An all-optical matrix multiplication scheme with non-linear material based switching system

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Optics is a potential candidate in information, data, and image processing. In all-optical data and information processing, optics has been used as information carrying signal because of its inherent advantages of parallelism. Several optical methods are proposed in support of the above processing. In many algebraic, arithmetic, and image processing schemes fundamental logic and memory operations are conducted exploring all-optical devices. In this communication we report an all-optical matrix multiplication operation with non-linear material based switching circuit.

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Several works in optical information processing have been reported in the last three decades. Some of them are on optical logic, arithmetic, and algebraic processing. In all-optical processing, the massive uses of non-linear photo-refractive materials as all-optical switches are seen to enhance the operational speed (above THz) of operation. Non-linear material has various aspects to be used as all-optical switches[1−6]. Here we use a special feature of such non-linear activity in optical switching. We can develop an all-optical system for conducting matrix multiplication with this switch. Various techniques of matrix multiplication have already been proposed[3,7−12]. In our earlier work, we have proposed a scheme for conducting matrix multiplication where we have used electrically controlled spatial light modulators or optical shutters as switches for encoding the inputs of elements of the matrix[3]. Here we are proposing a new technique for conducting all-optical matrix multiplication accommodating non-linear based optical switches to get the real time operation. In this proposal the reported scheme is fully all-optical in nature.

In matrix-matrix multiplication process, we generally represent the matrixes by two planer structure of same sizes. Here all the elements of the matrices are binary bits. Here two such matrices are considered as

$$[A] = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1p} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2p} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3p} \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mp} \end{bmatrix}_{m\times p}$$

$$\text{and} \quad [B] = \begin{bmatrix} b_{11} & b_{12} & b_{13} & \cdots & b_{1n} \\ b_{21} & b_{22} & b_{23} & \cdots & b_{2n} \\ b_{31} & b_{32} & b_{33} & \cdots & a_{3n} \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ b_{p1} & b_{p2} & b_{p3} & \cdots & b_{pn} \end{bmatrix}_{p\times n},$$

or simply $[A] = (a_{ij})_{m\times p}\{i = 1,2,3,\cdots,m; j = 1,2,3,\cdots,p\}$, and similarly $[B] = (b_{jk})_{p\times n}\{j = 1,2,3,\cdots,p; k = 1,2,3,\cdots,n\}$. Here the number of columns in A is the same as the number of rows in B. Then the product $[A][B]$ of order $m \times n$ is given by $(c_{ik})_{m\times n}$, where any element c_{ik} is $a_{i1}b_{1k} + a_{i2}b_{2k} + a_{i3}b_{3k} + \cdots + a_{ik}b_{ik}$ $(i = 1,2,3,\cdots,p; k = 1,2,3,\cdots,n)$. Since all the elements of $[A]$ and $[B]$ are represented either by 0 or 1 (in binary state), so the elements of the resultant (c_{ik}) could be expressed by binary digits if it considered as a binary matrix otherwise the elements of the matrix will be binary number.

For physical implementation of matrix-matrix multiplication, we can represent the matrices by two planes of equal size. Each plane consists of square shaped pixels. There are p number of rows and $p\times n$ number of columns in each plane. Representation technique of $[A]$ and $[B]$ by pixels for $m = 3$, $p = 3$, and $n = 3$ are shown in Fig. 1. As the elements of the matrices can have only 0 or 1 value, so we can represent 1 by presence of light, and 0 by absence of light in the pixels of the represented matrix. When any elements of the matrices (a_{ij} for matrix $[A]$ and b_{ij} for matrix $[B]$) get 1 value then light will present in the all a_{ij} and b_{ij} marked pixels. For example when a_{11} and b_{33} have the 1 value then first, fourth, and seventh pixels in the first row of first plane ($[A]$ matrix plane) and each pixels in the ninth column of the second plane ($[B]$ matrix plane) get light and the other pixels can be represented in such way.

Optical phase conjugation is a mechanism where we can generate a new beam from the interaction of three coherent beams with equal frequency[1,2,13,14]. Basically four-wave mixing (FWM) may be used to generate a phase conjugated wave. A common setup of FWM is shown in Fig. 2. Here forward and backward propagating pump beams (laser beams) are to be made incident in opposite direction to interact in a non-linear medium. To organize the FWM process and to generate a phase-conjugated wave with same frequency as that of the probe beam, we require at least cubic nature of non-linearity of the non-linear interacting medium[13,14]. So optical non-linear material (OPNLM) which has at least cubic type of non-linearity, may be used as interacted medium. When a third probe beam is introduced at a specific angle at the interacting point of the two above pump beams, then a reflected and conjugated beam is generated as a result, which flows just in opposite direction of probe beam. Usually the pump beams have high power while the probe beam is generally weak. Here the

Number of columns $(p \times n) = 3 \times 3$

Number of rows $(p) = 3$

a_{11}	a_{12}	a_{13}	a_{11}	a_{12}	a_{13}	a_{11}	a_{12}	a_{13}
a_{21}	a_{22}	a_{23}	a_{21}	a_{22}	a_{23}	a_{21}	a_{22}	a_{23}
a_{31}	a_{32}	a_{33}	a_{31}	a_{32}	a_{33}	a_{31}	a_{32}	a_{33}

