# Conditional quantum CNOT gate between two four-level atoms

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We propose a scheme to implement a quantum controlled-NOT (CNOT) gate between two four-level atoms inside the detuned optical cavity. The system state is evolved inside the decoherence-free (DF) subspace through stimulated Raman processes, which yields the desired unitary evolution operation for the CNOT. Our scheme is immune to decoherence due to dissipation of cavity excitation and spontaneous emission from the excited atomic level.

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Quantum computation has received a lot of attention recently. It has been shown that quantum computer (QC) can enable a striking exponential speedup over a purely classical computer on solving certain problems[1−3]. As we know, the single-qubit rotation and the controlled-NOT (CNOT) gate form a "universal" set of quantum gate. Any quantum computation can be reduced to a sequence of these gates. Thus the physical implementation of a high-fidelity quantum CNOT gate is a key to quantum computation. In recent years, many different schemes for CNOT gates using atoms or ions have been proposed [4−6], and some of them have been implemented experimentally [7,8].

Now the main obstacle for quantum gate implementation is decoherence. It is very difficult to isolate the quantum system completely from the environment. Uncontrollable environmental couplings lead to dissipation and the loss of information in general. In the atom-cavity setup, the two main types of dissipation are cavity photon decay and atomic spontaneous emission. To avoid decoherence, decoherence-free states (DFSs) have been proposed to be used as qubits, which are exempted from decoherence in principle[9−11]. Any arbitrary but sufficiently weak interaction can move the state of a system inside the DFS, while all non-DF states of the system couple strongly to the environment and populating them leading to an immediate photon emission[10]. Here the operation inside the DFS can be used to implement a quantum logic gate.

In this paper, we consider the four-level atoms behave like the $\Lambda$ systems inside an optical cavity to implement a CNOT gate. Compared with the original scheme[10], our setup is greatly simplified. Fewer laser pulses are required, which therefore improves the feasibility of the operation.

The two atoms ($i = 1, 2$) involved in the gate operation should be confined to fixed positions inside the optical cavity. This was achieved with the help of a linear ion trap in the experiment by Guthorlein *et al.*[12]. As shown in Fig. 1, each atom comprises a four-level system. The levels $|0\rangle$, $|1\rangle$, $|2\rangle$ correspond to the Zeeman sublevels of the $F = 1$ ground hyperfine level, and $|e\rangle$ corresponds to the $F = 1$ excited level, which is only virtually populated during the gate operation. The qubit of atom is encoded in $|0\rangle$ and $|1\rangle$. The $|e\rangle \rightarrow |1\rangle$ transition of each atom is coupled with the strength $g$ in the cavity mode. To manipulate the states of the atoms, laser pulses are applied, which address each atom individually. The transition $|2\rangle \rightarrow |e\rangle$ is driven by the laser field with Rabi frequency $\Omega_2$, in addition two laser fields are required, one laser couples with Rabi frequency $\Omega_1$ to the $|1\rangle \rightarrow |e\rangle$ transition of atom 1, while the other couples with Rabi frequency $\Omega_0$ to the $|0\rangle \rightarrow |e\rangle$ transition of atom 2. Here, we replace all transitions by Raman transitions. Thus the spontaneous emission from atomic excited state does not have a critical effect on our scheme.

To increase the precision of the operation, we use a broadband photondetector to continuously monitor the free radiation field outside the system. During the time intervals when no count is detected, i.e., no photon is emmited from the cavity during the operation, it is convenient to use a quantum trajectory description[13]. The evolution of the system's wave function is governed by a non-Hermitian effective Hamiltonian[11],

$$\begin{aligned} H_{\rm cond} = & i\hbar g \sum_i [|e\rangle_i\langle 1|b - H.c.] \\ & + \frac{1}{2}\hbar[\Omega_1|e\rangle_1\langle 1| + \Omega_0|e\rangle_2\langle 0| + H.c.] \\ & + \frac{1}{2}\hbar\sum_i[\Omega_2|e\rangle_i\langle 2| + H.c.] + h\Delta_2|e\rangle_i\langle e| \\ & - \frac{i}{2}\hbar k b^+ b - \frac{i}{2}\hbar\sum_i \Gamma_i|e\rangle_i\langle e|, \end{aligned} \tag{1}$$

where, $b$ and $b^+$ are the annihilation and creation operators for a photon inside the cavity mode, respectively.

