Partially coherent vortex beams propagation in a turbulent atmosphere

Yixin Zhang (张逸新)[1,3], Minxia Tang (汤敏霞)[2], and Chunkan Tao (陶纯堪)[3]

[1]*School of Science;* [2]*Institute of Intelligence, Southern Yangtze University, Wuxi 214036*

[3]*School of Electron-Optics, Nanjing University of Science and Technology, Nanjing 210014*

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Based on the Rytov approximation and the cross-spectral density approximation for the mutual coherence function of the partially coherent field, the propagation properties of the partially coherent beams with optical vortices in turbulent atmosphere are discussed. The average intensity and the mutual coherence function of the partially coherent vortex beams propagation in weak turbulent atmosphere are obtained. It is shown that the vortex structure of the average cross-spectral density of partially coherent beams has the same helicoidally shape as that of the phase of the fully coherent Laguerre-Gauss beams in free space and the relative intensity of the beam is degraded by optical vortex.

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Beam propagation through atmospheric turbulence is a subject of considerable importance in connection with free-space optical communications, targeting systems, and remote sensing, and imaging systems[1,2]. Laser beams are of special interest and employed due to their high directionality. However, fully coherent laser beams are very sensitive to the properties of the propagation media, as a consequence, the turbulence-induced spatial broadening of the beam is a limiting factor in most applications[1,2]. Several researchers have suggested that partially coherent beams have less sensitivity to some random media, so recent works have characterized beam spread[3−6] and bit-error rate[7] for partially coherent beams.

Recently, laser beams possessing wave-front singularities known as optical vortices, have become the focus of many investigations because of their interesting properties[8−10] as well as their potential applications[11]. Fully coherent beams with wave-front singularities have been extensively studied[12]. However, partially coherent beams that may carry optical vortices[8,13] are studied much less.

Ponomarenko[8] discovered that the cross-spectral density of a partially coherent beam with a separable phase that carries optical vortices is invariant with respect to the spatial Fourier transform in the transverse plane, a property that makes these beams similar to Gaussian Schell-model and twisted Gaussian Schell model beams. On the other hand, we know that the fully coherent beams with optical vortices are the familiar Laguerre-Gauss modes[14], which have a separable phase with a simple helicoidal structure. So, in this paper, we analyze the propagation properties of the partially coherent beams with optical vortices, which are constructed by the normalized Laguerre-Gauss modes of arbitrary order with the same phase dependence, in turbulent atmosphere.

Let us first consider that the propagation of the fully coherent Laguerre-Gauss beams in the turbulent atmosphere ($z > 0$). The field distribution of Laguerre-Gauss beam in the source plane ($z = 0$) is given by

$$E_{mn}(\boldsymbol{\rho},\omega) = \left(\sqrt{2}\rho/w_0\right)^m L_n^m\left(2\rho^2/w_0^2\right) \times \exp\left(-im\phi\right)\exp\left(-\rho^2/w_0^2\right), \quad (1)$$

where $\boldsymbol{\rho} = (\rho,\phi)$ is a position vector of a point in the source plane, w_0 is a spot size at the waist of the beam, m is the azimuthal mode index, and n is the order of the Laguerre polynomial $L_n^m(x)$. It is clearly seen from Eq. (1) that the phase dependent on each Laguerre-Gauss mode is specified by a factor $\exp(-im\phi)$; in other words, it has a separable phase. The functional form of the Laguerre-Gauss source mode is well known at any point $z > 0$ in free space. One has

$$E_{mn}(\boldsymbol{\rho},z) = (w_0/w)\left(\sqrt{2}\rho/w\right)^m L_n^m\left(2\rho^2/w^2\right) \times \exp\left\{-im\phi - \rho^2/w^2 + i\left[kz - (m+1)\Phi + \frac{1}{2}k\rho^2/R\right]\right\}, \quad (2)$$

where $w = \left(w_0^2 + \frac{4z^2}{k^2w_0^2}\right)^{1/2}$, $R = z + \frac{k^2w_0^4}{4z}$, $\Phi = \arctan\left(\frac{2z}{kw^2}\right)$.