Matrix $[A]$

Number of columns $(p \times n) = 3 \times 3$

Number of rows $(p) = 3$

b_{11}	b_{21}	b_{31}	b_{12}	b_{22}	b_{32}	b_{13}	b_{23}	b_{33}
b_{11}	b_{21}	b_{31}	b_{12}	b_{22}	b_{32}	b_{13}	b_{23}	b_{33}
b_{11}	b_{21}	b_{31}	b_{12}	b_{22}	b_{32}	b_{13}	b_{23}	b_{33}

Matrix $[B]$

Fig. 1. Representation of matrices $[A]$ and $[B]$ by pixels for $m = 3$, $p = 3$.

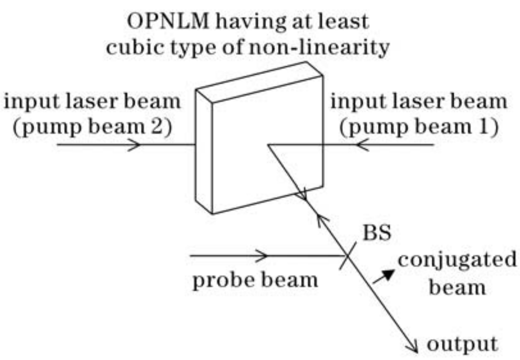

Fig. 2. Optical phase conjugate system.

switching mechanism can be discussed as follows. When two interacting pump beams are present then output beam will be present and when any one or both the pump beams are absent, no output beam will be received at the output.

To implement an all-optical matrix-matrix multiplication scheme, we take a non-linear material at the center and in one side we keep the input plane $[A]$ and in the other side we keep the other input plane $[B]$ as described in Fig. 4 coaxially. Now the laser beams are allowed to be incident on the OPNLM through these input planes and $(p \times n) \times p$ number of probe beams are allowed to incident in each cells on the OPNLM. Here in Fig. 3 $\{(3 \times 3) \times 3\} = 27$ probe beams are made incident on 27 spatially distributed cells on the OPNLM. The output channels coming from each pixel on the OPNLM are coupled following a principle. 1, 2, and 3 marked cells on the OPNLM are coupled together to get the element c_{11}, light from 4, 5, and 6 marked cells are coupled to get the element c_{12} and so on. To get this actual product of the two matrices $[A]$ and $[B]$, we must consider the graded level variation of the total light intensity corresponding to each element of the resultant matrix $[C]$. Nine spots are arrayed in three rows and three columns in the output plane. The whole operation will be clear from the following example. Two particular 3×3 binary matrices are considered as

$$[A] = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \quad [B] = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix},$$

i.e. a_{11}, a_{12}, a_{23}, a_{31}, a_{32}, and a_{33} have value 1, and a_{13}, a_{21}, and a_{22} have value 0, b_{12}, b_{21}, b_{22}, b_{32}, and b_{33} have value 1 and b_{11}, b_{13}, b_{23}, and b_{31} have value 0.

Hence only a_{11}, a_{12}, a_{23}, a_{31}, a_{32}, and a_{33} marked pixels in the $[A]$ plane get light and no light presents in the other pixels. Similarly light presents in the b_{12}, b_{21}, b_{22}, b_{32}, and b_{33} marked pixels and no light presents in the other pixels in the $[B]$ plane. Based the values of the elements (which may be 0 or 1) of the matrices $[A]$ and $[B]$, the pump beams fall on OPNLM from two opposite sides. Probe falls in all the cells of the OPNLM. Now based on switching mechanism described in Fig. 2, the conjugated output beam will appear. For example the second cell of the first row on the OPNLM gets the pump beams from two opposite sides, so the phase conjugated beam will come from this cell whereas no phase conjugated beam will come from the first and second cells of the first row as these cells are getting only single pump beam from the left. In the same way, some conjugated beams will come out from some cells on the OPNLM and no conjugated beams from the other cells. Coupling of the output channels from the cells of the OPNLM is done properly with mirrors and beam splitters so that the resultant output matrix is identified by the intensity variation. Though the elements of the input matrices are binary number (0 or 1), the resultant elements of the output matrix can have values 0, 01, 10, 11 or 0, 1, 2, 3 in decimal. In Fig. 3 each three channels is coupled to get one element of the output matrix. The elements of the resultant matrix are marked by 00 for no light, 01 for light with some prefixed intensity I_0, 10 for light with the intensity approximately by $2I_0$, and 11 for light with the intensity near approximately by $3I_0$. Following this procedure in the output plane, we will get no light (00) in the positions of 1st row 3rd element and 2nd row 1st element, near about double intensity of light ($2I_0 = 10$) in the position of 1st row 2nd element in comparison to the intensity ($I_0 = 01$) obtained in the positions of 1st row 1st, 2nd row 2nd and 3rd, 3rd row 1st and 3rd element and near about triple intensity of light ($3I_0 = 11$) in the position of 3rd row 2nd element in comparison to the intensity (01) position.

