

# Two-dimensional overlapping-image multimode interference couplers

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The overlapping-imaging effect of one-dimensional (1D) multimode interference (MMI) coupler is widened to study the two-dimensional (2D) MMI coupler. 2D overlapping-image MMI couplers permit uniform and nonuniform 2D power splitting. Analytical formulas are derived for the intensities and phases of the overlapping-images at the end of MMI section. The overlapping-imaging properties in 2D MMI couplers are also concluded. And the guided-mode propagation analysis method is used to confirm the analytical results.

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Recently there has been a growing interest in two-dimensional (2D) multimode interference (MMI) couplers<sup>[1-4]</sup>. Adding a new dimension to the one-dimensional (1D) MMI couplers would greatly increase the integration density<sup>[5]</sup>. Rajarajan has given an accurate analysis method for 2D MMI couplers<sup>[1]</sup>, but the imaging properties of 2D overlapping-image MMI couplers have not been discussed. In this letter, the formulas of the intensities and phase relations are presented and verified with numerical simulation of the guided-mode propagation analysis method (GMPAM)<sup>[6]</sup>.

When the input horizontal position  $x_0 \neq iW_x/N$  ( $i = 1, 2, \dots, N - 1$ ) and the input vertical position  $y_0 \neq jW_y/M$  ( $j = 1, 2, \dots, M - 1$ ),  $N \times M$  fold-images of equal intensities can be obtained at the distances<sup>[2]</sup>

$$L = \frac{s_x}{N}3L_{\pi x} = \frac{s_y}{M}3L_{\pi y}, \tag{1}$$

where  $N$  and  $M$  are positive integers without common divisors with the positive integers  $s_x$  and  $s_y$ , respectively.

$L_{\pi x} = \frac{4n_g W_x^2}{3\lambda_0}$  and  $L_{\pi y} = \frac{4n_g W_y^2}{3\lambda_0}$  are two beating lengths in the  $x$ - and  $y$ -direction,  $\lambda_0$  is the wavelength in vacuum,  $n_g$  is the core refractive index,  $W_x$  and  $W_y$  are the lateral and vertical widths.

If  $x_0 = iW_x/N$  or  $y_0 = jW_y/M$ , previously separated self-images overlap and a new output ratio arise at the distances  $L$ , which is called overlapping-imaging effect<sup>[7]</sup>.

In the case of  $x_0 = iW_x/N$  and  $y_0 \neq jW_y/M$ , overlapping-images only occur in the  $x$ -direction. The number of images in the  $x$ -direction reduces to  $K$ , and  $K \times M$  fold-images can be obtained at the output. Interference between two equal light distributions of intensity  $1/(N \times M)$  occurs. For two interfering images with phase  $\varphi_1$  and  $\varphi_2$ , which can be obtained from the equations in Ref. [8], intensity  $r^2$  and phase  $\Phi$  of the resulting images are

$$r^2 = \frac{4}{N \times M} \cos^2\left(\frac{\varphi_1 - \varphi_2}{2}\right),$$

$$\Phi = \begin{cases} \frac{\varphi_1 + \varphi_2}{2} & \cos\left(\frac{\varphi_1 - \varphi_2}{2}\right) > 0 \\ \frac{\varphi_1 + \varphi_2}{2} + \pi & \cos\left(\frac{\varphi_1 - \varphi_2}{2}\right) < 0 \end{cases} . \tag{2}$$

There are two especial cases: Input at the center

of the MMI coupler ( $x_0 = W_x/2$ ), with  $N = 2k$  ( $k = 1, 2, 3, \dots$ ); input at one third of the MMI coupler ( $x_0 = W_x/3$  or  $x_0 = 2W_x/3$ ), with  $N = 3k$  ( $k = 1, 2, 3, \dots$ ). In the two cases, MMI couplers of length  $L$  yield  $K \times M$  fold-images of equal intensities. All the other cases yield nonuniform splitting ratios in the  $x$ -direction.

To verify the previous results, the GMPAM is used. In the following examples, the parameters of the silicon-on-insulator (SOI) material are used. The cladding and core refractive index are 1.45 and 3.45, respectively. The wavelength is  $1.55 \mu\text{m}$ . Consider a 2D MMI coupler with  $W_x = W_y = 30 \mu\text{m}$  and  $L = 1619 \mu\text{m}$ . If the input field is centered at  $x_0 = 4 \mu\text{m}$  and  $y_0 = 4 \mu\text{m}$ , we get  $5 \times 5$  fold-images at the output. If  $x_0 = 6 \mu\text{m}$  and  $y_0 = 4 \mu\text{m}$ , overlapping-images only occur in the  $x$ -direction, and we get  $2 \times 5$  fold-images. The 2D MMI coupler splits the input power into ten images in two columns, as shown in Fig. 1, obtained by the GMPAM. The five images in the same column have equal intensities. The splitting ratio between the two images in the  $x$ -direction is 28 : 72. From Fig. 1, we can see that our analytical formulas agree very well with the GMPAM simulation results.

