Mode characteristics of hollow core Bragg fiber

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Analytical expression to calculate propagation constant and mode field of the hollow core Bragg fiber is derived. Numerical results are presented. It is shown that the fundamental mode of the hollow core Bragg fiber is circularly symmetric TE_{01} mode with no polarization degeneracy, while the higher order mode may be HE_{11}, TM_{01}, or TE_{02} etc.. This property is different from conventional optical fiber that its fundamental mode is the linearly polarized HE_{11} mode and is polarization degeneracy.

OCIS codes: 060.2310, 060.2430, 999.9999 (Hollow core Bragg fiber).

Photonic crystal fibers (PCFs)[1−4] have been received much attention in recent years for their special properties such as single mode operation over broadband, guidance in an air core, large and adjustable dispersion, much high or low nonlinearity. A variety of PCF has been fabricated and studied, such as solid core PCF utilizing total internal refraction (TIR), or hollow core PCF employing photonic bang gap effects (PBG). Here a special photonic crystal fiber with one dimension periodic structure, i.e., the hollow core Bragg fiber with radial periodic rings, is analyzed. Though some numerical results about the leakage loss of low and higher order modes in hollow core Bragg fibers have been presented by Bassett *et al.*[5,6], novel properties will be demonstrated in this paper through comprehensive research on the mode characteristics of the hollow core Bragg fiber.

As is shown in Figs. 1(a) and (b), the hollow core Bragg fiber is composed of hollow core with refractive index $n_c = 1$ (the darkest region), periodic rings with higher refractive index n_p (white rings) and lower refractive index n_n (dark rings) in turn, and outmost cladding with refractive index n_e. The core radius is designated by c, the thickness of the higher refractive index ring is designated by a, the thickness of the lower refractive index ring is designated by b, the number of the periodic rings is designated by N.

While cross section and refractive index distribution of a hollow core Bragg fiber with $N = 10$ is shown in Figs. 1(a) and (b) (only for simplicity and to see clear), the parameters $n_c = 1$, $n_p = 1.45$, $n_n = 1.1$, $n_e = 1.45$ and $c = 2$ μm, $a = 0.5$ μm, $b = 0.5$ μm, $N = 50$ are designed to demonstrate the properties of the hollow core Bragg fiber in the following.

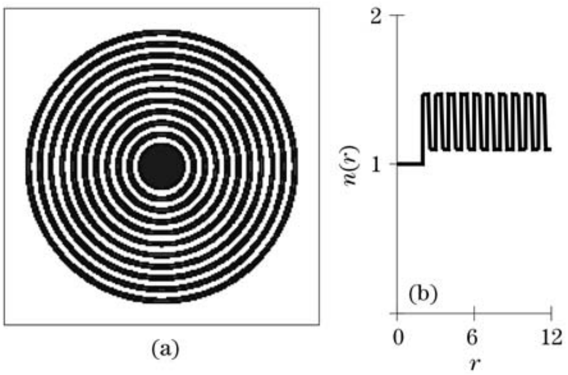

Fig. 1. (a) Cross section of the hollow core Bragg fiber. $c = 2$ μm, $a = 0.5$ μm, $b = 0.5$ μm, $N = 10$. (b) Refractive index distribution of the hollow core Bragg fiber. $n_c = 1$, $n_p = 1.45$, $n_n = 1.1$, $n_e = 1.45$.

Based on Maxwell equations and boundary conditions, following equations describing hollow core Bragg fiber are derived through some complicated calculation and analyzing:

$$T(m,r,n,\beta,k) = \begin{bmatrix} j & h & 0 & 0 \\ 0 & 0 & j & h \\ \frac{im\beta}{r\cdot u(n,\beta,k)^2}j & \frac{im\beta}{r\cdot u(n,\beta,k)^2}h & -\frac{k}{u(n,\beta,k)}j' & -\frac{k}{u(n,\beta,k)}h' \\ \frac{kn^2}{u(n,\beta,k)}j' & \frac{kn^2}{u(n,\beta,k)}h' & \frac{im\beta}{r\cdot u(n,\beta,k)^2}j & \frac{im\beta}{r\cdot u(n,\beta,k)^2}h \end{bmatrix},$$

