# Analysis of dispersion compensation for position-dependence in externally modulated CATV lightwave systems by using chirped fiber grating

**Qing Ye (叶　青)[1,2], Feng Liu (刘　峰)[1,2],**
**Haiwen Cai (蔡海文)[1], Ronghui Qu (瞿荣辉)[1], and Zujie Fang (方祖捷)[1]**

[1] *Laboratory of Information Optics, Shanghai Institute of Optics and Fine Mechanics,*
*Chinese Academy of Sciences, Shanghai 201800*
[2] *Graduate School of the Chinese Academy of Sciences, Beijing 100039*

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The dispersion compensation characteristics of the chirped fiber grating (CFG) for different dispersion compensation positions are analyzed in externally modulated cable television (CATV) lightwave system and the analytic expression of the composite second order (CSO) distortion is derived. The analyses give a reasonable explanation for the position-dependent effect of CFG dispersion compensator, which was found in practical systems. Moreover, the theoretical result is also verified by an experiment. It is believed that the theory will be helpful in designing optical CATV fiber links with nodes at proper positions both for intensity amplification and dispersion compensation.

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Transmission of amplitude-modulated sub-carrier multiplexed (AM-SCM) analog optical signals in single mode fiber is an important and useful technology. Amplitude-modulation vestigial-sideband (AM-VSB) cable television (CATV) systems have been widely used all over the world. Apart from 1310-nm optical CATV technology, 1550-nm systems have been used for longer transmission distance and more multiplexed channel numbers[1,2]. However, it is found that fiber dispersion, combined with the self-phase modulation (SPM) and the chirping of laser transmitter, is one of the most severe limiting factors to high video quality and long fiber transmission link. To mitigate the much higher fiber dispersion of 1550 nm than that of 1310 nm range, a widely accepted technique is using of an externally modulated transmitter because of its low chirping. Moreover, dispersion compensation fiber (DCF) and reverse dispersion fiber (RDF)[3] are also used to compensate the dispersion. However, because of the high insertion loss of these fibers, there are still difficulties for good composite second order (CSO) distortion and composite triple beat (CTB) distortion.

Chirped fiber gratings (CFGs) have been used as a dispersion compensation device both in digital fiber communication[4] and analog lightwave systems[5−7], and the results showed that CFG could greatly improve CSO performance of CATV systems. One of the issues for CFG dispersion compensator is the group delay ripple. In earlier work, we studied its effect on CSO in optical CATV systems, showing that the large ripple amplitude and small ripple period will deteriorate the system CSO performance seriously and must be improved[8].

One of the most attractive advantages for 1550-nm lightwave systems is that the erbium doped fiber amplifier (EDFA) is available both for digital and analog systems. Nodes with EDFA can be used to compose a large scale and long distance CATV network, and dispersion compensators can also be installed at these nodes. Recently it was observed in domestic long-distance optical CATV systems that the effect of CFG dispersion compensator depended on the position where the CFG installed. The phenomenon was also reported in Ref. [7], but without explanation. The characteristics were quite different from that of dispersion compensation by CFG in digital communication systems, where the compensation effect does not depend on the position in the fiber link in principle. A theoretical analysis is therefore needed to understand the phenomena and to improve the effect of CFG in applications. In this letter, the dispersion compensation theory for CFG is analyzed; the analytical expressions for CSO and an explanation for the position-dependence are presented. An experimental result is also given to verify our analysis.

To concentrate our analysis on the concerned devices, a system with one node in the fiber link is considered as shown in Fig. 1. The transmitter is externally modulated usually by lithium niobite modulator. The CSO and other related performances are measured at the terminal receiver. The middle node will contain an EDFA and a dispersion compensator. The CSO characteristics in the fiber transmission link can be discussed after the theory given by Refs. [9] and [10], which gave expressions for CSO and CTB of AM-SCM optical signals in fibers without dispersion compensation. The electric field envelope of the analog signal is

$$A = \sqrt{x(z,t)}\exp[iy(z,t)], \quad (1)$$

where $x(z,t) = P_0[1 + m\cos(\omega t)]$ is the modulated

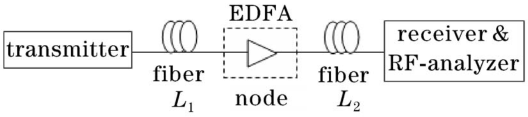

Fig. 1. Schematic diagram of the CATV transmission system.

