Bright-dark incoherently coupled soliton pairs composed of spatially incoherent solitons

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It is shown that bright-dark incoherently coupled soliton pairs can exist in photorefractive (PR) crystals under steady-state conditions, each soliton constituent of which is spatially incoherent. The characteristics of bright-dark incoherently coupled soliton pairs are studied by the coherent density approach and the intensity expressions of soliton pairs are obtained. The propagation properties of coherent components of each constituent in a soliton pair are also discussed in detail.

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In 1996, Christodoulides *et al.* showed theoretically firstly that the incoherently coupled screening soliton pairs are possible in biased photorefractive (PR) crystals provided that the carrier beams share the same polarization, wavelength, and are mutually incoherent[1]. Screening solitons can exist in biased non-photovoltaic PR crystals under steady-state conditions[2−4]. We call the two beams of a coupled soliton pair as soliton constituents. Soon afterwards, Chen *et al.* observed these incoherently coupled soliton pairs in a biased strontium barium niobate (SBN) crystal[5−7]. In their experiment, a beam originated from a laser is split into two beams by a polarizing beam splitter and they are mutually incoherent at the input face of the crystal because their optical path difference greatly exceeds the coherent length of the laser. At present, the study of soliton pairs is attracting more and more attention and being developed into a new branch of the optical soliton theory[1,5−10].

However, the two soliton constituents of a soliton pair mentioned above share the same wavelength, are mutually coherent, and each beam of them is still coherent light. In 1996, Mitchell *et al.* demonstrated experimentally the self-trapping of spatially incoherent beam in SBN crystal, which originated from an Ar^+ laser and passed through a rotating diffuser[11]. That beam is fully spatially incoherent, because the phase varied randomly in space/time across any wave plane and the spatial frequency $\vec{k}$ of the light wave is different at any point. The spatially incoherent solitons have characteristics of low launch power, steady propagation, maneuverable and so on, which are becoming the research hotspots currently. Now, the related experimental observations of soliton pairs composed of these spatially incoherent beams have not been reported. We have studied theoretically this new type of incoherently coupled soliton pairs in bright-bright and dark-dark configurations[12]. And in this paper, we show the characteristics of this type incoherent soliton pairs in bright-dark configuration by using the coherent density approach.

The coherent density approach[13−16] is useful for the study of spatially incoherent solitons. In this approach, the incoherent input beam is decomposed into infinite "coherent components" with different $\vec{k}$, all of which happen to be mutually incoherent. So the self-trapping of the beam can be described by an infinite set nonlinear Schrödinger-like equations, provided that the equations are initially appropriately weighted with respect to the incoherent angular power spectrum of light source.

To start, let us consider a planar photorefractive crystal with its optical c axis oriented in the x direction. Two (1+1) dimensional planar spatially incoherent beams 1 and 2 of the same frequency propagate collinearly along the z axis in the crystal, and they are permitted to diffract only along the x direction and their incoherences are the same. Furthermore, the external bias electric field is applied in the x direction. As mentioned above, every beam is spatially incoherent, so each can be looked at as the incoherent superposition of infinite "coherent components" with different $\vec{k}$. By the coherent density approach, the two optical beams satisfy the following envelope evolution equations

$$i\left(\frac{\partial f_1}{\partial z}+\theta\frac{\partial f_1}{\partial x}\right)+\frac{1}{2k}\frac{\partial^2 f_1}{\partial x^2}-\frac{k_0(n_e^3 r_{33}E_{SC})}{2}f_1=0, \quad (1)$$

$$i\left(\frac{\partial f_2}{\partial z}+\theta\frac{\partial f_2}{\partial x}\right)+\frac{1}{2k}\frac{\partial^2 f_2}{\partial x^2}-\frac{k_0(n_e^3 r_{33}E_{SC})}{2}f_2=0, \quad (2)$$

where f_1 and f_2 represent the envelops of coherent component θ of the beams 1 and 2, respectively, i.e., the so-called coherent density, and θ is an angle with respect to the z axis. And, f_1, f_2 can be given by

$$f_1(x,\theta,z)=r_1^{1/2}I_d^{1/2}G_N^{1/2}(\theta)\psi_1(x,\theta,z), \quad (3)$$

$$f_2(x,\theta,z)=r_2^{1/2}I_d^{1/2}G_N^{1/2}(\theta)\psi_2(x,\theta,z), \quad (4)$$