$$u(n,\beta,k) = \sqrt{k^2\cdot n^2 - \beta^2},$$

$$j = J(m, r\cdot u(n,\beta,k)), \quad h = H(m, r\cdot u(n,\beta,k)),$$

$$j' = \frac{J(m-1, r\cdot u(n,\beta,k)) - J(m+1, r\cdot u(n,\beta,k))}{2},$$

$$h' = \frac{H(m-1, r\cdot u(n,\beta,k)) - H(m+1, r\cdot u(n,\beta,k))}{2},$$

where $k = \frac{2\pi}{\lambda}$, λ is wavelength in free space, β is propagation constant along z axis (axial direction), $J(m,x)$ and $H(m,x)$ represent Bessel function and first kind of Hanker function of order m respectively, n represents refractive index in the related region, $T(m,r,n,\beta,k)$ represents transformation matrix of the boundary conditions (continuity of the tangential electromagnetic field components at interface), m is an integer indicated mode order.

$$T_p(r) = T(m,r,n_p,\beta,k), \quad T_n(r) = T(m,r,n_n,\beta,k),$$

$$T_e = T(m, c+N\cdot(a+b), n_e, \beta, k),$$

$$T_c(m,\beta,k) = T(m,c,n_c,\beta,k),$$

$$S(m,\beta,k) = \left[\prod_{i=1}^{N} T_p(r_i)\cdot T_p(r_i+a)^{-1}\cdot T_n(r_i+a)\cdot T_n(r_{i+1})^{-1}\right]\cdot T_e,$$

$$r_i = c+(i-1)\cdot(a+b), \quad i = 1,2\cdots N,$$

$$G(m,\beta,k) = \begin{bmatrix} T_c(\cdot)_{0,0} & -S(\cdot)_{0,1} & T_c(\cdot)_{0,2} & -S(\cdot)_{0,3} \\ T_c(\cdot)_{1,0} & -S(\cdot)_{1,1} & T_c(\cdot)_{1,2} & -S(\cdot)_{1,3} \\ T_c(\cdot)_{2,0} & -S(\cdot)_{2,1} & T_c(\cdot)_{2,2} & -S(\cdot)_{2,3} \\ T_c(\cdot)_{3,0} & -S(\cdot)_{3,1} & T_c(\cdot)_{3,2} & -S(\cdot)_{3,3} \end{bmatrix},$$

$$G(m,\beta,k)\cdot\begin{bmatrix}A\\C\\B\\D\end{bmatrix}=0. \tag{1}$$

Equation (1) is the eigenmode equation of the hollow core Bragg fiber. Matrix $G(m,\beta,k)$ is the eigenmode matrix, we use $(\cdot)$ to represent (m,β,k) for simplicity.

The axial electromagnetic component in the core region $(r\le c)$ is

$$E_z(r,\Theta)=A\cdot J(m,r\cdot u(n_c,\beta,k))\cdot \mathrm{e}^{i\cdot m\cdot\Theta},$$
$$K_z(r,\Theta)=B\cdot J(m,r\cdot u(n_c,\beta,k))\cdot \mathrm{e}^{i\cdot m\cdot\Theta}.$$

The correspondent in the outmost cladding $[r\ge c+N\cdot(a+b)]$ is

$$E_z(r,\Theta)=C\cdot H(m,r\cdot u(n_e,\beta,k))\cdot \mathrm{e}^{i\cdot m\cdot\Theta},$$
$$K_z(r,\Theta)=D\cdot H(m,r\cdot u(n_e,\beta,k))\cdot \mathrm{e}^{i\cdot m\cdot\Theta},$$

where E_z and K_z represent axial electro and magnetic components, respectively, A, B, C, D represent the corresponding mode field coefficients.