intensity, $\omega$ is the frequency of the sub-carrier and $m$ is modulation depth. The phase factor $y(z,t)$ is caused by intensity modulation with different forms related to the modulation mechanism. In the case of external modulation, the original phase can be neglected. After propagation for a distance of $L_1$, the intensity and phase factor can then be expressed as

$$x(L_1,t) = P_0 \mathrm{e}^{-\alpha L_1}(1 + m\cos\omega t + m^2\beta_2 S_1^2\omega^2\gamma P_0\cos 2\omega t), \tag{2a}$$

$$y(L_1,t) = \gamma P_0 \mathrm{e}^{-\alpha L_1} L_{1\mathrm{eff}}(1 - m\cos\omega t), \tag{2b}$$

where $\alpha$ is the fiber loss, $\beta_2$ is the second-order dispersion coefficient of the propagation constant, $\gamma$ is the fiber nonlinear coefficient, and $L_{1\mathrm{eff}} = (1 - \mathrm{e}^{-\alpha L_1})/\alpha$ is the effective length of the fiber. $S_1^2 = (L_1 - L_{1\mathrm{eff}})/\alpha$. Equation (2) may give the second-order harmonic distortion (2HD), defined as the ratio of component of $2\omega$ to component of $\omega$, due to the combined effect of SPM and fiber dispersion,

$$2\mathrm{HD} =< x(2\omega) > / < x(\omega) >= m\beta_2\omega^2 S_1^2\gamma P_0. \tag{3}$$

A linearly chirped fiber grating is now supposed to be connected at the point $z = L_1$. The CFG can be considered as a short section of a special fiber with much higher dispersion, which is denoted by a dispersion parameter $D_\mathrm{g}$ in ps/nm. The second derivative $\beta_{2\mathrm{g}}$ of the propagation constant over optical frequency for the CFG can then be written as

$$\beta_{2\mathrm{g}} = \lambda^2 D_\mathrm{g}/(2\pi c l), \tag{4}$$

where $l$ is an effective length of CFG section corresponding to the signal linewidth $\Delta\lambda$, and can be expressed as $l = \Delta\lambda/(\lambda_0\rho)$ with the grating chirping rate $\rho$ in 1/m. The intensity and phase variation of analog signals in CFG should also satisfy the same equation as in Ref. [9]

$$\frac{\partial X}{\partial z} = -\beta_{2\mathrm{g}}\left(\frac{\partial X}{\partial \tau}\frac{\partial Y}{\partial \tau} + X\frac{\partial^2 Y}{\partial \tau^2}\right), \tag{5a}$$

$$\frac{\partial Y}{\partial z} = \beta_{2\mathrm{g}}\left[\frac{1}{2}\left(\frac{\partial Y}{\partial \tau}\right)^2 + \frac{1}{4X}\frac{\partial^2 X}{\partial \tau^2} - \frac{1}{8X^2}\left(\frac{\partial X}{\partial \tau}\right)^2\right], \tag{5b}$$

where $X$ and $Y$ stand for the intensity and phase factor, and $\tau = t - \beta_1 z$ is the time in the coordinate travelling with the optical wave; the nonlinearity and loss of CFG are neglected because of its very short length. The signal described by Eq. (2) is now taken as the input for the CFG. To solve Eq. (5), a perturbation expansion is applied to the fiber grating effective length $l$ to first order:

$$X(z,\tau) = X_0(z,\tau) + lX_1(z,\tau) + o(l^2), \tag{6a}$$

$$Y(z,\tau) = Y_0(z,\tau) + lY_1(z,\tau) + o(l^2). \tag{6b}$$

The complex amplitude of the signal after being reflected from the CFG is now denoted as $A = \sqrt{X(l,\tau)}\exp[iY(l,\tau)]$ with the expressions

$$X = P_0\mathrm{e}^{-L_1\alpha}\{1 + m[1 - \gamma P_0\omega^2(\beta_2 S_1^2 - \beta_{2\mathrm{g}}L_{1\mathrm{eff}}l)]\cos\omega\tau + m^2\gamma P_0\omega^2[\beta_{2\mathrm{g}}L_{1\mathrm{eff}}l + \beta_2 S_1^2(1 - \omega^2\beta_{2\mathrm{g}}\gamma P_0 L_{1\mathrm{eff}}l)]\cos 2\omega\tau\}, \tag{7a}$$

$$Y = \gamma P_0 L_{1\mathrm{eff}}\mathrm{e}^{-\alpha L_1}[1 - m\cos\omega\tau + (1/2)m^2\gamma P_0\omega^2\beta_{2\mathrm{g}}L_{1\mathrm{eff}}l\sin\omega\tau]. \tag{7b}$$

