Analysis and simulation of XPM intensity modulation

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Received June 30, 2004

Based on the split-step Fourier method and small signal analysis, an improved analytical solution which describes the cross-phase modulation (XPM) intensity is derived. It can suppress the spurious XPM intensity modulation efficiently in the whole transmission fiber. Thus it is more coincidence with the practical result. Furthermore, it is convenient, because it is independent of channel separation and the dispersion and nonlinear effects interact through the XPM intensity. A criterion of select the step size is described as the derived XPM intensity modulation being taken into account. It is non-uniform distribution and is the function of average signal power $\langle P(z)\rangle$ (or z). Compared with the conventional split-step method, the simulation accuracy is improved when the step size is determined by the improved XPM intensity.

OCIS codes: 060.0060, 190.4370.

The split-step method (SSM) is a usual method to obtain the approximate solution of nonlinear Schrodinger equation (NLSE), but the step size is too small, for a wavelength of 1 μm, a good accuracy is gained as $z < 40$ μm[1]. The process is very complicated, especially for the multi-channel system, the calculated time is proportional to N^2, N is the channel number. The most time consuming procedure is the estimate of channel's power which the nonlinear crosstalk should be considered, to gain the sufficient accuracy, an iterative calculation is required. Recently, there are some studies about the improvement of split-step Fourier method which are based on physical principles. These studies focus on the relation between nonlinear effects (four-wave mixing, cross-phase modulation and so on) and step size. To derive an exact expression of the system, and for a relatively larger step size, the nonlinear effect can be estimated in the numerical simulation. Researchers discover that the improper distribution of the step sizes may lead to the four-wave mixing (FWM) power to be overestimated. For a given accuracy of FWM power, the maximum constant step size in SSM is analyzed in Ref. [2]. To efficiently suppress the numerical artifact of FWM, a logarithmic distribution of the step size is used to keep the spurious FWM components below a certain level[3]. For a system in which nonlinearity plays a major role, the step size is selected to make the phase shift caused by nonlinearity not exceeds a certain value[4]. In the case which dispersion plays a dominant role, the step size is determined by the largest group velocity difference among the channels[5]. In intensity modulation-direct detection (IM-DD) systems, the power fluctuation of one optical wave propagating in an optical fiber can modulate the phase of other co-propagating waves through cross-phase modulation (XPM), and the group velocity dispersion (GVD) converts the XPM-induced phase modulation (PM) to IM. Usually, the step size selection criterion neglects the conversion of PM into IM as discussed in Ref. [6]. It seems that the conversion within one split step is weak. However, some detailed calculations show the split step is limited by many factors and cannot be too long[2−5,7], using one split step to simulate the system is impossible, especially in IM-DD systems, when the nonlinear and dispersion effects play equal important role (i.e. $L \geq L_{\rm NL}$, $L \geq L_{\rm D}$, L, $L_{\rm NL}$, $L_{\rm D}$ are fiber length, nonlinear length, and dispersion length, respectively), the XPM intensity modulation is even bigger than signal power if the step size selected improperly (in Fig. 1(a))[7]. For a typical system, the step size is limited in several hundred meters. The process requires a long time.

This letter will present a revised analysis result of the XPM intensity modulation (in Fig. 1(b)). It is based on the SSM, so the spurious XPM intensity modulation can be suppressed along the whole transmission fiber.

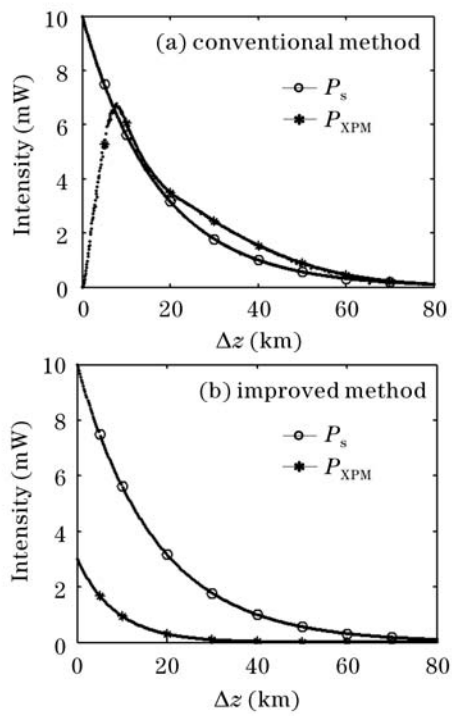

Fig. 1. The XPM intensity modulation versus step size. $D = 17$ (ps/(km·nm)), $D_{\rm slope} = 0.08$ (ps/(km·nm^2)), $\gamma = 15$ (W^{-1}km^{-1}), $P_0 = 10$ dBm, $\alpha = 0.21$ (dB/km), $\lambda = 1550$ nm, $\Delta\lambda = 0.5$ nm.

