Analysis on the effect of nonlinear polarization evolution in nonlinear amplifying loop mirror

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By considering the cross phase modulation (XPM) between the two orthogonal poparization components, the nonlinear birefringence and nonlinear polarization evolution (NPE) in highly-nonlinear fiber (HNLF), as well as the unequal evolutions of the state of polarization (SOP) between the clockwise (CW) and counter-clockwise (CCW) waves in a nonlinear amplifying loop mirror (NALM) are analyzed. It is pointed out that the traditional cosine expression is no longer valid for the power transmission of NALM due to uncompleted interference under the high power condition. The analytical expression considering NPE effect is derived, and the experimental result is presented.

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There has being great interest in the development of some optical subsystems where nonlinear amplifying loop mirror (NALM)[1] is widely used, such as optical switches[2,3], ultra-short pulses compression[4], all-optical wavelength conversion[5], all optical signal reshaping/regeneration[6], and amplified spontaneous emission reduction. The traditional analytical expression for NALM's power transmission characteristics is essentially a cosine function with zero points when the phase difference between clockwise (CW) and counter-clockwise (CCW) waves is odd times of π. However, the experimantal results show that, the transmission cannot return to zero even if the phase difference is properly π. This feature has serious effect in some applicatons, and was explained in Ref. [7] as the inaccurate fiber parameters with omitting the group velocity dispersion (GVD) in the theoretical expression.

Actually, in order to achieve large enough phase difference, relatively high power and high nonlinear coefficient fiber are usually used in NALM, and usually the residual birefringence in real fiber is unavoidable, because the fiber section cannot be perfectly circular. As the result, in addition to the desired self phase modulation (SPM) difference between the CW and CCW waves, different refractive index for each polarization wave and cross phase modulation (XPM) between the two orthogonal components, that result in nonlinear birefringence and nonlinear polarization evolution (NPE), must be considered. NPE effect had been discussed in Ref. [8, 9] as the passively mode-locking mechanism for the generation of picosecond and femtosecond pulses, but to our knowledge, no discussion was carried out on NALM.

In this paper, the magnitude of nonlinear birefringence and NPE in a highly nonlinear fiber (HNLF) is evaluated based on the theoretical analysis, and the analytical expression for the power transmission of NALM considering NPE effect is derived. It is demonstrated numerically and experimentally that uncompleted interference from the unequal polarization evolution between the CW and CCW waves is one of the reasons that the transmission characteristic of NALM is distorted from a cosine curve.

Considering a linear polarized light that enters and propagates in a HNLF, assuming that the amplitude of the input light is A, the polarization angle with respect to the fast axis (the x-axis) of fiber is θ. The accumulated nonlinear phase shifts over the fiber length L for the two orthogonal polarization components can be expressed as[10]

$$\begin{cases} \varphi_x = \gamma\left(|A_x|^2 + \frac{2}{3}|A_y|^2\right)A_{\mathrm{eff}}L \\ \quad\ = \gamma P\left(\cos^2\theta + \frac{2}{3}\sin^2\theta\right)L \\ \varphi_y = \gamma\left(|A_y|^2 + \frac{2}{3}|A_x|^2\right)A_{\mathrm{eff}}L \\ \quad\ = \gamma P\left(\sin^2\theta + \frac{2}{3}\cos^2\theta\right)L \end{cases}, \qquad (1)$$

where A_x, A_y are the amplitude of each polarization components, γ is the nonlinear coefficient of the fiber, $P = |A|^2 A_{\mathrm{eff}}$ is the total input optical power, and A_{eff} is effective area of the fiber. The first term and second term in Eq. (1) represent SPM and XPM effect of the components, respectively.

For a given HNLF, we are more interested in the relative magnitude of NPE strength, which was defined as the ratio of $\Delta\varphi_{\mathrm{NL}} = \varphi_x - \varphi_y$ to φ_x or φ_y. From Eq. (1), this ratio can be derived as

$$\begin{cases} \alpha_x = \left|\frac{\Delta\varphi_{\mathrm{NL}}}{\varphi_x}\right| = \left|\frac{\frac{\gamma PL}{3}|\cos 2\theta|}{\gamma PL(\cos^2\theta + \frac{2}{3}\sin^2\theta)}\right| = \frac{|\cos 2\theta|}{3\cos^2\theta + 2\sin^2\theta} \\ \alpha_y = \left|\frac{\Delta\varphi_{\mathrm{NL}}}{\varphi_y}\right| = \left|\frac{\frac{\gamma PL}{3}|\cos 2\theta|}{\gamma PL(\sin^2\theta + \frac{2}{3}\cos^2\theta)}\right| = \frac{|\cos 2\theta|}{3\sin^2\theta + 2\cos^2\theta} \end{cases}. \qquad (2)$$