To have nonzero solution, the eigenmode matrix $G(m,\beta,k)$ must satisfy following equation:

$$|G(m,\beta,k)|=0. \tag{2}$$

From Eqs. (1) and (2), propagation constant β and mode field coefficients A, B, C, D can be solved if wave vector $\mathbf{k}$ is given. Transverse components of the mode field in the core region can be expressed as

$$\begin{matrix}E_x(r,\Theta)=\cos(\Theta)\cdot E_r(r,\Theta)-\sin(\Theta)\cdot E_\Theta(r,\Theta)\\ E_y(r,\Theta)=\sin(\Theta)\cdot E_r(r,\Theta)+\cos(\Theta)\cdot E_\Theta(r,\Theta)\end{matrix}, \tag{3a}$$

$$\begin{matrix}K_x(r,\Theta)=\cos(\Theta)\cdot K_r(r,\Theta)-\sin(\Theta)\cdot K_\Theta(r,\Theta)\\ K_y(r,\Theta)=\sin(\Theta)\cdot K_r(r,\Theta)+\cos(\Theta)\cdot K_\Theta(r,\Theta)\end{matrix}, \tag{3b}$$

$$E_r(r,\Theta)=$$
$$i\cdot\left[\frac{\beta}{u_c(\beta,k)}\cdot J_c'(*)\cdot A+\frac{i\cdot m\cdot k}{r\cdot u_c(\beta,k)^2}\cdot J_c(*)\cdot B\right]\cdot \mathrm{e}^{i\cdot m\cdot\Theta},$$
$$E_\Theta(r,\Theta)=$$
$$i\cdot\left[\frac{i\cdot m\cdot\beta}{r\cdot u_c(\beta,k)^2}\cdot J_c(*)\cdot A-\frac{k}{u_c(\beta,k)}\cdot J_c'(*)\cdot B\right]\cdot \mathrm{e}^{i\cdot m\cdot\Theta},$$
$$K_r(r,\Theta)=$$
$$i\cdot\left[\frac{\beta}{u_c(\beta,k)}\cdot J_c'(*)\cdot B-\frac{i\cdot m\cdot k\cdot n_c^2}{r\cdot u_c(\beta,k)^2}\cdot J_c(*)\cdot A\right]\cdot \mathrm{e}^{i\cdot m\cdot\Theta},$$
$$K_\Theta(r,\Theta)=$$
$$i\cdot\left[\frac{i\cdot m\cdot\beta}{r\cdot u_c(\beta,k)^2}\cdot J_c(*)\cdot B+\frac{k\cdot n_c^2}{u_c(\beta,k)}\cdot J_c'(*)\cdot A\right]\cdot \mathrm{e}^{i\cdot m\cdot\Theta},$$
$$u_c(\beta,k)=u(n_c,\beta,k),\quad J_c(m,r,\beta,k)=J(m,r\cdot u_c(\beta,k)),$$
$$J_c'(m,r,\beta,k)=\frac{J_c(m-1,r,\beta,k)-J_c(m+1,r,\beta,k)}{2},$$

where E_x, E_y, K_x, K_y represent x and y components of the transverse electro and magnetic field, and E_r, E_Θ, K_r, K_Θ represent the radial and azimuthal ones. Here we use $(*)$ to represent (m,r,β,k) for simplicity.

Transmitting power density along axial direction of the fiber in the core region can be expressed as

$$S(r,\Theta)=$$
$$\mathrm{Re}\left[E_x(r,\Theta)\cdot\overline{K_y(r,\Theta)}-E_y(r,\Theta)\cdot\overline{K_x(r,\Theta)}\right]. \tag{4}$$

From Eqs. (3a), (3b), and (4), mode field distribution and transmitting power distribution can be drawn when wavelength λ in free space is given and corresponding β is derived from Eq. (2). Loss coefficient α that arises from waveguide structure can be determined by imaginary part of the propagation constant β:

$$\alpha=2\mathrm{Im}(\beta). \tag{5}$$

Attenuation of transmitting power in the fiber can be expressed as

$$P(z)=P(0)\cdot \mathrm{e}^{-\alpha\cdot z}, \tag{6}$$

where $P(0)$ represents initial power of the input light, $P(z)$ represents the power of transmitting light at distance z from input end.

Above all, the property of the hollow core Bragg fiber can be acquired precisely by solving analytical equations.