The output signal from CFG will suffer the distortion again in the second fiber length $L_2$, which can be analyzed by the same method as the previous used. The final expression of 2HD distortion is given directly here for simplicity

$$2\mathrm{HD} = \frac{X(2\omega)}{X(\omega)} = \frac{\lambda^2}{2\pi c}m\omega^2\gamma P_0[D_\mathrm{g}L_{1\mathrm{eff}} + D(S^2 + S_2^2)], \tag{8}$$

here $D = \frac{2\pi c}{\lambda^2}\beta_2$ is the fiber dispersion coefficient, and parameters $S^2 = \frac{\alpha L + \exp(-\alpha L) - 1}{\alpha^2}$ and $S_2^2 = \frac{\alpha L_2 + \exp(-\alpha L_2) - 1}{\alpha^2}$ with $L = L_1 + L_2$. It is indicated that a CFG with dispersion parameter in opposite sign of the fiber dispersion, $D_\mathrm{g} = -D(S^2 + S_2^2)/L_{1\mathrm{eff}}$, can improve the system second-order distortion by compensating the fiber transmission link dispersion.

To demonstrate the preceding expression, simulations are presented with $m = 0.04$, $\alpha = 0.24$ dB/km, $P_0 = 40$ mW, $\gamma = 1\ \mathrm{W}^{-1}\mathrm{km}^{-1}$, $\beta_2 = -22\ \mathrm{ps}^2/\mathrm{km}$, $\omega = 550$ MHz, $\lambda = 1550$ nm, and $L = 100$ km. Figure 2(a) shows the

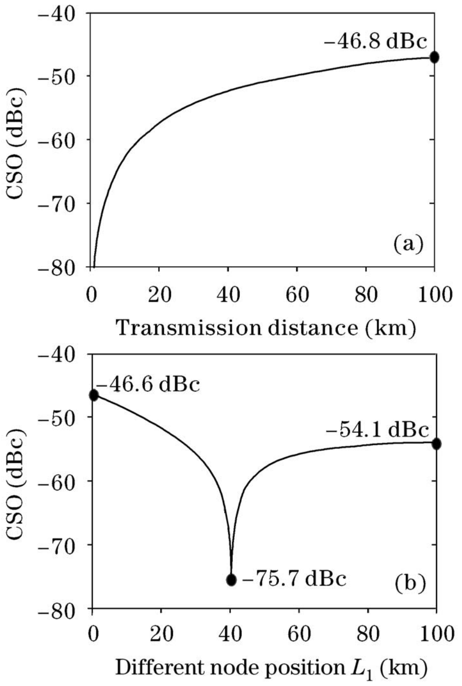

Fig. 2. CSO versus the transmission distance without dispersion compensation (a) and CSO measured at $L = 100$ km versus dispersion node position with $D_\mathrm{g} = 1650$ ps/nm (b).

CSO variations in the fiber transmission distance without dispersion compensation. The system CSO is −46.8 dBc when the fiber transmission distance is 100 km. Figure 2(b) is the CSO measured at the terminal of the link (i.e. in the position of $L_1 + L_2 = 100$ km) with CFG installed at the different node position $L_1$. The dispersion compensation amount of the CFG keeps the same as 1650 ps/nm in the simulation. It is shown that the compensation effect reaches its best at $L_1 \approx 40$ km with CSO $< -70$ dBc at the terminal. Moreover, it is worthy to notice that CFG cannot play the dispersion compensation effect if it is put at the beginning of the link by comparing Figs. 2(a) and (b), because CFG cannot change the linewidth of the source, nor can it change the dispersion of the fiber afterwards. When the CFG is placed after the optimal position, the compensation effect will also be weakened, for example, it is placed at the end of the fiber link. Although the CSO has a certain improvement at the receiver, it will be still beyond the CATV performance requirement (i.e. −60 dBc), as shown in Fig. 2(b). These characteristics differ from that of digital fiber link, in which the pulse distortion is related to the net dispersion and can be compensated at different positions.

In CATV transmission system, a large number of nodes with optical amplifiers are necessary to get longer transmission distance and larger scale of the network. It is surely beneficial to install dispersion compensators at the nodes simultaneously. The proper node position should be designed in considerations both of intensity amplification and dispersion compensation.

Figure 3 shows our experimental setup of an optical CATV transmission system with 59-channel (PAL-D) fully loaded externally modulated signals by using a 10-cm CFG ($D_g = 1500$ ps/nm) as a fiber dispersion compensator. Radio-frequency (RF) sub-carriers are generated from a RF carrier signal generator and are fed into an externally modulated transmitter with a modulation index $m$ of ∼4%. The launched optical power into the fiber is kept at $<$ 17 dBm by using a variable optical attenuator (VOA) to avoid degradation caused by the stimulated Brillouin scattering (SBS) effect. The system link with a transmission length of 125 km consists of two single mode fiber (SMF) spans (50+75 km, with an attenuation of 0.24 dB/km and a dispersion coefficient of 17 ps/(km·nm)). The power input to the receiver is adjusted to about 1 dBm using a VOA. All CATV RF parameters are measured by HP RF-spectrum analyzer at the terminal.