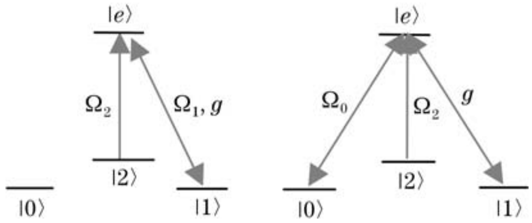


Fig. 1. Level configuration of the atoms inside the cavity. Each qubit is obtained from a pair of ground states $|0\rangle$ and $|1\rangle$ of one atom. The transition between the ground states is realized by Raman transition via the excited state $|e\rangle$.

The spontaneous decay rate of a single photon inside the cavity is denoted by $k$ and the spontaneous emission of excited atomic level $|e\rangle$ by $\Gamma_i$, the corresponding detuning of the laser is denoted by $\Delta_j$ $(j=0,1,2)$.

In order to suppress spontaneous emission from the excited state, we should assume that the detuning $\Delta_2 \gg \Omega_i, g, \Gamma_i$. In this case, one can adiabatically eliminate the excited states $|e\rangle$, thus one has $\Gamma_i = 0$. The new Hamiltonian for the dynamics of the three ground states interacted with the cavity becomes

$$\begin{aligned} H_{\text{cond}} &= i\hbar g_{\text{eff}} \sum_i [|2\rangle_i\langle 1|b - H.c.] \\ &+\frac{1}{2}\hbar[\Omega_{1\text{eff}}|1\rangle_1\langle 2| + \Omega_{0\text{eff}}|0\rangle_2\langle 2| + H.c.] \\ &-\frac{i}{2}\hbar k b^+ b - \hbar\frac{g^2}{\Delta_1}\sum_i |1\rangle_i\langle 1|b^+ b - \frac{\hbar\Omega_2^2}{4\Delta_2}\sum_i |2\rangle_i\langle 2| \\ &-\frac{\hbar\Omega_0^2}{4\Delta_2}|0\rangle_2\langle 0| - \frac{\hbar\Omega_1^2}{4\Delta_2}|1\rangle_1\langle 1|. \end{aligned} \quad (2)$$

The first term is the familiar Jaynes-Cummings interaction with an effective coupling constant $g_{\text{eff}} = -\frac{g\Omega_2}{2\Delta_2}$, the second term is the laser interaction with the effective Rabi frequencies $\Omega_{j\text{eff}} = -\frac{\Omega_j\Omega_2}{2\Delta_j}$. The final four terms all represent level shifts.

Since the DF states of the system must be decoupled from the environment and unable to excite the cavity, they are of the form $|\psi\rangle = |0\rangle_{\text{cav}} \otimes |\varphi\rangle$, where $|0\rangle_{\text{cav}}$ denotes the vacuum state of the cavity field mode, and $|\varphi\rangle$ can be an arbitrary superposition of the atomic ground states $|00\rangle$, $|01\rangle$, $|10\rangle$, $|11\rangle$, and the antisymmetric state $|a\rangle = [|12\rangle - |21\rangle]/\sqrt{2}$. Population that transfers between those states is achieved with the help of stimulated Raman processes. It suggests that the weak laser pulse does not move the state of the system out of the DFS. Nevertheless, the time evolution inside the DFS is not inhibited and is now governed by the effective Hamiltonian $H_{\text{eff}}$. The effective Hamiltonian is the projector of the conditional Hamiltonian $H_{\text{cond}}$ with the projector $P_{\text{DFS}}$, and the relation between them is[10]

$$H_{\text{eff}} = P_{\text{DFS}} H_{\text{cond}} P_{\text{DFS}}, \quad (3)$$

where, $P_{\text{DFS}} = \sum_{i:|\lambda_i\rangle\in\text{DFS}} |\lambda_i\rangle\langle\lambda_i|$ is the projection onto the decoherence-free subspace.