It can be seen that, α_x and α_y are related to the input angle θ only, no matter the fiber parameters and the optical power. If we set $\theta = 20°$, α_x and α_y can be calculated to be 0.362 and 0.230 respectively, indicating that NPE is about a few tenths of the accumulated nonlinear phase shift. An exception may happen at $\theta = 45°$, where $\alpha_x = 0$, $\alpha_y = 0$. However, for a real fiber, not only birefringence is unavoidable, but also the two principle axes of the fiber would rotate irregularly. As the result, the state of polarization (SOP) cannot preserve along the fiber, thus the effect by the input angle θ would be averaged out and the special case of $\alpha_x = 0$, $\alpha_y = 0$ would not happen.

On the other hand, one can also evaluate the nonlinear strength by comparing the linear and nonlinear beat length

$$\frac{L_{\mathrm{B}}^{\mathrm{NL}}}{L_{\mathrm{B}}^{\mathrm{L}}}=\frac{\frac{\lambda}{|\Delta n_x^{\mathrm{NL}}-\Delta n_y^{\mathrm{NL}}|}}{\frac{\lambda}{|n_{x\mathrm{eff}}-n_{y\mathrm{eff}}|}}=\frac{3|n_{x\mathrm{eff}}-n_{y\mathrm{eff}}|}{n_2|A|^2\cos 2\theta}$$

$$\sim\frac{3|n_{x\mathrm{eff}}-n_{y\mathrm{eff}}|}{n_2|A|^2\langle\cos 2\theta\rangle|_{\theta=[-\pi/4,\pi/4]}}=\frac{3\pi B_m^{\mathrm{L}}A_{\mathrm{eff}}}{2n_2P}, \tag{3}$$

where $L_{\mathrm{B}}^{\mathrm{L}}$ and $L_{\mathrm{B}}^{\mathrm{NL}}$ are the linear and nonlinear beat length, λ is the optical wavelength, $n_{x\mathrm{eff}}$, $n_{y\mathrm{eff}}$ and Δn_x^{NL}, Δn_y^{NL} are effective refractive indices and the nonlinear refractive indices on x-axis and y-axis, respectively. $B_m^{\mathrm{L}}=|n_{x\mathrm{eff}}-n_{y\mathrm{eff}}|$ is the linear birefringence and n_2 is the nonlinear index coefficient. The factor of $\langle\cos 2\theta\rangle|_{\theta=[-\pi/4,\pi/4]}=\frac{2}{\pi}$ in denominator represents the average effect of θ. Here the limitation of integral is set to be $\pm\pi/4$, because same SOP would repeat beyond this range. If $B_m^{\mathrm{L}}=10^{-6}$, $n_2=3.2\times10^{-20}$, peak power P = 500 mW and $A_{\mathrm{eff}}=10\ \mu\mathrm{m}^2$, $L_{\mathrm{B}}^{\mathrm{NL}}$ is about 2945 times of $L_{\mathrm{B}}^{\mathrm{L}}$, while $L_{\mathrm{B}}^{\mathrm{L}}$ is generally ~ 1 m. It implies that, for most applications, where the HNLF length is about a few kilometers, the NPE could cause almost a period of the SOP variation.

A typical configuration of NALM is illustrated in Fig. 1, there are also the labels for two principle axes of the loop fiber and the SOP of CW and the CCW waves. Here, the CCW wave is amplified before transmitting through the HNLF, while the CW wave is in the reverse process, so different SOP variations appear for the two waves, which affects the interference result and the power transmission characteristics of NALM. In order to deduce the analytical expression, we assume that A is the amplitude of input wave, θ is the input polarization angle with respect to the fast axis of HNLF. The gain of bidirectional erbium doped fiber amplifier (EDFA) (Bi-EDFA) is clamped at G. The length of HNLF is L. The ratio of coupler is $k:(1-k)$. GVD is neglected during the deduction. For the CW wave, the x- and y-components at input end after the $k:(1-k)$ coupler are

$$\begin{cases} A_{x\mathrm{CW}}=A\sqrt{k}\cos\theta \\ A_{y\mathrm{CW}}=A\sqrt{k}\sin\theta \end{cases}. \tag{4}$$

