Mathematical model for light scanning system based on circular laser

Peiquan Xu (徐培全)[1], **Shun Yao (姚 舜)**[1], **Fenggui Lu (芦凤桂)**[1],
Xinhua Tang (唐新华)[1], **and Wei Zhang (张 伟)**[2]

[1]*Welding Research Institute, Shanghai Jiao Tong University, Shanghai 200030*
[2]*Shanghai Institute of Laser Technology, Shanghai 200233*

A novel light scanning system based on circular laser trajectory for welding robot is developed. With the help of image processing technique, intelligent laser welding could be realized. According to laser triangulation algorithm and Scheimpflug condition, mathematical model for circular laser vision is built. This scanning system projects circular laser onto welded seams and recovers the depth of the welded seams, escapes from shortcomings of less information, explains ambiguity and single tracking direction inherent in "spot" or "line" type laser trajectory. Three-dimensional (3D) model for welded seams could be recognized after depth recovery. The imaging error is investigated also.
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During the course of intelligent welding, it is a crucial problem for welding robot system to be superior in quality and guarantee repeatable accuracy. Mismachining tolerance, welding stress, and deformation will lower the positioning accuracy of welded seams deviating from position preset by robot controller. However, robot system cannot identify such a deviation. In order to meet the requirement of seam location and seam tracking, sensing technique is developed to identify error and send it back to robot controller, so that the robot can adjust the seam trajectory in real time. In general, the methods for welded seams location and tracking include touch sensing, through arc seam tracking, supersonic sensing, passive vision sensing, and active vision system[1−6]. Vision sensing technique based on laser attracts much interest due to its contactless detection characteristic and depth recovery ability.

We propose a kind of circular laser vision system and build mathematical model. Laser as active light source is projected onto welded seams, and circular laser as scanning light overcomes the difficult of depth recovery for passive vision sensing. This vision system has significant advantages compared with other sensors. High performance, intelligent vision software enables scanning of all joint types including butt, lap, fillet, corner, and V grooves etc..

Traditionally, the laser triangular algorithm[7] is used to calculate the depth values; however, this algorithm cannot adapt to the proposed circular laser vision system. The algorithm for the depth recovery based on circular vision system is investigated and error analysis is also carried out by image processing method.

Vision sensing system based on circular laser consists of laser diode, motor with coder, charge-coupled device (CCD) camera, and workpiece platform, as illustrated in Fig. 1. Laser is projected onto welded seams as a cone. Image acquisition could be realized by the application of area-array CCD camera and the angle of CCD camera to laser axis is preset.

In the laser triangular model, the angle of laser source to camera is vitally significant. Three defined angles are laser scanning cone angle α, deflection angle β of laser axis with camera axis, and off-axis angle γ of laser spot vector with x axis, as shown in Fig. 2. Because the circular vision system is fixed on the robot arm, height between laser source and workpiece and radius of circular laser are constant, so α is constant. β has an effect upon imaging scope of welded seams in CCD sensitivity plane and imaging quality[7]. The scope of β is $30° − 60°$. In this paper, β is chosen as $15°$, $30°$, $45°$, $60°$, and $75°$. γ is changed from 0 to $360°$.

The principle of circular vision system is algorithm based on laser triangular method. This method[7] developed from traditional light triangular principle could be used to detect three-dimensional (3D) change of workpiece. The principle is illustrated in Fig. 3. According to Scheimpflug condition, we have

$$a_0 \cot \beta = b_0 \cot \theta. \tag{1}$$

CCD camera is fixed and perpendicular to light axis, that is to say $\beta = 90°$. So only if laser axis is perpendicular to CCD axis, this kind of light path could abide by Scheimpflug condition[7]. When laser is projected onto
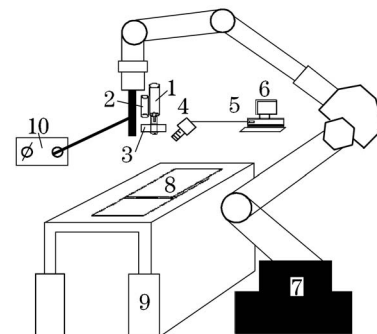