There are no photons in the cavity, so the first level shift term of Eq. (2) can be negligible. As a simplification, we assume that detuning $\Delta_j$ is the same for all $j$, if we choose $\Omega_2 \ll g$, $\Omega_0 \ll \Omega_2$ and $\Omega_1 \ll \Omega_2$, then $\Omega_2^2/\Delta_2$ becomes negligible compared with $g_{\text{eff}}$. $\Omega_0^2/\Delta_2$ and $\Omega_1^2/\Delta_2$ are much smaller than $\Omega_{0\text{eff}}$ and $\Omega_{1\text{eff}}$, respectively, thus the final four terms of Eq. (2) may be neglected. The first term of Eq. (2) can drive the state outside the DFS, which has no contribution to the effective Hamiltonian. So only the laser interaction term is left. This allows us to simplify the Eq. (3), we find

$$H_{\text{eff}} = P_{\text{DFS}} H_{\text{laser}} P_{\text{DFS}}, \quad (4)$$

here $H_{\text{laser}}$ describes the laser interaction and is given in Eq. (2) with the form

$$H_{\text{laser}} = \frac{1}{2}\hbar[\Omega_{1\text{eff}}|1\rangle_1\langle 2| + \Omega_{0\text{eff}}|0\rangle_2\langle 2| + H.c.]. \quad (5)$$

Choosing the effective Rabi frequencies $\Omega_{0\text{eff}} = \Omega_{1\text{eff}} = \Omega$ lead to

$$H_{\text{eff}} = \frac{1}{2\sqrt{2}}\hbar\Omega[|10\rangle\langle a| - |a\rangle\langle 11| + H.c.] \otimes |0\rangle_{\text{cav}}\langle 0|. \quad (6)$$

To perform a CNOT gate, one has to realize a unitary operation between two qubits involved. With a laser pulse of duration $T = 2\pi/\Omega$, the unitary evolution operator of the system is

$$U_{\text{eff}}(T,0) = |010\rangle\langle 011| + H.c. \quad (7)$$

At this particular interaction time $T$, the evolution of the system exchanges the qubit states $|010\rangle$ and $|011\rangle$, while $|000\rangle$ and $|001\rangle$ remain unchanged, thus one can perform the following operation

$$\begin{array}{l} |000\rangle \to |000\rangle \\ |001\rangle \to |001\rangle \\ |010\rangle \to |011\rangle \\ |011\rangle \to |010\rangle \end{array}. \quad (8)$$

This transformation flips the value of the target qubit conditional on the control qubit being in the state $|1\rangle$, which demonstrates the conditional implementation of quantum CNOT gate operation between two atomic states,

$$U_{\text{CNOT}} = |00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 11| + |11\rangle\langle 10|. \quad (9)$$

Now we present a brief discussion on the fidelity of the operation. The no-photon time evolution of the system over a time interval $T$ plays an important role in the operation. If no photon is detected, the fidelity in principle is very close to unity. If an emission is counted, the operation failed and has to be repeated. Therefore the dissipation only decreases the possibility of success, without any influence on the fidelity of the expected operation.

In summary, we have presented a scheme similar to that shown in Ref. [10], but our setup has been optimized for simplicity and its construction is more feasible. To avoid dissipation, we populate only the DF states including all stable states of the atoms while the cavity is in vacuum state. Nevertheless, the interactions between qubits through the simulated Raman process, thus the excited atomic levels are virtually populated.

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## References

1. D. Deutsch, Proc. R. Soc. Lond. A **400,** 97 (1985).
2. D. Deutsch, Proc. R. Soc. Lond. A **425,** 73 (1989).
3. L. K. Grover, Phys. Rev. Lett. **79,** 4709 (1995).
4. S.-B. Zheng and G.-C. Guo, Phys. Rev. Lett. **85,** 2392 (2000).
5. I. E. Protsenko, G. Reymond, N. Schlosser, and P. Grangier, Phys. Rev. A **66,** 062306 (2002).

6. J. I. Cirac and P. Zoller, Phys. Rev. Lett. **74,** 4091 (1995).
7. A. Rauschenbeutel, G. Nogues, S. Osnaghi, P. Bertet, M. Brune, J. M. Raimond, and S. Haroche, Phys. Rev. Lett. **83,** 5166 (1999).
8. S. Osnaghi, P. Bertet, A. Auffeves, P. Maioli, M. Brune, J. M. Raimond, and S. Haroche, Phys. Rev. Lett. **87,** 037902 (2001).
9. A. Beige, D. Braun, B. Tregennan, and P. L. Knight, Phys. Rev. Lett. **85,** 1762 (2000).
10. B. Tregenna, A. Beige, and P. L. Knight, Phys. Rev. A **65,** 032305 (2002).
11. A. Beige, D. Braun, and P. L. Knight, New J. Phys. **2,** 22 (2000).
12. G. R. Guthörlein, M. Keller, K. Hayasaka, W. Lange, and H. Walther, Nature **414,** 49 (2001).
13. M. B. Plenio and P. L. Knight, Rev. Mod. Phys. **70,** 101 (1998).