After transmitting through the loop, the two components before the coupler become

$$\begin{cases} A'_{x\mathrm{CW}}=\sqrt{G}A_{x\mathrm{CW}}\exp(-j\varphi_{x\mathrm{CW}}) \\ A'_{y\mathrm{CW}}=\sqrt{G}A_{y\mathrm{CW}}\exp(-j\varphi_{x\mathrm{CW}}) \\ \qquad\times\exp[-j(\Delta\varphi_{\mathrm{LCW}}+\Delta\varphi_{\mathrm{NLCW}})] \end{cases}, \tag{5}$$

where $\varphi_{x\mathrm{CW}}=(n_{x\mathrm{eff}}+n_{x\mathrm{CW}}^{\mathrm{NL}})\beta L$ is the accumulated phase shift for x-component during the propagation through the loop, the nonlinear refractive index $n_{x\mathrm{CW}}^{\mathrm{NL}}=n_2k|A|^2(\cos^2\theta+\frac{2}{3}\sin^2\theta)$ is determined by the power intensity and the input angle. The linear and nonlinear phase difference between x- and y-components are $\Delta\varphi_{\mathrm{LCW}}=(n_{y\mathrm{eff}}-n_{x\mathrm{eff}})\beta L$ and $\Delta\varphi_{\mathrm{NLCW}}=\frac{\beta L}{3}n_2(\sin^2\theta-\cos^2\theta)k|A|^2=-\frac{\gamma kPL}{3}\cos 2\theta$, respectively. Similarly, we can get all the equations for CCW wave.

Considering the phase-jumping at the $k:(1-k)$ coupler, the x-component of output amplitude at transmission port is

$$A_{\mathrm{T}x}=\sqrt{k}A'_{x\mathrm{CW}}+\sqrt{1-k}A'_{x\mathrm{CCW}}\exp(-j\pi). \tag{6}$$

Similar expressions can be got for y-component ($A_{\mathrm{T}y}$) and for reflection port ($A_{\mathrm{R}x}$, $A_{\mathrm{R}y}$). Finally, the effective transmittivity T (at the transmission port) and reflectivity R (at the reflection port) are achieved,

$$\begin{aligned} T=\frac{P_{\mathrm{T}}}{GP}&=\frac{|A_{\mathrm{T}x}|^2+|A_{\mathrm{T}y}|^2}{G|A|^2} \\ &=|k-(1-k)\exp\{-j[n_2(G(1-k)-k) \\ &\quad\times|A|^2(\cos^2\theta+\frac{2}{3}\sin^2\theta)]\beta L\}|^2\cos^2\theta \\ &\quad+|k-(1-k)\exp\{-j[n_2(G(1-k)-k) \\ &\quad\times|A|^2(\cos^2\theta+\frac{2}{3}\sin^2\theta)]\beta L\} \\ &\quad\times\exp\{j\frac{\gamma[G(1-k)-k]PL}{3}\cos 2\theta\}|^2\sin^2\theta, \end{aligned} \tag{7}$$

$$R=\frac{P_{\mathrm{R}}}{GP}=\frac{|A_{\mathrm{R}x}|^2+|A_{\mathrm{R}y}|^2}{G|A|^2}, \tag{8}$$

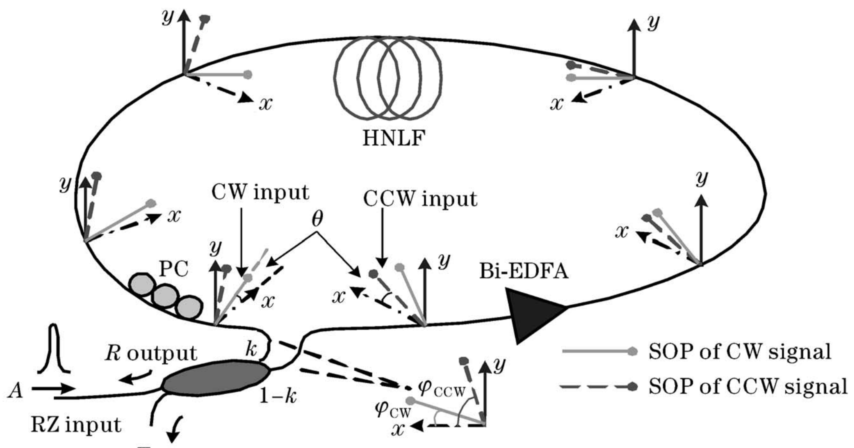

Fig. 1. The typical configuration of NALM.

here the term of 'effective' is used because the input power P is replaced by GP, so a normalized characteristic is expressed. It can be seen from Eqs. (7) and (8) that, θ and n_2 are the key parameters. The larger the n_2, the more obvious the NPE, implying that NPE is the concomitant of SPM. If θ is set to 0° or 90°, Eqs. (7) and (8) will degenerate to the traditional cosine expressions. As discussed above, the residual birefringence is unavoidable and the principle axes are rotated irregularly, so NPE will definitely appear and zero point of transmittion would not happen.