Under the Rytov approximation[1], the field of an optical wave propagating in weak fluctuation turbulent atmosphere at distance z from the source is represented by

$$\tilde{E}_{mn}(\boldsymbol{\rho},z) = \left(\frac{w_0}{w}\right)\left(\frac{\sqrt{2}\rho}{w}\right)^m L_n^m\left(\frac{2\rho^2}{w^2}\right) \times \exp\left\{-im\phi - \frac{\rho^2}{w^2} + \psi(\boldsymbol{\rho},z) + i\left[kz - (m+1)\Phi + \frac{k\rho^2}{2R}\right]\right\}, \quad (3)$$

where $\psi(\boldsymbol{\rho},z) = \chi(\boldsymbol{\rho},z) + iS(\boldsymbol{\rho},z)$ is the complex phase perturbation caused by the atmospheric turbulence, χ is the log-amplitude, and S is the phase.

The cross-spectral density $W(\boldsymbol{\rho},\boldsymbol{\rho}',\omega)$ of a partially coherent beam can be represented as a Mercer-type series of spatially completely coherent modes $E_{mn}(\boldsymbol{\rho},\omega)$ at a given frequency ω by means of the expression[15]

$$W(\boldsymbol{\rho},\boldsymbol{\rho}',\omega)=\sum_{mn}\lambda_{mn}E^*_{mn}(\boldsymbol{\rho},\omega)E_{mn}(\boldsymbol{\rho}',\omega), \quad (4)$$

where the subscript mn stands for a set of integers labeling the modes, and $\lambda_{mn}\,(\lambda_{mn}\geq 0)$ is the eigenvalue corresponding to the orthonormal mode E_{mn}[15].

The cross-spectral density of a beam at a pair of points $(\boldsymbol{\rho},\omega)$ and $(\boldsymbol{\rho}',\omega)$ in the half-space $z>0$ is related, in the paraxial domain, to the cross-spectral density at a pair of points in the source plane through the double Fresnel transform[15]

$$W(\boldsymbol{\rho},\boldsymbol{\rho}',z,z')=\frac{k^2}{4\pi^2 zz'}\exp\left[ik(z-z')\right]\times\int d^2\rho_1\int d^2\rho_2 W(\boldsymbol{\rho},\boldsymbol{\rho}',0)\times\exp\left[\frac{ik}{2z}(\boldsymbol{\rho}-\boldsymbol{\rho}_1)^2\right]\exp\left[-\frac{ik}{2z'}(\boldsymbol{\rho}'-\boldsymbol{\rho}_2)\right]. \quad (5)$$

Substituting the mode expansion (4) into Eq. (5), we obtain the expression of the cross-spectral density

$$W(\boldsymbol{\rho},\boldsymbol{\rho}',z,z')=\sum_{mn}\lambda_{mn}E^*_{mn}(\boldsymbol{\rho},z)E_{mn}(\boldsymbol{\rho}',z'), \quad (6)$$

where $E_{mn}(\boldsymbol{\rho},z)$ is the Fresnel transform of the source mode $E_{mn}(\boldsymbol{\rho},0)$. By the Eq. (3), we have the cross-spectral density of such beams in weakly fluctuation turbulence-atmosphere

$$\tilde{W}(\boldsymbol{\rho},\boldsymbol{\rho}',z,z')=\sum_{mn}\lambda_{mn}\tilde{E}^*_{mn}(\boldsymbol{\rho},z)\tilde{E}_{mn}(\boldsymbol{\rho}',z'). \quad (7)$$

Consider the following summation formula for Laguerre polynomials[16]

$$\sum_{n=0}\frac{n!}{(m+n)!}\xi^n L^n_m\left(\frac{2\rho^2}{w^2}\right)L^n_m\left(\frac{2\rho'^2}{w^2}\right)=\frac{\xi 4\rho^2\rho'^2}{w^4(1-\xi)}\exp\left[-\frac{2\xi\left(\rho^2+\rho'^2\right)}{(1-\xi)w^2}\right]I_m\left(\frac{4\rho\rho'\sqrt{\xi}}{1-\xi}\right), \quad (8)$$

where $I_m(x)$ is a modified Bessel function of order m, ξ specified the spectral degree of coherence of the field[8], with $\xi\to 0$ corresponding to the fully coherent case and $\xi\to 1$ corresponding to the completely incoherent case. The cross-spectral density at an arbitrary pair of points is given by

