [14] in the simplicity of manipulation. Our protocol needs to teleport only two particles for single qubit state, succeeds with $2\sin^2\theta \cdot \cos^2\theta$ probability, which is determined by both Schmidt coefficients of the entangled pair. This is of great advantage to the situation where the m is large in constructing quantum logic gates. Such a technique reduces the difficulty of constructing a quantum logic gate to prepare a special partially entangled state $|\chi\rangle = (I_m \otimes U)|\varphi^m\rangle$ that is used as quantum channel. Creating $|\chi\rangle$ may appear, at first sight, to be as difficult as performing U. However, constructing specific known states is easier than doing operations on unknown states. This technique allows us to teleport an arbitrary single-qubit or multiple-qubit gate with unit fidelity and unit probability (when $\theta_1 = \theta_2 = \cdots = \theta_m = \pi/4$) or reduced probability (when $\theta_1 = \theta_2 = \cdots = \theta_m \neq \pi/4$), as demonstrated in our construction of the H gate, phase gate S , $\pi/8$ gate T, Toffoli gate, and the remote CNOT gate.

On the other hand, although the probabilistic operations may be not so useful in the context of quantum computation, as it may change the complexity class of the problem and may thus destroy the (exponential) speedup of the quantum algorithm in question. However, probabilistic gates are useful for processes such as entanglement distillation[19,20], which itself is already a probabilistic process. For example, this may help in the implementation of quantum repeaters[21,22] using photons only (i.e., for quantum communication over arbitrary distances). Due to the fact that photons are ideal candidates for quantum information processing (due to their fast propagation), it is highly desirable to manipulate them directly rather than mapping their states on the states of another physical system, e.g. of an ion or an atom, and *vice versa*.

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