Probe spectrum in photonic crystal

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The probe absorption spectrum of a four-level atom embedded in a double-band photonic crystal is discussed in this paper. The double V-type transitions from the two upper levels to the two lower levels interact with the free vacuum modes and the photonic band gap (PBG) modes synchronously. We investigate the quantum interferences in the probe absorption spectrum. And the new features of transparency are also predicted.

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Quantum interference in spontaneous emission and in probe absorption of an atom depends not only on the properties of the excited atomic system but also on the nature of the surrounding environment[1,2]. As photonic band gap (PBG) structures show different density of state (DOS) compared with the vacuum field[1−4], the study of quantum and nonlinear optical phenomena of atoms embedded in such PBG materials leads to many interesting effects. Such as localization of light[5,6], photon-atom bound states [1,2,5,7], suppression and even complete cancellation of spontaneous emission[8,9], enhancement of spontaneous emission[10], electro-magnetically induced transparency and other phenomena[11−13].

In this paper, the quantum interference in the probe absorption spectrum of an atom embedded in a photonic crystal is investigated based on a double V-type system model. We assume that the transitions from the two upper levels to level $|1\rangle$ are synchronously coupled by the free vacuum modes (ω_λ) and a weak probe field ($\omega_{\rm p}$), and to level $|0\rangle$ coupled by the double-band isotropic (anistropic) PBG modes (ω_k) or the free vacuum modes as shown in Fig. 1. The Hamiltonian describing the dynamics of this system in the interaction picture and the rotating wave approximation can be written as

$$\begin{aligned} H_I &= \hbar\left(\Omega_{31}b_1^+b_3{\rm e}^{i\mu_p^{31}t}+\Omega_{21}b_1^+b_2{\rm e}^{i\mu_p^{21}t}+C.C.\right)\\ &+\hbar\sum\nolimits_{\lambda e}\left(g_{\lambda e}^{31}a_{\lambda e}^+b_1^+b_3{\rm e}^{i\mu_\lambda^{31}t}+g_{\lambda e}^{21}a_{\lambda e}^+b_1^+b_2{\rm e}^{i\mu_\lambda^{21}t}+C.C.\right)\\ &+\hbar\sum\nolimits_{ke}\left(g_{ke}^{30}a_{ke}^+b_0^+b_3{\rm e}^{i\mu_k^{30}t}+g_{ke}^{20}a_{ke}^+b_0^+b_2{\rm e}^{i\mu_k^{20}t}+C.C.\right), \end{aligned} \tag{1}$$

where b_i^+, b_i $(i=0,1,2,3)$ are respectively creation and annihilation operators for an atom in level i; $a_{\lambda e}^+$ $(a_{\lambda e})$ is the creation (annihilation) operator for the λth vacuum mode with frequency ω_λ; a_{ke}^+ (a_{ke}) is the creation (annihilation) operator for the kth PBG mode with frequency ω_k; Ω_{31} and Ω_{21} are the Rabi frequencies of probe field coupling respectively with the transitions from the upper two levels to the lower level $|1\rangle$; $g_{\lambda e}^{j1}$ $(j=2,3)$ is the frequency-dependent coupling constant between the atomic transition $|j\rangle-|1\rangle$ and the free vacuum modes ω_λ, and g_{ke}^{j0} $(j=2,3)$ is another coupling constant between the atomic transition $|j\rangle-|0\rangle$ and PBG modes ω_k. More precisely,

$$\begin{cases} g_{\lambda e}^{j1}=(\omega_{j1}d_{j1}/\hbar)\,(\hbar/2\varepsilon_0\omega_\lambda V)^{\frac{1}{2}}\,\vec{e}_{\lambda e}\cdot\vec{d}_{j1}\\ g_{ke}^{j0}=(\omega_{j0}d_{j0}/\hbar)\,(\hbar/2\varepsilon_0\omega_k V)^{\frac{1}{2}}\,\vec{e}_{ke}\cdot\vec{d}_{j0}\\ \mu_p^{j1}=\omega_p-\omega_{j1},\quad (j=2,3)\\ \mu_\lambda^{j1}=\omega_\lambda-\omega_{j1},\quad (j=2,3)\\ \mu_k^{j0}=\omega_k-\omega_{j0},\quad (j=2,3) \end{cases}. \tag{2}$$