Fig. 1. Illustration of circular vision sensor system for laser welding robot. 1, 2, 3: circular laser source; 4: area-array CCD; 5: image grabber; 6: PC; 7: robot; 8: zinc-coated plateform; 9: operation platform; 10: YAG laser.
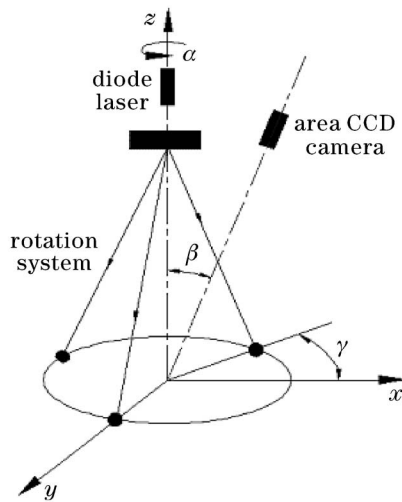
Fig. 2. Definition of angles for rotatable laser vision system configuration.
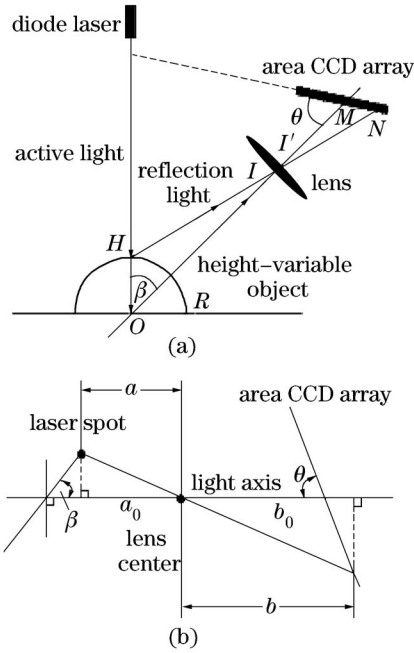

Fig. 3. Principle of the traditional laser triangular method. (a) Principle of traditional laser triangulation algorithm, (b) diffuse reflection and CCD camera light path.

the welded seams as a circle, ideal image cannot form in image sensitivity plane. This could be solved by pre-image processing and depth compensation algorithm.

Traditional laser triangular rule is shown in Fig. 3(a). β is angle of light axis with laser axis, θ is angle of CCD sensitivity plane with light axis. O point is intersection of laser axis across light axis. H is random point in object, I and I' are inlet and outlet of light in the lens, respectively. H is imaged on N point of CCD sensitivity plane, the deviation distance l between N point and plane center M point is

$$l = x_M - x_N. \tag{2}$$

According to the simple geometry rules, the relation

between depth of object OH and deviation distance l is

$$Z = \frac{(|OI| - f)l}{f\sin\beta + l\cos\beta}, \tag{3}$$

where f is focal distance of imaging system. When laser spot travels along a certain direction, line-scanning comes true, and more information could be acquired.

The traditional simple triangular method cannot be used to detect the depth in the circular vision system. Here we propose an algorithm based on analytic geometry, Oxz light path plane can be built. This mathematical model will be studied in the following step when $\gamma = 0°$, $90°$, $180°$, respectively.

When $\gamma = 180°$, the principle of light path is shown in Fig. 4 (far most distance of workpiece to CCD camera). From Fig. 4, we can get

$$|IJ| = l\sin\theta. \tag{4}$$

The coordinates of lens center I are

$$x_I = |OI|\sin\beta, \tag{5}$$

$$z_I = |OI|\cos\beta, \tag{6}$$

$$\sqrt{(|OI|\sin\beta - x_J)^2 + \left[|OI|\cos\beta + (x_J - \frac{|OI|}{\sin\beta})\tan\beta\right]^2}$$
$$= l\sin\theta. \tag{7}$$

That is to say

$$\sec^2\beta x_J^2 - 2\frac{\sin\beta}{\cos^2\beta}|OI|x_J$$
$$+ |OI|^2\tan^2\beta - l^2\sin^2\theta = 0. \tag{8}$$