As is shown in Figs. 2(a) and (b), the fundamental mode corresponding to hollow core Bragg fiber with parameters $n_c=1$, $n_p=1.45$, $n_n=1.1$, $n_e=1.45$, $c=2$ μm, $a=0.5$ μm, $b=0.5$ μm, $N=50$, and $\lambda=2$ μm is a TE_{01} mode $(m=0)$ $(\beta=2.512653735168542+0.000000000000000i)$. This mode with no polarization degeneracy is azimuthally polarized and circularly symmetric. At center of the core it is dark, and the first light ring appears nearby.

Mode field distribution and transmitting power distribution of the first higher order mode are shown in Figs. 3(a) and (b) $(\beta=2.006915177525921+0.003901190714363i)$. Mode field distribution near center of the core closely resembles HE_{11} mode $(m=1)$ of conventional optical fibers, while mode field far away from center of the core is drastically modified by multiple reflections of the Bragg rings. Being different from TE_{01} mode, this HE_{11} mode is polarization degeneracy and is the brightest at center of the core.

Mode field distribution and transmitting power distribution of the second higher order mode are shown in Figs. 4(a) and (b) $(\beta=1.780628925877693+0.02443324409651i)$. It is a TM_{01} mode $(m=0)$. This

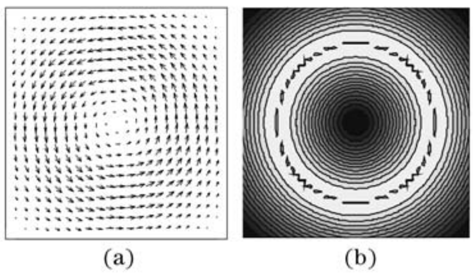

Fig. 2. (a) Mode field distribution and (b) transmitting power distribution of the fundamental mode in the hollow core Bragg fiber. Parameters are the same as in Figs. 1(a) and (b) except $N=50$, $\lambda=2$ μm; fundamental mode is TE_{01} mode.

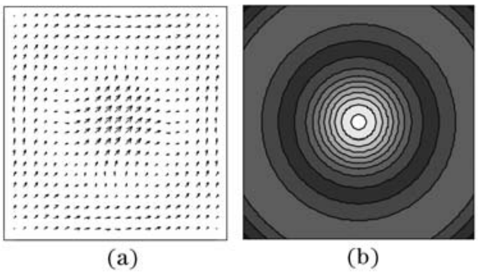

Fig. 3. (a) Mode field distribution and (b) transmitting power distribution of the first higher order mode in the hollow core Bragg fiber. Parameters are the same as in Fig. 2; first higher order mode is HE_{11} mode.

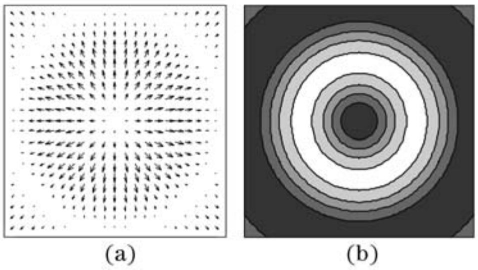

Fig. 4. (a) Mode field distribution and (b) transmitting power distribution of the second higher order mode in the hollow core Bragg fiber. Parameters are the same as in Fig. 2; second higher order mode is TM_{01} mode.

mode with no polarization degeneracy is radially polarized and circularly symmetric. At center of the core there is a dark point, and the light rings appear nearby.

From Figs. 2 to 4 the mode characteristics are shown only at wavelength $\lambda = 2$ μm. In fact we have calculated the prorogation constant β and the correspondent mode at a broadband wavelength from $\lambda = 0.5$ to $\lambda = 2$ μm. As is expected, the fundamental mode is always TE_{01} mode, while the higher order modes may be HE_{11}, TM_{01}, TE_{02}, HE_{12}, TM_{02}, etc..

In Fig. 5 relationship between loss coefficient and wavelength of the fundamental mode in the hollow core Bragg fiber is shown. From this figure we see that this hollow core Bragg fiber can transmit light at a broadband wavelength from $\lambda = 0.5$ to 1.1 μm and $\lambda = 1.5$ to 2 μm, though the light transmission is banned when wavelength is between 1.1 μm and 1.5 μm.