Here $\vec{d}_{j1}$ and $\vec{d}_{j0}$ $(j=2,3)$ are the atomic dipole moment unit vectors for the transitions $|j\rangle-|1\rangle$ and $|j\rangle-|0\rangle$, respectively. The state vector of the system in the interaction picture can be written as

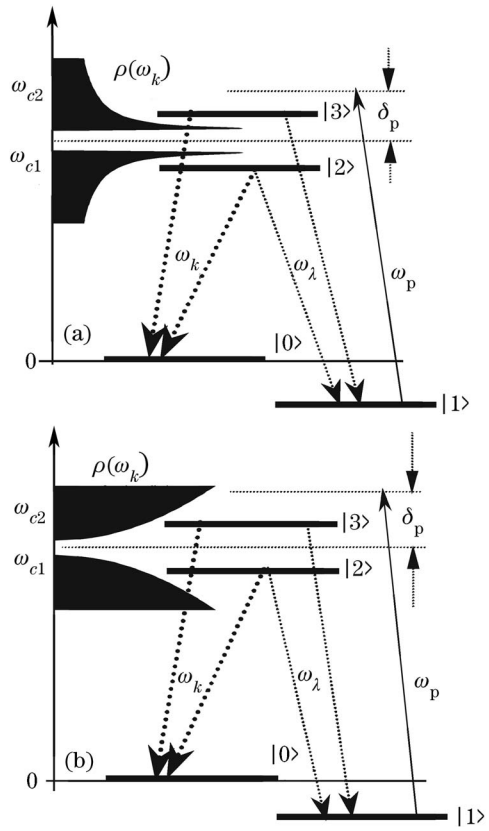

Fig. 1. Schematic diagram of the four-level atom in isotropic (a) and anistropic (b) double photonic band structure. $\rho(\omega_k)$ denotes the density of state of the PBG modes. ω_{c2} and ω_{c1} are the upper and lower frequencies of the forbidden gap, respectively. $\delta_{\rm p}$ is the detuning of frequency of probe field from the middle of the forbidden gap.

$$
\begin{aligned}
|\Psi(t)\rangle &= A_3(t)\mathrm{e}^{-i\mu_{\mathrm{p}}^{31}t}\,|3,\{0\}\rangle + A_2(t)\mathrm{e}^{-i\mu_{\mathrm{p}}^{21}t}\,|2,\{0\}\rangle \\
&\quad + A_1(t)\,|1,\{0\}\rangle + \sum\nolimits_{\lambda} A_{1\lambda}\,|1,\{\lambda\}\rangle \\
&\quad + \sum\nolimits_{k} A_{0k}\,|0,\{k\}\rangle, \qquad (3)
\end{aligned}
$$

where the amplitude $A_iA_i^*$ $(i=0,1,2,3)$ represents the probability of an atom occurring in the level i. The state vector $|3,\{0\}\rangle$ describes the atom in its excited state $|3\rangle$ and no photons present, whereas the state vector $|0,\{k\}\rangle$ describes $|0\rangle$ with a single photon in mode $\{k\}$. Substituting Eqs. (1) and (3) into Schrödinger equation, and using the Laplace transforms[1], we have

$$
\begin{cases}
\tilde{A}_3(\infty) = s\tilde{A}_3(s)_{s\to 0} = \frac{Y_1D_4 - Y_2D_2}{D_1D_4 - D_2D_3} \\
\tilde{A}_2(\infty) = s\tilde{A}_2(s)_{s\to 0} = \frac{Y_2D_1 - Y_1D_3}{D_1D_4 - D_2D_3} \\
\tilde{A}_1(\infty) = 1.0
\end{cases}, \qquad (4)
$$