The case of  $x_0 \neq iW_x/N$  and  $y_0 = jW_y/M$  just resembles the case of  $x_0 = iW_x/N$  and  $y_0 \neq jW_y/M$ . Overlapping-images only occur in the  $y$ -direction, and the number of images in the  $y$ -direction reduces to  $R$ . We get  $N \times R$  fold-images at the output. Except the two

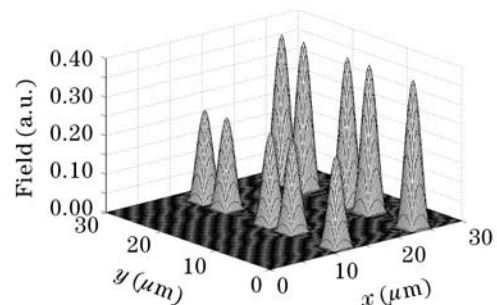


Fig. 1. The output field distribution at  $L = 1619 \mu\text{m}$  with  $x_0 = 6 \mu\text{m}$  and  $y_0 = 4 \mu\text{m}$ .

especial cases, overlapping-image MMI couplers yield nonuniform splitting ratios in the  $y$ -direction.

In the case of  $x_0 = iW_x/N$  and  $y_0 = jW_y/M$ , overlapping-images occur in both  $x$ - and  $y$ -direction. The number of images in the  $x$ -direction reduces to  $K$ , and that in the  $y$ -direction reduces to  $R$ . Interference among four equal light distributions of intensity  $1/(N \times M)$  occurs. For four interfering images with phase  $\varphi_1, \varphi_2, \varphi_3$ , and  $\varphi_4$ , intensity  $r^2$  and phase  $\Phi$  of the resulted images are

$$r^2 = \frac{16}{N \times M} \cos^2\left(\frac{\varphi_1 - \varphi_2}{2}\right) \cos^2\left(\frac{\varphi_1 - \varphi_3}{2}\right),$$

$$\Phi = \begin{cases} \frac{\varphi_1 + \varphi_2 + \varphi_3 + \varphi_4}{4} & \cos\left(\frac{\varphi_1 - \varphi_2}{2}\right) \cos\left(\frac{\varphi_1 - \varphi_3}{2}\right) > 0 \\ \frac{\varphi_1 + \varphi_2 + \varphi_3 + \varphi_4}{4} + \pi & \cos\left(\frac{\varphi_1 - \varphi_2}{2}\right) \cos\left(\frac{\varphi_1 - \varphi_3}{2}\right) < 0 \end{cases} \quad (3)$$

From Eq. (3), we can find that intensity of the resulted image is the product of intensities of the two interfering images in  $x$ - and  $y$ -direction, which means the 2D overlapping-imaging effect can be regarded as the superposition coupling of two orthogonal but independent 1D overlapping-imaging effects. If neither of the two directions satisfies the two especial cases mentioned above, MMI couplers yield nonuniform splitting ratios in two directions. If only one direction satisfies the two especial

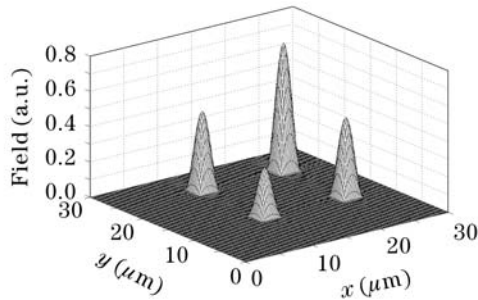


Fig. 2. The output field distribution at  $L = 1619 \mu\text{m}$  with  $x_0 = y_0 = 6 \mu\text{m}$ .

cases, MMI couplers yield nonuniform splitting ratios in one direction. If both of the two directions satisfy the two especial cases, MMI couplers yield uniform splitting ratios.

If the input is centered at  $x_0 = y_0 = 6 \mu\text{m}$ , we get  $2 \times 2$  fold-images. The 2D MMI coupler splits the input power into four images in two columns, as shown in Fig. 2, obtained by GMPAM. The intensity distribution at the four outputs is 8 : 20 : 20 : 52.

As a new class of devices, 2D overlapping-image MMI couplers permit uniform power splitting in two directions and nonuniform power splitting in one or two directions, as long as the input waveguides are placed properly. To get the same splitting ratios, 2D overlapping-image MMI couplers have more compact structure as compared with general 2D MMI couplers.

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