In Figs. 6(a) and (b), the relationship between loss coefficient and wavelength of the higher order modes in the hollow core Bragg fiber is shown. As is expected, the loss coefficient of the higher order modes is much greater than that of the fundamental mode (usually is five orders of magnitude greater). So this fiber is truly a single mode optical fiber with TE_{01} as its guided mode, and is no polarization degeneracy.

As is shown above, the hollow core Bragg fiber with the designed parameters is truly a single mode optical fiber when wavelength of the incident light is between 0.5 μm and 1.1 μm or 1.5 μm and 2 μm. Its fundamental mode is the circularly symmetric TE_{01} mode that is no polarization degeneracy. Its higher order modes are HE_{11}, TM_{01}, or TE_{02} mode, etc.. This is entirely different from conventional optical fibers where the fundamental mode is the polarization degeneracy HE_{11} mode.

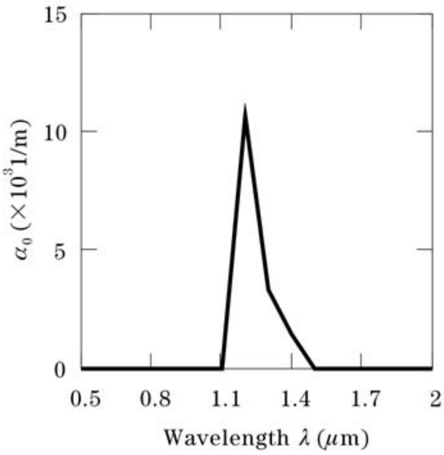

Fig. 5. Relationship between loss coefficient and wavelength of the fundamental mode in the hollow core Bragg fiber. α_0 is loss coefficient of the fundamental mode. Parameters are the same as in Figs. 1(a) and (b) except $N = 50$.

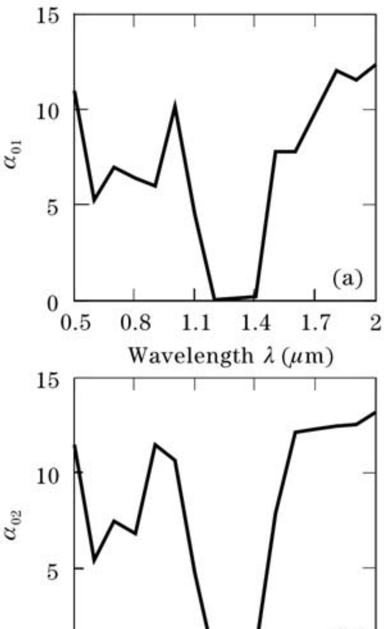

Fig. 6. Relationship between loss coefficient and wavelength of (a) the first and (b) the second higher order modes in the hollow core Bragg fiber. α_{01} (α_{02}) is relative loss coefficient of the first (second) higher order mode to fundamental mode, $\alpha_{01} = \log(\alpha_1/\alpha_0)$, $\alpha_{02} = \log(\alpha_2/\alpha_0)$, α_1 (α_2) is loss coefficient of the first (second) higher order mode. Parameters are the same as in Fig. 5.

In Figs. 5, 6(a) and (b), we also note that no matter it is the fundamental mode or the higher order modes, their loss coefficients are so large that they can not be travelled in the fiber when incident light wavelength is between 1.1 μm and 1.5 μm. This peculiar characteristic is hard to understand at first, because there is no such phenomenon in conventional optical fibers. In fact this arises from the multiple reflections of the periodic rings that result in band gap effects. When light wavelength is fall in the band gap of the periodic rings, the light will be confined in the hollow core to travel. But when light wavelength is fall in conducting band of the periodic rings, the light will be able to travel in the periodic rings and cannot be confined in the hollow core. The energy of the incident light will be lost passing through some distance. This is why there is a wavelength band where no light can travel in the hollow core of the Bragg fiber.

From above results we can see that the hollow core Bragg fiber is not only being able to use as a single mode optical fiber with no polarization degeneracy, but also can be used as a band filter.

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