where

$$
\begin{cases}
Y_1 = -i\Omega_{31}A_1(0) \\
Y_2 = -i\Omega_{21}A_1(0) \\
D_1 = -i\mu_{\mathrm{p}}^{31} + \tilde{G}_{33}^{\lambda}(s\to -i\mu_{\mathrm{p}}^{31}) + \tilde{G}_{33}^{k}(s\to -i\mu_{\mathrm{p}}^{31}) \\
D_2 = \tilde{G}_{23}^{\lambda}(s\to -i\mu_{\mathrm{p}}^{31}) + \tilde{G}_{23}^{k}(s\to -i\mu_{\mathrm{p}}^{31}) \\
D_2 = \tilde{G}_{32}^{\lambda}(s\to -i\mu_{\mathrm{p}}^{31}) + \tilde{G}_{32}^{k}(s\to -i\mu_{\mathrm{p}}^{31}) \\
D_4 = -i\mu_{\mathrm{p}}^{21} + \tilde{G}_{22}^{\lambda}(s\to -i\mu_{\mathrm{p}}^{31}) + \tilde{G}_{22}^{k}(s\to -i\mu_{\mathrm{p}}^{31})
\end{cases}. \qquad (5)
$$

Here, $\tilde{G}_{jl}^{k(\lambda)}$ $(j,l=2,3)$ is the Laplace transform of the delayed Green's function which can be derived in the forms of

$$
\begin{cases}
\tilde{G}_{jl}^{k}(s) = \eta_{jl}^{k}\frac{1}{2}\sqrt{\beta_{j0}^{3/2}\beta_{l0}^{3/2}}\left[\frac{i}{\sqrt{is+\delta_{l1c1}}} + \frac{1}{\sqrt{is+\delta_{l1c2}}}\right] \\
\tilde{G}_{jl}^{k}(s) = \eta_{jl}^{k}\frac{1}{2}\sqrt{\alpha_{j0}\alpha_{l0}}\left[-i\sqrt{is+\delta_{l1c1}} + \sqrt{is+\delta_{l1c2}}\right] \\
\tilde{G}_{jl}^{k}(s) = \eta_{jl}^{k}\frac{1}{2}\sqrt{\gamma_{j0}\gamma_{l0}}, \quad (j,l=2,3) \\
\tilde{G}_{jl}^{k}(s) = \eta_{jl}^{k}\frac{1}{2}\sqrt{\gamma_{j1}\gamma_{l1}}, \quad (j,l=2,3)
\end{cases} \qquad (6)
$$

for isotropic PBG, anisotropic PBG and free space reservoirs, respectively. Here the definitions of the detuning parameters are

$$
\begin{cases}
\mu_{\lambda}^{21} = \mu_{\lambda}^{31} + \omega_{32} = \mu_{\lambda}^{31} + \delta_{31c2} + \delta_{c1c2} - \delta_{21c1} \\
\delta_{31c1} = \delta_{31c2} + \delta_{c2c1} \\
\delta_{21c2} = \delta_{21c1} - \delta_{c2c1} \\
\delta_{31c2} = \omega_{31} - \omega_{c2} \\
\delta_{21c1} = \omega_{21} - \omega_{c1}
\end{cases}, \qquad (7)
$$

where δ_{j1cl} is the difference between the upper level j $(j=2,3)$ and the lower edge $(l=1)$ or the upper edge $(l=2)$ of the forbidden gap. The calculation of a probe absorption spectrum can be performed based on Eqs. (4), (5), and (6).