Therefore, it is more accurate in describing the XPM. Utilizing the revised analytical result, the corresponding step size in SSM can be chosen to estimate the channel power ($|A|^2$, $|A'|^2$, A, A' are the field amplitudes)in the nonlinear factor.

Considering co-propagating two optical fields in a single-mode fiber, in the slowly-varying-envelope approximation, the field amplitudes $A(z,t)$ and $A'(z,t)$ satisfy the nonlinear Schrodinger equation which is modified to account for XPM by the addition of a cross-coupling term[8]:

$$\frac{\partial A(z,t)}{\partial z} + \beta_1 \frac{\partial A(z,t)}{\partial t} + \frac{i}{2}\beta_2 \frac{\partial^2 A(z,t)}{\partial t^2} + \frac{\alpha}{2}A(z,t) = i\gamma(|A(z,t)|^2 + 2|A'(z,t)|^2)A(z,t), \quad (1)$$

where β_1, β_2 describe the dispersion, α is the loss of fiber, and γ is nonlinear coefficient. According the principle of split step Fourier method, we utilize the dispersion and nonlinear operators $\hat{D}$, $\hat{N}$:

$$\hat{D} = -\frac{i}{2}\beta_2 \frac{\partial^2}{\partial t^2} - \frac{\alpha}{2} - \beta_1 \frac{\partial}{\partial t}, \quad (2)$$

$$\hat{N} = i\gamma[|A|^2 + 2|A'|^2]. \quad (3)$$

There is

$$A(z + \mathrm{d}z, t) \approx \exp(\mathrm{d}z\hat{D})A(z,t)\exp\{i\gamma[|A|^2 + 2|A'|^2]\mathrm{d}z\}. \quad (4)$$

In Eq. (4), the operators $\hat{D}$, $\hat{N}$ cannot be exchanged. $\mathrm{d}z$ is a small segment of the whole length L. Usually, the field amplitude can be written as

$$A(z,t) = \sqrt{P(z,t)}\exp[i\phi(z,t)]. \quad (5)$$

So we can write Eq. (4) as

$$\begin{aligned} A(z + \mathrm{d}z, t) &= \sqrt{P(z + \mathrm{d}z, t)}\exp[i\phi(z + \mathrm{d}z, t)] \\ &= \mathrm{e}^{-\frac{\alpha}{2}\mathrm{d}z}\exp(-\beta_1 \mathrm{d}z\frac{\partial}{\partial t})exp(-\frac{i}{2}\beta_2 \mathrm{d}z\frac{\partial^2}{\partial t^2}) \\ &\quad \times\sqrt{P(z,t)}\mathrm{e}^{i\phi(z,t)+i\gamma[P(z,t)+2P'(z,t)]\mathrm{d}z}. \end{aligned} \quad (6)$$

We depart the intensity into two parts $\langle P(z)\rangle$, $\Delta P(z,t)$, $\langle P(z)\rangle$ is the average signal power. $\Delta P(z,t)$ is fluctuated with time, which is caused by the dispersion and nonlinear effects in length z. The small signal analysis implies $P(z,t) = \langle P(z)\rangle + \Delta P(z,t)$, with $\langle P(z)\rangle \gg \Delta P(z,t)$.

Finally (similar to Ref. [9]), in the frequency domain, we can obtain the intensity and phase of $A(z + \mathrm{d}z, t)$:

$$\begin{pmatrix} \frac{\Delta P(z+\mathrm{d}z,i\omega)}{2\langle P(z)\rangle} \\ \phi(z + \mathrm{d}z, i\omega) \end{pmatrix} = \mathrm{e}^{-a\mathrm{d}z/2-i\beta_1\mathrm{d}z\omega}\mathrm{e}^{i\gamma[\langle P(z)\rangle+2\langle P'(z)\rangle]\mathrm{d}z} \begin{pmatrix} \cos(\frac{1}{2}\beta_2\mathrm{d}z\omega^2) & -\sin(\frac{1}{2}\beta_2\mathrm{d}z\omega^2) \\ \sin(\frac{1}{2}\beta_2\mathrm{d}z\omega^2) & \cos(\frac{1}{2}\beta_2\mathrm{d}z\omega^2) \end{pmatrix} \begin{pmatrix} \frac{\Delta P(z,i\omega)}{2\langle P(z)\rangle} \\ \phi(z, i\omega) \end{pmatrix}. \quad (7)$$