$$\tilde{W}(\boldsymbol{\rho},\boldsymbol{\rho}',z,z')=\frac{A\xi^{-m/2}}{1-\xi}\left(\frac{w_0^2}{ww'}\right)\times\exp\left\{im\left[\phi-\phi'\right]+i\left[k(z-z')-(m+1)(\Phi-\Phi')\right]\right\}\times\exp\left[i\left(\frac{k\rho^2}{2R}-\frac{k\rho'^2}{2R'}\right)\right]\exp\left[-\frac{1+\xi}{1-\xi}\left(\frac{\rho^2}{w^2}+\frac{\rho'^2}{w'^2}\right)\right]\times I_m\left(\frac{4\sqrt{\xi}}{1-\xi}\frac{\rho\rho'}{ww'}\right)\exp\left[\psi^*(\boldsymbol{\rho},z)+\psi(\boldsymbol{\rho}',z')\right]. \quad (9)$$

It is seen from Eq. (9) that if one chooses a reference point $(\boldsymbol{\rho}',z')$, the overall phase of the cross-spectral density of the partially coherent beam has the same helicoidal shape as that of the phase of the fully coherent Laguerre-Gauss beam of Eq. (3). Therefore the wave front of the former beam is endowed with a vortex structure similar to that of latter, with the azimuthally mode index m being a topological charge of the optical vortex.

The average cross-spectral density at an arbitrary plane $z=z'$ is given by

$$\left\langle\tilde{W}(\boldsymbol{\rho},\boldsymbol{\rho}',z)\right\rangle=\frac{A\xi^{-m/2}}{1-\xi}\left(\frac{w_0^2}{ww'}\right)\exp\left\{im\left[\phi-\phi'\right]\right\}\times\exp\left[-\frac{1+\xi}{1-\xi}\left(\frac{\rho^2}{w^2}+\frac{\rho'^2}{w'^2}\right)-\frac{1}{2}D_\psi(\boldsymbol{\rho},\boldsymbol{\rho}',z)\right]\times I_m\left(\frac{4\sqrt{\xi}}{1-\xi}\frac{\rho\rho'}{ww'}\right), \quad (10)$$

where $\langle\cdots\rangle$ denote the ensemble average (or long-time-average) of the refractive index fluctuations, $D_\psi(\boldsymbol{\rho},\boldsymbol{\rho}',z)=\left[0.545k^2zC_n^2\right]^{5/6}\left[2-0.206\,(3/8)^{-5/6}\kappa_0^{1/3}\right]\left(\rho^2+\rho'^2+\boldsymbol{\rho}\cdot\boldsymbol{\rho}'\right)$, $k=2\pi/\lambda$, λ is the wavelength of the propagating wave, C_n^2 is the turbulence strength along the propagation path, $\kappa_0=2\pi/L_0$, L_0 is the outer scale of turbulence. It is seen from Eq. (10) that the overall phase of the average cross-spectral density of the partially coherent beam has the same helicoidal shape as that of the phase of the fully coherent Laguerre-Gauss beam of Eq. (2), which is propagating in free space. Therefore the wave front of the former beam is endowed with a vortex structure similar to that of latter, with the azimuthal mode index m being a topological charge of the optical vortex.

The cross-spectral density function, which obeys the Helmholtz equation, is a measurement of the correlation between the fluctuations of two field components at the same frequency. If the field is strictly monochromatic or sufficiently narrow band, then one can have[7]

$$\Gamma(\boldsymbol{\rho},\boldsymbol{\rho}',z)\cong\left\langle\tilde{W}(\boldsymbol{\rho},\boldsymbol{\rho}',z)\right\rangle. \quad (11)$$

By Eqs. (10) and (11), we have

$$\Gamma(\boldsymbol{\rho},\boldsymbol{\rho}',z)=\frac{A\xi^{-m/2}}{1-\xi}\left(\frac{w_0^2}{ww'}\right)\exp\{im[\phi-\phi']\}$$
$$\exp\left[-\frac{1+\xi}{1-\xi}\left(\frac{\rho^2}{w^2}+\frac{\rho'^2}{w'^2}\right)\right]I_m\left(\frac{4\sqrt{\xi}}{1-\xi}\frac{\rho\rho'}{ww'}\right)$$
$$\exp\left[-\tfrac{1}{2}D_\psi(\boldsymbol{\rho},\boldsymbol{\rho}',z)\right]. \quad (12)$$