We study the case using symmetric values of parameters. In Fig. 2(a), the two peaks correspond to the transitions of an atom from upper two levels $|2\rangle$ and $|3\rangle$ to the lower level $|1\rangle$. Whereas, in Fig. 2(c), the two transitions from upper levels to level $|1\rangle$ merge into only one peak because of narrow width of the two upper levels. There are three transparencies in the case of isotropic PBG reservoir (Fig. 2(a)), but only one transparency in the case of anisotropic PBG (Fig. 2(b)) and free vacuum reservoir (Fig. 2(c)). The transparency at the center of $\delta_{\mathrm{p}}=0$ in all the three cases is similar to that reported by Zhou[14], and has been attributed to the two types of quantum interference that exist simultaneously in the system. It is further seen that the two transparencies at symmetric sides of $\delta_{\mathrm{p}}=0$ in Fig. 2(a) result from the two singularities in the Laplace transform of the delayed Green's function as indicated in Eq. (6), which is equivalent to the DOS of the isotropic PBG modes. It is also clear that the interference effects on probe absorption spectrum in the case of the isotropic PBG or anisotropic PBG reservoir is weaker than that in the case of free-space vacuum reservoir.

The probe absorption spectrum of a double V-type four-level atom embedded in a double-band photonic crystal has been discussed in this paper. In the case of isotropic PBG reservoir, there are three transparencies

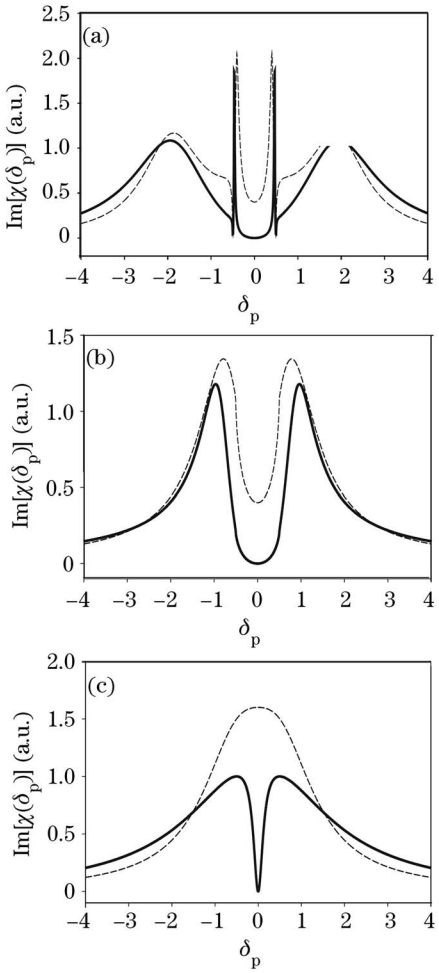

Fig. 2. The absorption spectrum $\mathrm{Im}[\chi(\delta_{\mathrm{p}})]$ as a function of probe detuning δ_{p}, (a) for double-band isotropic PBG reservoir and $\beta_{30}=\beta_{20}=\gamma_{21}=1.0$, (b) for double-band anisotropic PBG reservoir and $\alpha_{30}^2=\alpha_{20}^2=\gamma_{21}=1.0$, (c) for free vacuum reservoir and $\omega_{32}=\gamma_{30}=\gamma_{20}=\gamma_{21}=1.0$. The other parameters used are $\delta_{c2c1}=\delta_{31c2}=-\delta_{21c1}=1.0$, $A_{\lambda e}(0)=A_{ke}(0)=A_2(0)=A_3(0)=0.0$; $A_1(0)=1.0$. $\eta_{32}^{\lambda}=\eta_{32}^{k}=0.0$ (short dash curves), $\eta_{32}^{\lambda}=\eta_{32}^{k}=1.0$ (solid curves). All parameters in this paper are in units of γ_{31} except that η_{32}^{λ} and η_{32}^{k} are dimensionless.

on probe absorption spectrum. Whereas, only one transparency in the cases of anisotropic PBG and free vacuum reservoirs. And the interference effect in the case of PBG reservoir is weaker than that in the case of free-space vacuum reservoir. The symmetric transparencies in the case isotropic PBG result from the two singularities of DOS of PBG modes.

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