where r_1, r_2 are the intensity ratios of coherent component θ of the beams 1 and 2, respectively, $r_1=I_{1\max}/I_d$, $r_2=I_{2\max}/I_d$, and I_d is the dark irradiance of the crystal. $G_N(\theta)$ is the normalized angular power spectrum of incoherent source[17] and used to describe the incoherence of beams, and let us assume that is Gaussian, i.e., $G_N(\theta)=\frac{1}{\sqrt{\pi}\theta_0}\exp(-\theta^2/\theta_0^2)$, θ_0 is the spectral width of the angular power spectrum. $\psi_1(x,\theta,z)$ and $\psi_2(x,\theta,z)$ are the envelops of coherent component θ of the two beams, which are normalized by r_1 and r_2, respectively. n_e is the extraordinary index of refraction and r_{33} is the index of electrooptics, the steady-state charge electric field is approximately given by

$$E_{\mathrm{SC}}(x,z)=E_0\frac{I_\mathrm{d}+I_\infty}{I_\mathrm{d}+I(x,z)}, \tag{5}$$

where E_0 is the value of the space charge electric field caused by the bias voltage, $I(x,z)$ is the total power density of the two beams,

$$I(x,z)=\int_{-\infty}^{\infty}|f_1(x,\theta,z)|^2\,\mathrm{d}\theta+\int_{-\infty}^{\infty}|f_2(x,\theta,z)|^2\,\mathrm{d}\theta$$
$$=I_\mathrm{d}(r_1|\psi_1(x,z)|^2+r_2|\psi_2(x,z)|^2), \tag{6}$$

and $I_\infty=I(x\to\pm\infty,z)$.

Substituting Eqs. (3) and (4) into Eqs. (1) and (2) and employing these above transformations, the following dimensionless coordinates and variables are $\xi=z/kx_0^2$, $s=x/x_0$, $\alpha=kx_0\theta$, $\beta=(k_0x_0)^2n_\mathrm{e}^4r_{33}E_{\mathrm{SC}}/2$, $\rho=I_\infty/I_\mathrm{d}$, the normalized planar envelops ψ_1 and ψ_2 are found to satisfy

$$i\left(\frac{\partial\psi_1}{\partial\xi}+\alpha\frac{\partial\psi_1}{\partial s}\right)+\frac{1}{2}\frac{\partial^2\psi_1}{\partial s^2}$$
$$-\beta(1+\rho)\frac{\psi_1}{1+r_1|\psi_1|^2+r_2|\psi_2|^2}=0, \tag{7}$$

$$i\left(\frac{\partial\psi_2}{\partial\xi}+\alpha\frac{\partial\psi_2}{\partial s}\right)+\frac{1}{2}\frac{\partial^2\psi_2}{\partial s^2}$$
$$-\beta(1+\rho)\frac{\psi_2}{1+r_1|\psi_1|^2+r_2|\psi_2|^2}=0. \tag{8}$$

Now, let us consider the case of the bright-dark soliton pair, which is composed of one bright soliton constituent 1 and one dark soliton constituent 2. According to the intensity profiles of bright and dark solitons, we know $I_\infty=I_{2\max}$, so we can set $r_1=r$, $r_2=\rho$, where $r=I_{1\max}/I_\mathrm{d}$, $\rho=I_\infty/I_\mathrm{d}$. The steady bright-dark soliton pair solutions are expressed as

$$\psi_1(s,\theta,\xi)=u^{1/2}(s)\exp[i\phi_1(s,\theta,\xi)], \tag{9}$$

$$\psi_2(s,\theta,\xi)=v^{1/2}(s)\exp[i\phi_2(s,\theta,\xi)], \tag{10}$$

where $u+v=1$. $u(s)$ represents the coherent component envelop of the incoherent bright soliton constituent, which satisfies $0\le u(s)\le 1$, $u(0)=1$, $\dot{u}(0)=0$, $u(s\to\pm\infty)=0$, $u^{(n)}(s\to\pm\infty)=0$. $v(s)$ represents the case of the dark soliton constituent and satisfies $0\le v(s)\le 1$, $v(0)=0$, $v(s\to\pm\infty)=1$, $v^{(n)}(s\to\pm\infty)=0$. And ϕ_1, ϕ_2 are the phase evolutions of any coherent component in bright and dark soliton constituents, respectively.

Substituting Eqs. (9) and (10) into Eqs. (7) and (8), we get

$$\left(\frac{\mathrm{d}u}{\mathrm{d}s}\right)^2-(8\gamma_1-4\alpha^2)u^2$$
$$-\frac{8\beta(1+\rho)}{r-\rho}ul_n\left(1+\frac{r-\rho}{1+\rho}u\right)=0, \tag{11}$$

$$-\frac{1}{8v^2}\left(\frac{\partial v}{\partial s}\right)^2+\frac{1}{4v}\frac{\partial^2 v}{\partial s^2}+\frac{1}{2}\alpha^2$$
$$-\gamma_2-\beta\frac{1}{1+\delta(1-v)}=0. \tag{12}$$