Based on the coupled nonlinear Schrödinger (CNLS) equations and split fast Fourier transform (FFT) algorithm[10], the evolution of SOP along the fiber length for CW and CCW waves was numerically simulated, assuming that HNLF was 1450 m, the gain of Bi-EDFA was 17 dB, the ratio of coupler was 1:1 and the linearly polarized input was at $\theta = 20°$, the input averaged power was 2 mW and the duty ratio of pulse was 1:10, respectively. The results are shown in Fig. 2. It is clear that the SOP of CW wave keeps almost linear over the entire fiber length (Fig. 2(a)), but that of CCW wave is changed from linear to elliptical, and then to linear and elliptical again (Fig. 2(b)). Thus, the interference at the coupler will never be completed.

Figure 3 shows the comparison of the transmission characteristics among the analytical curve (discrete squares) by Eq. (7), the simulated curve (solid line) based on CNLS and FFT and the experimental measurement (dots). $\theta = 20°$ was set for analytical curve and the simulation curve was the average for $\theta = 20°$, 35°, 55°, and 70°. The experiment was carried out when the length of HNLF was 1450 m, the input return-to-zero (RZ) pulse train was 10 GHz with duty ratio of $\sim 1:10$, provided by a passively mode-locking (PML) fiber laser and boosted by an EDFA. The curves show satisfactory consistency, the little discrepancy between the analytical and simulated curves may come from neglecting of GVD during the deduction. The experimental curve has the same tendency but there is obvious discrepancy around the cupped range. The further analysis indicated that, in addition to the residual linear birefringence and the irregular rotation of fiber principle axes, the uncompleted polarization state and the chirp of input light would also affect the transmission. The experimental curve in Ref. [7] was more consistent with the theoretical curves in Fig. 3. The output pulse-forms and optical spectra from NALM were measured for different power levels, showing no obvious stimulated Brillouin scattering (SBS) or pulse degradation.

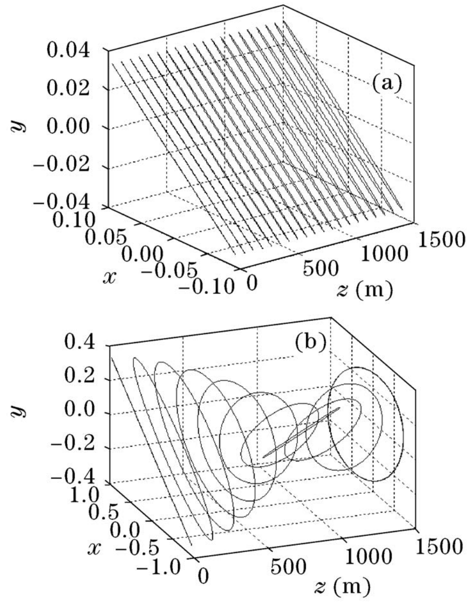

Fig. 2. Evolutions of the SOP along the HNLF in NALM. (a) CW wave signal polarization evolution; (b) CCW signal polarization evolution.

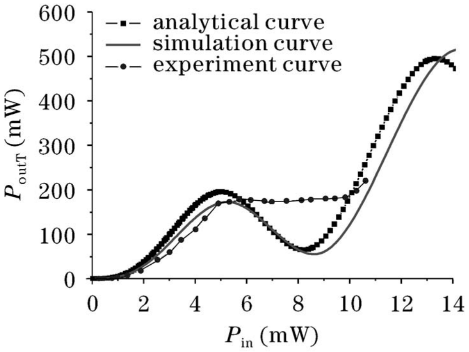

Fig. 3. Comparison between the analytical, simulated, and experimental transmission characteristics of a NALM.

In conclusion, the nonlinear phase difference between the two orthogonal polarization components is estimated to be a few tenths of accumulated nonlinear phase for single component and the nonlinear beat length is about a few kilometers for HNLF. Asymmetrical NPE process for CW and CCW waves results in different final SOP, so the interference can never be completed and the transmission curve is distored. The analytical expression considering the NPE effect for the transmission of NALM is derived.

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