Then, the intensity modulation $\Delta P(z+\mathrm{d}z, i\omega)$ at the dispersion, nonlinear fiber due to FM-AM conversion can be given as

$$\Delta P(z + \mathrm{d}z, i\omega) = -2\langle P(z)\rangle\mathrm{e}^{-a\mathrm{d}z/2-i\beta_1\mathrm{d}z\omega} \sin(\frac{1}{2}\beta_2\mathrm{d}z\omega^2)\phi(z, i\omega)\mathrm{e}^{i\gamma[\langle P(z)\rangle+2\langle P(z)'\rangle]\mathrm{d}z}. \quad (8)$$

The parametric gain caused by XPM is

$$g(z,\omega) = \frac{|\Delta P(z + \mathrm{d}z, i\omega)|}{\langle P(z)\rangle\mathrm{d}z} = 2\mathrm{e}^{-a\mathrm{d}z/2-i\beta_1\mathrm{d}z\omega} \sin(\frac{1}{2}\beta_2\mathrm{d}z\omega^2)\phi(z, i\omega)\mathrm{e}^{i\gamma[\langle P(z)\rangle+2\langle P(z)'\rangle]\mathrm{d}z}/\mathrm{d}z. \quad (9)$$

Figure 2 gives the gain spectra of Eq. (9) comparing with the perturbation approach[8] $g_{\mathrm{MI}}(\omega) = 2\mathrm{Im}(K)$. In Fig. 2, the curves are different but have the same maximum of g_{MI}, which indicates that the two approaches are coincident, because similar to FWM effect, only the maximum of g can exist in the fiber, and makes noise amplifying and signal-to-noise ratio (SNR) decrease[9]. From Eq. (8), we can gain the existing XPM intensity in the fiber is

$$P_{\mathrm{XPM}}(\Delta z) = \int_0^{\Delta z} \langle P(z)\rangle g_{\mathrm{MAX}}(z)\mathrm{d}z. \quad (10)$$

This XPM intensity P_{XPM} and signal power ($P_{\mathrm{S}} = P_0\exp(-a\Delta z)$) are plotted in Fig. 1(b), Fig. 1(a) gives the results of Ref. [6]. $\Delta\lambda$, γ, D, and α are the channel separation, nonlinear coefficient, dispersion, and fiber loss, respectively. λ and P_0 are the signal wavelength and input power. In Fig. 1(b), the XPM intensity modulation is even bigger than signal power if the step size is improperly selected. For a typical system, this step size is limited to several hundred meters. Equation (10) overcomes the numerical errors of XPM in Ref. [6]. The reason is as follows: in Ref. [6], Δz is departed into two parts, in the first part, only the phase shift caused by XPM (not including the dispersion conversion) is considered; then in the second part, dispersion changes this phase into intensity. In fact, the nonlinear and dispersion effects exist simultaneously, therefore, using Eq. (10) to estimate the XPM intensity is more reasonable.

Equation (8) is independent of channel separation, so the dispersion and nonlinear effects interact through the XPM intensity. Replacing the parameters (γ, β_1, β_2), the XPM intensity of any channel can be determined, so it is convenient to simulate XPM in a multi-channel system.

In the system numerical simulation, Eq. (10) can be used to estimate the values of $|A|^2$, $|A'|^2$ in the nonlinear factor $\hat{N}$. The corresponding step size Δz must meet the small signal assumption (see Eq. (5)) which the Eq. (8) derived from

$$P_{\mathrm{XPM}} \ll P_{\mathrm{S}}. \quad (11)$$

This criterion is suitable for the system which the nonlinear and dispersion effects play the same important roles. This step size is the function of $\langle P(z)\rangle$ (or z). Figure 3 gives the connection between maximum step size and average signal power, the step size is non-uniform distribution.