In the above discussed scheme, we can not define properly the state of the resultant element of the output matrix. To get proper result, we can introduce a non-linear material based system as shown in Fig. 4. The equation of refractive index for some type of non-linear material can be written as

$$n_{\mathrm{NL}} = n_0 + n_i \cdot I,$$

where n_0 and n_i are two constants for the non-linear material and I is the intensity of the laser beam passing through it. According to the above equation the direction of the output beam will change if we change the intensity of the light beam passing through it. To display the output of the resultant matrix in Fig. 3, we can pass each light beam corresponding each element through such isotropic non-linear material having the refractive index n_{NL} $(= n_0 + n_i \cdot I)$. The laser beams of intensity

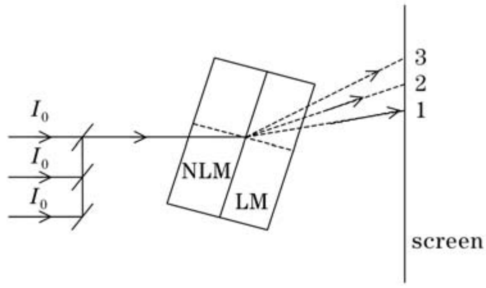

Fig. 3. Optical non-linear material based switching system (NLM indicates non-linear material and LM indicates linear material).

I_0, $2I_0$, $3I_0$ will pass through different directions being refracted from the non-linear material. If the directions of the output beams being refracted from such non-linear materials are represented by 1, 2, 3 (as the direction of I_0 intensity by 1, direction of $2I_0$ intensity beam by 2 and that of $3I_0$ by 3), then we can detect each element of the resultant matrix properly by the graded variation of the intensity of light which is obtained corresponding to the element.

Following this process, if we used this linear-nonlinear combination system before each element in the output matrix which is shown in Fig. 3, then the result will be appeared at the output screen as

$$[C] = [A] \times [B] = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 01 & 10 & 00 \\ 00 & 01 & 01 \\ 01 & 11 & 01 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 1 & 3 & 1 \end{bmatrix}$$

(equivalent resultant decimal matrix).

The value 1, 2, 3 in the screen are marked by light spot position after the refraction from the linear-nonlinear material combination. 0 is marked here by absence of light,

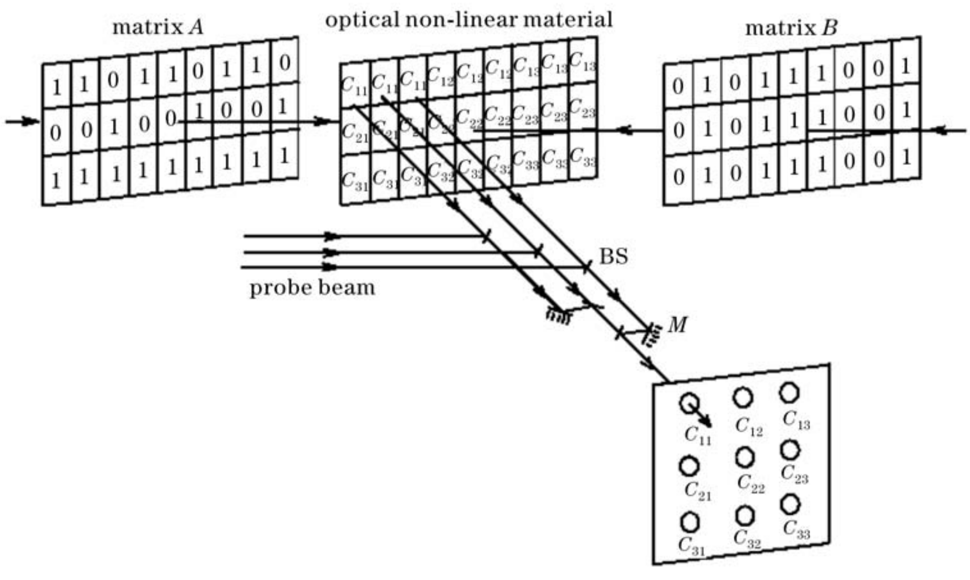

Fig. 4. An all-optical matrix-matrix multiplication ($[A] \times [B]$) scheme.

i.e. for 0 element. It is important to say that each beam in the input side should take a constant intensity I_0, otherwise the spot position in the screen will change.

Here in the above method, we have used two types of switching of OPNLM. In the first type (having at least cubic type of non-linearity) phase conjugated beam comes when the two pump beams interact with the probe beam and in the second type different intensity of light flows in different directions after reflection from OPNLM. The first type is used in organizing the multiplication whereas the second type is used in the detection of the elements of the resultant matrix. In the above scheme we have cited the example of conducting multiplication of any two 3×3 matrices, but in real field we can conduct the multiplication between any two higher ordered binary matrices. It is important to note that the coherency of the laser beams and direction of the both pump and probe beams are the essential requirement to hold the phase conjugation or the four-wave coupling mechanism. Some experiments can be referred in this regard[6,15]. In these experiments $LiNbO_3$, $LiIO_3$, $BaTiO_3$, etc. are used as photo-refractive crystal and Nd:YAG laser as pump beam for conducting four wave mixing process.

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