Because $\Delta = 4\frac{l^2\sin^2\theta}{\cos^2\beta} \geq 0$, in Oxz plane, there are two values for x_I, corresponding to near CCD camera spot and far CCD camera. By Eqs. (3)—(7), the coordinates of J point are

$$x_J = \sin\beta |OI| \pm l\sin\theta\cos\beta, \tag{9}$$

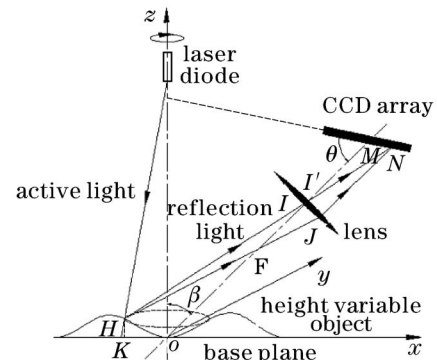$$z_J = \cos\beta |OI| \mp l\sin\theta\sin\beta, \tag{10}$$

Fig. 4. Principle of circular laser vision based on analytic geometry.

the position is shown in Fig. 4.

In addition, the coordinates of focal point F are defined as

$$x_F = |OI - f| \sin \beta, \tag{11}$$

$$z_F = |OI - f| \cos \beta. \tag{12}$$

The geometry formula for line FJ can be described as

$$z - \frac{f \cos \beta - l \sin \theta \sin \beta}{f \sin \beta + l \sin \theta \cos \beta} x = (|OI| - f) \cos \beta$$

$$- \frac{f \cos \beta - l \sin \theta \sin \beta}{f \sin \beta + l \sin \theta \cos \beta} (|OI| - f) \sin \beta, \tag{13}$$

while $\frac{f \cos \beta - l \sin \theta \sin \beta}{f \sin \beta + l \sin \theta \cos \beta} = K$, $(|OI| - f) \cos \beta - K(|OI| - f) \sin \beta = B$, then

$$z - Kx = B. \tag{14}$$

The geometry formula for scanning line $O'H$ can be described as

$$z = \cot \frac{\alpha}{2} \cdot (x + 16), \tag{15}$$

where α is laser scanning cone angle.

Combining Eqs. (14) and (15), the intersect coordinates are as follows, where the vertical coordinate of the intersect is the depth value of welded seams

$$h = \frac{16K - B}{K \cdot \tan \frac{\alpha}{2} - 1}. \tag{16}$$

Given welded seams plane information, we could build welding 3D model of welded seams.

Because CCD camera adopted here is area-array CCD, when $\gamma = 90°$ and $270°$, the light path principle is the same as the above. The plane for analytic geometry changes with γ. For the laser spot in right position, the light path abides Scheimpflug condition. However, when the height changes, clear imaging cannot be acquired in CCD sensitivity plane.

In the whole position of laser scanning, if only β accords with

$$\frac{\pi}{2} > \beta \geq \arctan \frac{R}{r}, \tag{17}$$

where R stands for the height of scanning cone, r is radius of standard circle. Then when $\gamma = -\frac{\pi}{2}$—$\frac{\pi}{2}$, there must be a couple of points abiding by Scheimpflug condition. As shown in Fig. 5, the model accords with that of laser-stripe scanning. There are a couple of points in the circle instead of one stripe. Here plane PKH is the scanning plane, combined with light axis OO' and CCD axis, composing the whole light path system, and abided by Scheimpflug condition. The method for depth resolution is the same as above. The verification and resolution of γ are as follows. $KH \perp Ox$ axis, $KH \perp PO \Rightarrow OH \perp$ coordinates plane Oxz, light axis $OO' \in Oxz$, so $KH \perp OO'$, moreover, $PH \perp OO' \Rightarrow OO' \perp PKH$. That is to say, the entire light path in plane PKH abides by the imaging rule. Laser light could meet the requirement of Scheimpflug condition. Here γ is defined as

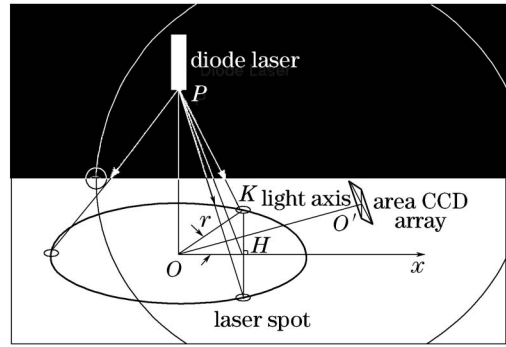$$\gamma = \arccos \frac{R \cot \beta}{r}, \tag{18}$$

