Analytical formulas for calculating the blocking probability of a dynamic star network

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For a dynamic routing and wavelength assignment (RWA) a star topology is shown to be more efficient in comparison with a ring topology. Analytical formulas for a dynamic RWA in a star network are presented and verified with virtual simulation.

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Routing and wavelength assignment (RWA) has become one of the key problems for today's complicated wavelength division multiplexing (WDM) networks. In general, RWA can be classified either into the dynamic case and the static case according to the wavelength allocation[1], or into the star, ring and mesh networks according to the topology. Although extensive research works[2,3] have been done on the RWA and many algorithms have been developed for the case of dynamic rings[4], little has been done for the dynamic RWA in a star network. The dynamic RWA of star type keeps the advantages of both the simplicity of a star topology and the efficiency of a dynamic wavelength assignment strategy. Protection for star type is also provided against path failures, link failures and node failures[5,6]. Therefore, a dynamic star topology is very attractive for improving the performance of a WDM network.

We focus on a dynamic wavelength-routing network of star topology without any wavelength conversion. A central node, which has an $N \times N$ wavelength router with a wavelength cross-connector (WXC), is connected through fiber pairs to N peripheral nodes with the function of optical add-drop multiplexing (OADM). Each peripheral node communicates with all the other $(N-1)$ peripheral nodes only through the central node. In a dynamic case, if any traffic occurs between two of the peripheral nodes, a wavelength is selected dynamically from all the available wavelengths and assigned to the corresponding lightpath. In this letter we use a random (RD) algorithm for wavelength allocation, i.e., the working wavelength for a required lightpath is selected randomly from the wavelengths available. Unlike a ring topology of same situation, the information about the available wavelengths for each peripheral node in a star network can be updated in time since it is transferred only through the central node and two-hop fibre links. Figure 1 shows the comparison of the blocking probability for a uniform traffic between the present RD star algorithm and various ring algorithms[4] such as the RD, FP (fixed priority), MB (minimal blocking), MC (maximal sum of channel capacities), and LB (lower bound for a network with full wavelength conversion capability) ring algorithms. Usually the RD algorithm is the simplest but not efficient (as seen in the ring case of Fig. 1). Nevertheless, the RD algorithm for a star network already performs much better than any ring algorithm in terms of the blocking probability. The blocking probability can be reduced further when some advanced algorithm (FP, MB, MC, LB) is used in the dynamic star network (as seen in the star case of Fig. 1). Furthermore, a star network has an equal number (2) of hops for all the lightpaths, and thus does not have the problem (existing in a ring network) of hop-number-dependent blocking probability for different lightpaths.

Assume we have W wavelengths in total for the dynamic wavelength assignment in an N-node star network. If the traffic arrival rate for each node pair is λ_0, then the arrival rate λ for one fiber link to a certain node is $(N-1) \cdot \lambda_0$. The traffic persistence time is a random variable whose probability satisfies an exponential distribution with an expectation value of T. Obviously, the leaving rate for a traffic (on an available wavelength) is $\mu = 1/T$. In a star network each lightpath has two hops (i.e., fiber links). One is from the source node to the central node, and the other is from the central node to the destination node. Let P_i^{S} and P_i^{D} denote the

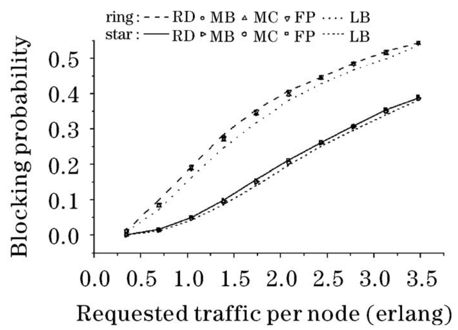

Fig. 1. Comparison of the average blocking probabilities between the various star and ring algorithms for the same total numbers of nodes ($N = 15$) and wavelengths ($W = 4$).

probabilities of having i available wavelengths in these two fiber links, respectively. For simplicity, we assume that the dependence of P_i^{S} and P_i^{D} on each other can be neglected (the influence of this dependence on the blocking probability is small, as can be seen from Fig. 3 below). Let P_{mn} denotes the probability of the traffic acceptance for a lightpath (with two fiber links) under the condition that one fiber link has m available wavelengths while the other has n available wavelengths. Then we have the following formula

$$P_{mn} = \gamma_{mn} P_m^{\mathrm{S}} P_n^{\mathrm{D}}, \quad (1)$$

where the probability of having at least one common available wavelength for this lighpath is given by (through the theory of combinatorics)

$$\gamma_{mn} = \begin{cases} 1, & W < m+n \\ 1 - \frac{(W-m)!(W-n)!}{W!(W-n-m)!}, & W \geq m+n \end{cases} . \quad (2)$$

Thus, the total probability of acceptance for a traffic between two specific peripheral nodes is

$$P = \sum_{m,n=0}^{W} P_{mn} = \sum_{m,n=0}^{W} \gamma_{mn} P_m^{\mathrm{S}} P_n^{\mathrm{D}}. \quad (3)$$