Using appropriate boundary conditions, we get

$$\gamma_1=\frac{1}{2}\alpha^2-\frac{\beta}{\delta}l_n(1+\delta), \tag{13}$$

$$\gamma_2=\frac{1}{2}\alpha^2-\beta, \tag{14}$$

where $\delta=\frac{r-\rho}{1+\rho}$. Substituting Eqs. (13) and (14) into Eqs. (11) and (12), and integrating, then we obtain

$$s=\pm\int_1^u\frac{\mathrm{d}u}{u\sqrt{-\frac{8\beta}{\delta}l_n(1+\delta)+\frac{8\beta}{\delta u}l_n(1+\delta u)}}, \tag{15}$$

$$s=\pm\int_0^v\frac{\mathrm{d}v}{\sqrt{-8\beta(v^2-v)+\frac{8\beta}{\delta}vl_n[1+\delta(1-v)]}}. \tag{16}$$

From Eqs. (15) and (16), $u(s)$ and $v(s)$ can be easily obtained by simple numerical integration procedures, $rI_\mathrm{d}u(s)$ and $\rho I_\mathrm{d}v(s)$ are the intensity profiles of the bright and dark soliton pair constituents, respectively.

When $|\delta|\ll 1$, that is the peak intensities of the incoherent bright and dark soliton constituents are approximately equal, we can obtain the approximately analytical solutions of the intensity profile of a coupled soliton pair. By using the Taylor series, $\ln(1+\delta)\approx\delta-\delta^2/2$, Eq. (11) can be transformed into

$$\left(\frac{\mathrm{d}u}{\mathrm{d}s}\right)^2=-\frac{\beta}{\delta}(\delta-\delta^2/2)u^2+\frac{8\beta}{\delta}u(\delta u-\delta^2u^2/2), \tag{17}$$

then, we get

$$u=\mathrm{sech}^2(\sqrt{\beta\delta}s). \tag{18}$$

Equation (18) is the analytical intensity expression of incoherently bright soliton constituent for $|\delta|\ll 1$. Similarly, the intensity of incoherently dark soliton constituent is given by

$$v=\tanh^2(\sqrt{\beta\delta}s). \tag{19}$$

From Eqs. (18) and (19), we can see that $\beta\delta>0$ is necessary for the establishment of incoherently bright-dark coupled soliton pairs. When $\beta>0$, that means the direction of the external bias field accords to the optical c axis of crystals, $\delta>0$ is required, i.e., the intensity peak of bright soliton constituent must be slightly larger than that of dark soliton constituent. On the contrary, $\delta<0$ when $\beta<0$, soliton pair may also exist, but the intensity peak of bright soliton constituent should be slightly less than that of dark soliton constituent. Figure 1 depicts

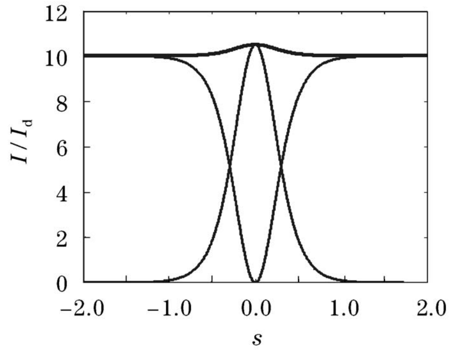

Fig. 1. Bright soliton constituent I_1/I_d, dark soliton constituent I_2/I_d (thin curves), and the total intensity (thick curve) for a bright-dark soliton pair.

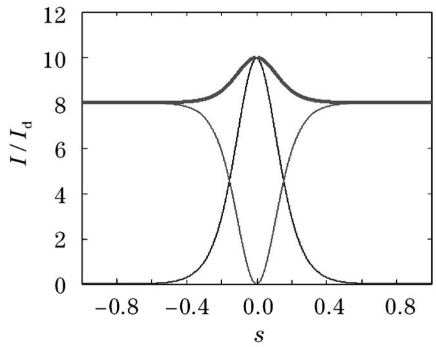

Fig. 2. Bright and dark soliton constituents (thin curves) and the total intensity (thick curve) for $\beta = 200$.

the intensity profiles of the bright-dark incoherent soliton pair when $\beta = 200$, $r = 10.1$, $\rho = 10$, and $\delta = 0.01$. Obviously, the intensity peak of bright soliton constituent is slightly higher than that of dark soliton constituent, so the total intensity of soliton pair at $s = 0$ is slightly higher than that at the edge. However, when $\beta < 0$, the intensity profiles of bright-dark incoherent soliton pair are similar to Fig. 1, but the total intensity of the soliton pair at $s = 0$ is slightly lower than that at the edge.