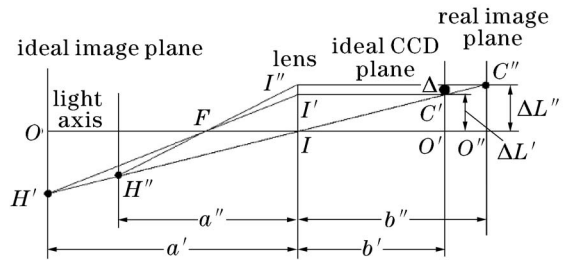Fig. 5. The scan position abides by Scheimpflug condition.

Fig. 6. Error analysis for visual system based on circular vision system.

where R is the height of scanning cone, r is the radius of scanning circle.

In real production, ideal spot forms in area-array CCD when Scheimpflug condition could be met. Here we define the ideal spot as geometry point. According to above analysis, error analysis of ideal condition (abiding by Scheimpflug condition) and real light path are shown in Fig. 6. In this figure, H', a', b', and $\Delta l'$ are ideal object point, ideal object length, ideal imaging length, and ideal deviation in sensitivity plane. H'', a'', b'', $\Delta l''$ stand for real object point, real object length, real image length, and real deviation distance in sensitivity plane. C' and C'' are ideal imaging point and real point respectively. Δ is the real light spot in CCD sensitivity plane. Therefore, error for Δ cannot be avoided when light path cannot abide by Scheimpflug condition, which is the source of the detection error. Image processing method is used to correct such an error.

Thinning processing is necessary when extracting information of welded seams from acquired image, aiming at describing edge of active laser scanning track and welded seams using curve expressed by single pixels. At present, the edge thinning algorithm includes Hilditch algorithm and Rosenfeld algorithm[8]. However, both of them take center of strip as the default edge, while edge in the proposed model is located at the exterior side rather than center of strip. Taking exterior side of strip, we can lower even eliminate the error caused by falling short of Scheimpflug condition. Projection thinning algorithm is investigated to solve this problem, whose principle is shown in Fig. 7.

In binary image for upper half circle (see Fig. 7(c)), in one certain column direction, if gray value of current point (x, y) is 0, check gray value of next point $(x, y + 1)$,
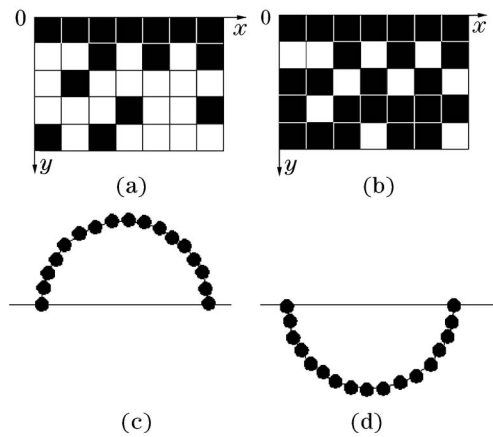
Fig. 7. Theory of the projection thinning.

if gray value of point $(x, y + 1)$ is 0, find the next until gray value of 255 is hunted and set it to 0. Essence of this algorithm is vertical projection pixels with discontinuous point in all columns onto the place where discontinuity exists. Connectivity for edge of original image is kept to the whole extent and exterior edge is extracted. And *vice versa* (see Fig. 7(d)).

In real vision system, we build look-up table (LUT) (index) about depth change value Δh and corresponding imaging change value by pre-calibration method. According to calculating the distance of real imaging position to CCD sensitivity center, real depth value can be obtained. Meanwhile, compensating for kinds of optical aberration caused by CCD plane will be realized by calibrating LUT.

This paper proposes vision-sensing model based on circular laser detection. The mathematical model for circular laser vision system is built and the error caused by falling short of Scheimpflug condition is analyzed. This proposed model could be used to recover the depth of the welded seams and 3D model could be built, which lay a foundation for seam distinguishing and tracking.

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