Figure 4 shows the measured CSO RF spectra of the channel 22 (CH-22) after the 125-km SMF link for different dispersion compensation positions. The central frequency is 543.256 MHz and the span is 8 MHz. From the figure, we can see that the CFG has no dispersion compensation effect basically when it is placed at the beginning of the fiber link (e.g. Fig. 4(b)). In the end of fiber transmission link, the CSO performance has about

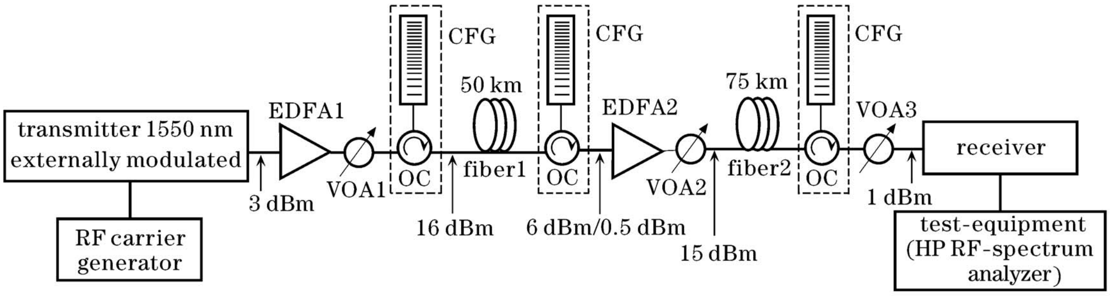

Fig. 3. Experimental setup (the dashed frames are positions of CFG).

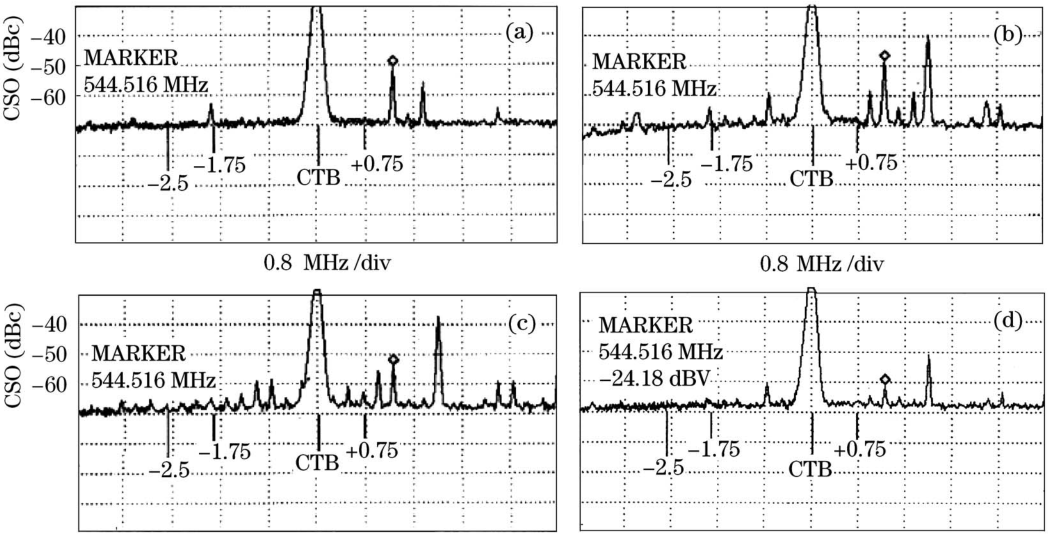

Fig. 4. The CSO RF spectra for different dispersion compensation positions in the 125-km SMF link, (a) without CFG; (b) at the beginning; (c) at the receiver; (d) at the middle node. Central frequency 543.256 MHz, offset 1.26 MHz.

2.6 dB improvement (e.g. Fig. 4(c)). However, when the CFG is placed in 50 km, the CSO performance is improved obviously and reaches 62.1 dB (e.g. Fig. 4(d)). The result is consistent with the above theory and simulation.

It is noticed that an additional distortion component at about 545.756 MHz emerged in the spectra. Primary experiment suggested that it may be attributed to the imperfect performances of the optical circulator. Further work is needed to identify and remove the impairment.

In summary, a dispersion compensation theory of CFG has been analyzed for position dependence in externally modulated CATV lightwave system and the analytic expressions of the second order distortion have been derived. The analytical result gives a satisfactory explanation for the position-dependent effect of the CFG dispersion compensator, which is also verified by the experiment. It is suggested that optimal node positions in the CATV network should be carefully designed under considerations both for intensity amplification and dispersion compensation in case of CFG being used.

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