The analytical solutions of bright-dark soliton pair, Eqs. (18) and (19), only adapt to the case that the peak intensities of incoherent bright and dark soliton constituents are approximately equal. When the difference of their peak intensities is large, we need to use Eqs. (15) and (16) to express the intensity of bright-dark soliton pairs. Figure 2 depicts the intensity profiles of bright-dark incoherent soliton pair for a larger intensity difference when $\beta = 200$, $r = 10$, and $\rho = 8$. Obviously, the intensity peak of bright soliton constituent is much higher than that of dark soliton constituent, so the total intensity of soliton pair at $s = 0$ is the maximum. When $\beta = -200$, $r = 10$, $\rho = 12$, we get the contrary result, and the total intensity of soliton pair at $s = 0$ is the minimum, as depicted in Fig. 3.

We mentioned above that spatially incoherent beam can be decomposed into infinite "coherent components", any of which is mutually incoherent. Since each soliton constituent can keep steady propagation, how does every coherent component propagate? From Eqs. (3) and (4), we can get the intensity profile of any coherent component θ in bright or dark soliton constituent.

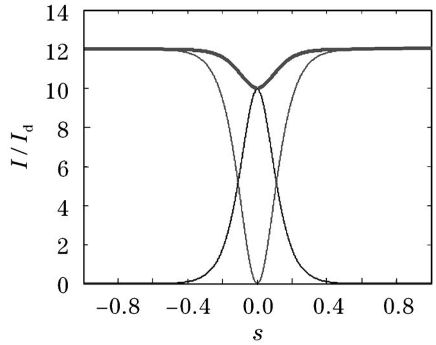

Fig. 3. Bright and dark soliton constituents (thin curves) and the total intensity (thick curve) for $\beta = -200$.

The intensity profiles of any coherent component θ in incoherent bright and dark soliton constituents are given by

$$|f_1(x,\theta,z)|^2 = \frac{rI_d}{\sqrt{\pi}\theta_0}\exp(-\theta^2/\theta_0^2)u(s), \qquad (20)$$

$$|f_2(x,\theta,z)|^2 = \frac{\rho I_d}{\sqrt{\pi}\theta_0}\exp(-\theta^2/\theta_0^2)v(s). \qquad (21)$$

Obviously, the intensity profiles of coherent components relate not only with the coherent degree θ_0 of an incoherent beam, but also with the position of coherent component θ. That is, the intensity peak will go smaller when coherence of beams turns weaker, i.e., θ_0 grows larger, or when coherent component is far away the z axis, i.e., θ turns larger. The three curves from the top to the bottom in the Figs. 4 and 5 represent respectively the intensity profiles of the coherent component $\theta = 0$, 2, 3 mrad in bright and dark soliton constituents, when $\theta_0 = 3$ mrad and other parameters are the same as those in Fig. 1. Evidently, bright and dark coherent components with the same θ can establish a steady state coupled coherent component soliton pair.

In conclusion, we have found that a new kind of incoherently coupled bright-dark soliton pair is possible in PR crystals under steady-state conditions, two soliton constituents composing soliton pair are mutually incoherent and each of them is spatially incoherent. We study the propagation properties of soliton pairs and get the intensity profiles. And the approximately analytical solutions of soliton pairs are also obtained when the peak intensities of incoherent bright and dark soliton constituents

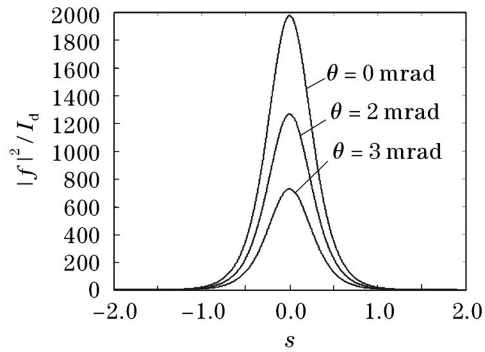

Fig. 4. Intensity profiles of coherent components $\theta = 0$, 2, 3 mrad in the bright soliton constituent.

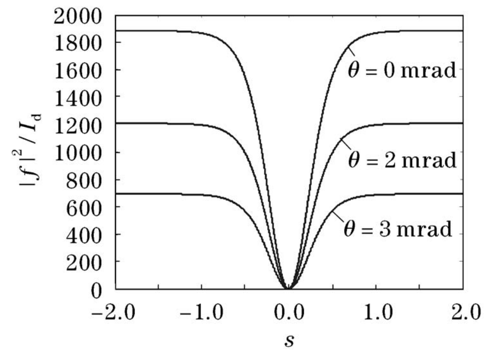

Fig. 5. Intensity profiles of coherent components $\theta = 0, 2, 3$ mrad in the dark soliton constituent.

are approximately equal. According to the definition of the coherent density, we know incoherent soliton constituent is made up with coherent components, and we also discuss the characteristics of coherent components and find the coherent components with the same θ of soliton pair can also establish a steady state coupled coherent component soliton pair.

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