Following Eqs. (10) and (11), the simulation of the system turns easy. We assume: the capacity of the system is 16—10 Gb/s with $2^7 - 1$ pseudorandom binary sequence (PRBS), and the whole length of standard single-mode fiber is 80 km. Other parameters are the same as those

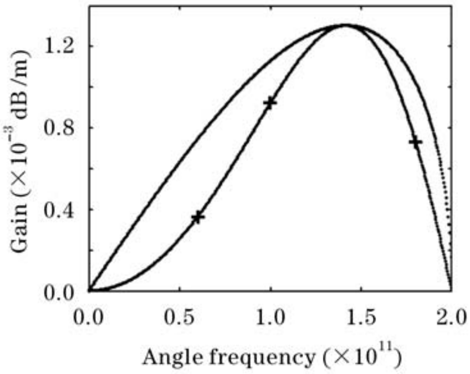

Fig. 2. The gain spectra of improved method and the perturbation approach. Line with + is the result of improved method. The parameters are the same as Fig. 1.

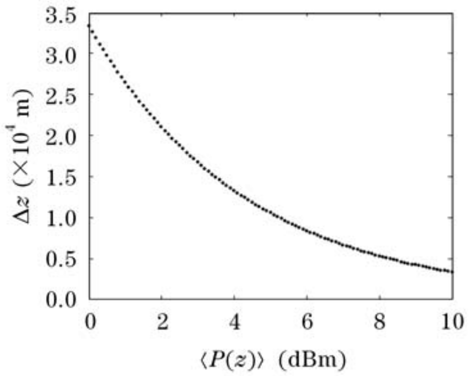

Fig. 3. Maximum step size versus average signal power. The parameters are the same as Fig. 1.

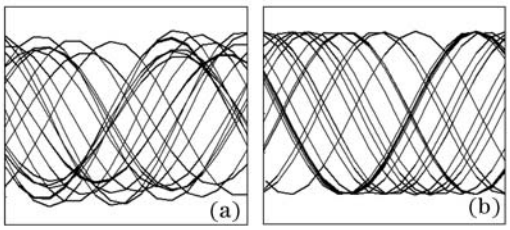

Fig. 4. Eye diagrams for $\Delta z = 1$ km. (a) Δz determined by Ref. [6]; (b) Δz determined by improved method.

used in Fig. 1. Generally, the eye diagram is a way to illustrate the signal transmission by the approximating evaluation $(P_0 \exp(-aL) - \sum_{i=1}^{M} P_{\mathrm{XPM}}(\Delta z_i))$, but for the exactly account, SSM is the usual approach. Figure 4 shows the eye diagrams in which the $P_{\mathrm{XPM}}(\Delta z_i)$ are determined by Ref. [6] and the improved XPM intensity. Note that, for the improved method, $\Delta z = \sum_{i=1}^{M} \Delta z_i / M$, M is the simulation times. The result of the revised method is more coincidence with the practical system. Using Eq. (10) to estimate the nonlinear factor $\hat{N}$ in the split-step Fourier method will make the procedure from $A(z,t) \rightarrow A(z + \mathrm{d}z, t)$ quicker and simpler[4,10]. Figure 5 gives the simulation accuracy when the signals in above system transmit 80 km. The solid lines is the result of the improved SSM in which the improved XPM intensity

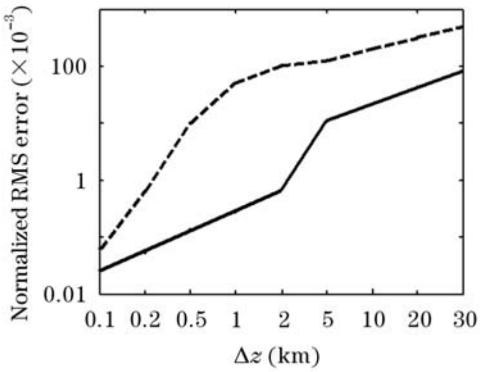

Fig. 5. Average normalized RMS errors for conventional method (dashed line) and improved method (solid line).

is considered, while the dashed line corresponds to the conventional method[11]. Compared with the conventional SSM method, the simulation accuracy is improved as the criterion for the step size is determined by the improved XPM intensity.

In conclusion, based on the SSM and small signal analysis, the XPM intensity has been derived. It is demonstrated that the spurious XPM intensity can be efficiently suppressed along the whole transmission fiber. The eye diagrams are consistent with the practical system. In the system which the nonlinear and dispersion effects are equally important, the step size must satisfy: $P_{\mathrm{XPM}} \ll P_{\mathrm{S}}$. This step size is the function of signal power, and it has a non-uniform distribution. The simulation accuracy in SSM is improved when the step sizes are determined by the improved XPM intensity.

This work was supported by the National Natural Science Foundation of China (No. 60278001) and the Science & Technology Cooperation Foundation of Nakai University, the Ministry of Education. J. Huang's e-mail address is huanggesheng@tom.com.

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