Obviously, $P_i^{\mathrm{S}} = P_i^{\mathrm{D}}$ and thus we drop their superscripts hereafter. Now consider a fiber link with i available wavelengths. The conditional probability of traffic acceptance (in the state with i available wavelength if the traffic arrives) is denoted P_i^*. Then it follows from Eq. (3) that

$$P_i^* = \sum_{j=0}^{w} \gamma_{ij} P_j, \quad i = 0, 1, \cdots, W. \quad (4)$$

The corresponding traffic leaving rate (in the state with i available wavelength) is denoted by μ_i^*. We note that in an infinitesimal time interval Δt the probability of transferring from a state with i available wavelengths to a state with $(i-1)$ available wavelengths is $\lambda \cdot \Delta t \cdot P_i^*$ and the probability of transferring from a state with i available wavelengths to a state with $(i+1)$ available wavelengths is $\mu_i^* \cdot \Delta t$. Figure 2 shows a Markov chain for $i = 0, 1, \cdots, W$. With the help of this figure, we can obtain the following set of equations

$$\sum_{i=0}^{W} P_i = 1, \quad (5)$$

$$\lambda \cdot P_1^* \cdot P_1 = \mu_0^* \cdot P_0, \quad (6)$$

$$\lambda \cdot P_W^* \cdot P_W = \mu_{W-1}^* \cdot P_{W-1}, \quad (7)$$

$$(\lambda \cdot P_i^* + \mu_i^*) \cdot P_i = \lambda \cdot P_{i+1}^* \cdot P_{i+1} + \mu_{i-1}^* \cdot P_{i-1},$$
$$i = 1, 2, \cdots, W-1. \quad (8)$$

Obviously, we have

$$\mu_i^* = (W - i) \cdot \mu. \quad (9)$$

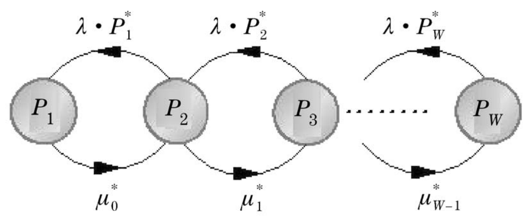

Fig. 2. The Markov chain corresponding to the RD algorithm for the dynamic star network.

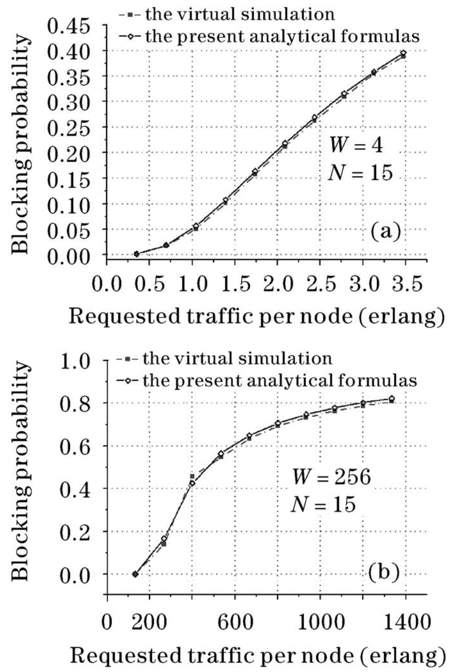

Fig. 3. Comparison of the blocking probabilities calculated with the present analytical formulas and the virtual simulation for a dynamic star network with (a) $W = 4$ and (b) $W = 256$. The total number of nodes is fixed on $N = 15$.

Equations (4)—(8) form $(2W+3)$ nonlinear equations (actually one of the $(W+2)$ equations in Eqs. (5)—(8) is reducible from the other equations) for $(2W+2)$ unknowns (P_i, P_i^*, $i = 0, 1, \cdots, W$). The set of Eqs. (4)—(8) can be easily solved with e.g. a standard iterative method (The complexity of the iterative method is relative to the number of unknowns, $O(2W+2)$).

To verify our analytical formulas, we compare the numerical solution for Eqs. (4)—(8) (indicated by triangles connected with dash lines in Fig. 3) with simulation under a virtual environment. In the virtual simulation, the time is discretized with minimal time unit. Traffics are generated with a specified arrival interval rate (of exponential distribution) and traffic length (of exponential distribution) by time unit. The RD algorithm is then used to assign to each trafficing a wavelength (if there are wavelengths available; otherwise the traffic is blocked). The blocking probability is calculated and its statistic mean value (indicated by squares connected with solid lines in Fig. 3) is obtained after a few million of time steps. Figures 3(a) and (b) (with $W = 4$ and $N = 15$ for the case of small number of wavelengths and underload, and with $W = 256$ and $N = 15$ for the case of large number of wavelengths and overload, respectively) show that the blocking probability predicted with the present analytical formulas agrees quite well with that obtained through virtual simulation for different total numbers of wavelengths and traffic loads. The total number $(2W+3)$ of the equations in Eqs. (4)—(8) is independent of the total number N of the peripheral nodes, whereas the computational time for the virtual simulation (which processes all the traffics between each node pair) increases with N (with a time complexity of $O(N^2)$).

We have analyzed the dynamic RWA for a WDM network of star topology. A dynamic star network with a RD wavelength assignment algorithm has been shown to have a much lower blocking probability than a dynamic ring network (of same situation) with any existing algorithm. Some analytical formulas for a dynamic RWA in a star network have been introduced to calculate the blocking probability. These formulas have been verified numerically by comparing with the results of virtual simulation. The present analytical formulas also provide a deeper insight into the mechanism of the dynamic wavelength assignment process as compared with the direct virtual simulation for a dynamic star network.

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