The average intensity $\langle I(\boldsymbol{\rho},z)\rangle$ for a partially coherent quasi-monochromatic Gaussian laser beam propagating in atmospheric turbulence is obtained from Eq. (12) when $\boldsymbol{\rho}=\boldsymbol{\rho}'$,

$$\langle I(\boldsymbol{\rho},z)\rangle=\frac{A\xi^{-m/2}}{1-\xi}\left(\frac{w_0^2}{w^2}\right)$$
$$\exp\left\{-\left[\frac{1+\xi}{1-\xi}\left(\frac{2}{w^2}\right)+a\right]\rho^2\right\}I_m\left(\frac{4\sqrt{\xi}}{1-\xi}\frac{\rho^2}{w^2}\right), \quad (13)$$

where $a=3\left[0.545k^2zC_n^2\right]^{5/6}\left[1-0.103\,(3/8)^{-5/6}\,\kappa_0^{1/3}\right]$.

We define the relative intensity of partially coherent beams as

$$\tilde{I}(\boldsymbol{\rho},z)=\langle I(\boldsymbol{\rho},z)\rangle/\langle I(0,z)\rangle. \quad (14)$$

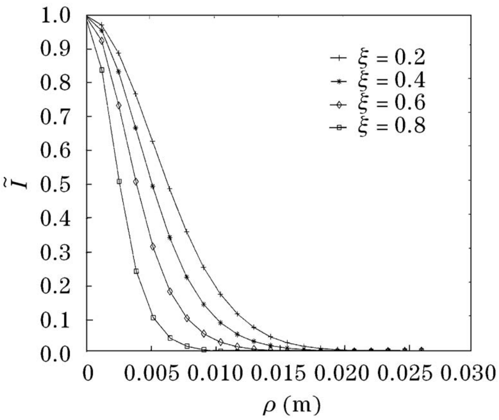

Fig. 1. Dependence of the $\tilde{I}$ on the transverse coordinate ρ. $\lambda=10.6$ μm, $z=1000$ m, $w_0=0.01$ m, $L_0=0.6$ m, $m=2$, $C_n^2=10^{-16}$.

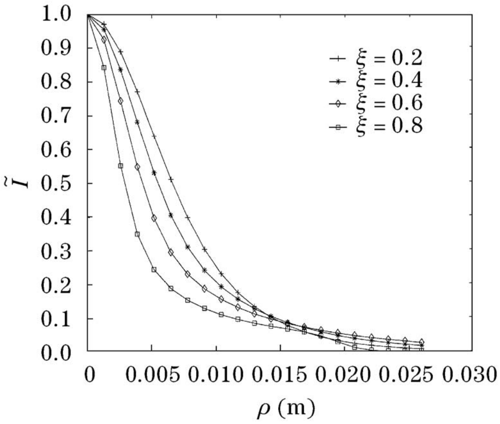

Fig. 2. Dependence of the $\tilde{I}$ on the transverse coordinate ρ. $\lambda=10.6$ μm, $z=1000$ m, $w_0=0.01$ m, $L_0=0.6$ m, $m=0$, $C_n^2=10^{-16}$.

To describe effects of the vortex structure and the coherent property of beam on the relative intensity of partially coherent beams, we performed numerical calculation. The results are presented in Figs. 1 and 2. In these figures, the relative intensity is plotted as a function of the parameter ρ for the vortex-free beam with the angular indices $m=0$ and $m=2$, the spectral degree of coherence of the field $\xi=0.2$ together with that of the beams with the degrees of coherence $\xi=0.4$, $\xi=0.6$, and $\xi=0.8$, respectively. It is also seen that the presence of a vortex degrades the relative intensity of the beam.

In summary, we have studied theoretically the propagating properties of a new class of partially coherent beams whose cross-spectral densities at a pair of points in any transverse plane are separable in polar coordinates. We have theoretically demonstrated that the vortex structure of the average cross-spectral density of such beam has the same helicoidally shape as that of the phase of the fully coherent Laguerre-Gauss beam which is propagating in free space. It is also shown that the relative intensity of the beam is degraded by vortex.

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