Numerical simulation of laser-induced plasma expansion in air

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A numerical study of the expansion of a laser-induced plasma is given by using of techniques of computational fluid dynamics. According to the experiment results reported, the density of electrons is assembled on the surface of the ionization area. In the present simulation, the model is considered as an expansion of plasma bubble in air. The laser energy is absorbed by the surface of the plasma bubble facing to the laser beam. Numerical results show the elongation of the bubble along the laser beam direction, which agrees with the experiments.

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Under the action of intense focused laser beam, the air that it is considered transparent usually absorbs laser energy and produce optical breakdown, i.e., the air is ionized seriously to produce plasma. In the early time, Kroll *et al.*[1] studied the ionization of air by intense laser pulses by solving the Boltzmann equation for electrons in both classical and quantum forms. Plasma absorbs laser energy strongly so it has high temperature and pressure and then expands rapidly. Lu and Ni[2] measured the expansion of the plasma in air by using interferometric method. The experiments showed that the expansion speed of plasma is larger in the directions of inverse laser propagating than other directions at the time interval of laser energy supplied. The plasma bubble was elongated in the direction of inverse laser propagating. After laser energy is stopped, the expansion speeds along every direction tend to identical gradually. In order to explain the effect, a simplified physical model is proposed in this paper according to the theories of laser-induced plasma and using the techniques of computational fluid dynamics. The results agree with the experiments qualitatively.

According to the theory of interaction between laser and the air, the molecules of the air can be ionized to produced free electrons (called seed electrons) first as the laser energy density exceeds the breakdown threshold. Seed electrons absorb laser energy by favorable collisions with heavier particles, i.e., molecules and ions[3]. If the electrons sustain enough favorable collisions, they will eventually gain sufficient energy to impact ionize other molecules in a geometric increase in the free electron density and form plasma[4]. The electrons in plasma bubble absorb laser energy continuously. So the bubble has high temperature and pressure and then expands rapidly. We assume that the plasma bubble absorbs laser energy and expands in the space follows the rule below.

The free electrons on the surface of plasma bubble that the laser beam irradiates firstly absorb laser energy prior. Hence the electron density, temperature and pressure near the irradiated surface by laser beam firstly are larger than other area of the bubble. The electrons on this part of the surface absorb most part of laser energy and transfer the energy to other electrons by the ways of collision and diffusion during the plasma bubble expanding.

For simplification of computing, we adopt non-viscous ideal gas dynamic equation. The two-dimensional (2D) Euler equation to describe the expanding of plasma can be written as

$$\frac{\partial U}{\partial t}+\frac{\partial \vec{f}(U)}{\partial x}+\frac{\partial \vec{g}(U)}{\partial y}=H(U), \tag{1}$$

where

$$U=\begin{bmatrix}\rho\\ \rho u\\ \rho v\\ e\end{bmatrix}, \vec{f}(U)=\begin{bmatrix}\rho u\\ \rho u^2+p\\ \rho uv\\ u(e+p)\end{bmatrix},$$
$$\vec{g}(U)=\begin{bmatrix}\rho v\\ \rho uv\\ \rho v^2+p\\ v(e+p)\end{bmatrix}, H(U)=\begin{bmatrix}0\\ 0\\ 0\\ E_{\mathrm{L}}\end{bmatrix},$$

here, the variable ρ is the density, u and v are velocities in x and y directions, e is the specific internal energy, p is the pressure, E_{L} is the spatial distribution of laser energy. And the Eq. (1) can be closed through the equation of state for ideal gas

$$p=(\gamma-1)\left(e-\rho\frac{u^2+v^2}{2}\right), \tag{2}$$

where $\gamma=1.4$ is the ratio of specific heat.

Here, we adopt the second order accurate SCB scheme developed in Ref. [5]. This is a high order accurate finite difference scheme for the equation of gas dynamics. It can be written as

$$U_{j.k}^{n+1}=U_{j,k}^{n}-\lambda\left(\vec{F}_{j+1/2,k}^{n}-F_{j-1/2,k}^{n}\right)$$
$$-\mu\left(\vec{G}_{j,k+1/2}^{n}-\vec{G}_{j,k+1/2}^{n}\right), \tag{3}$$

where the numerical flux

$$\vec{F}^n_{j+1/2,k} = \frac{1}{2}\left(\vec{F}^n_{j,k} + \vec{F}^n_{j+1,k}\right) - \frac{1}{2}R^x_{j+1/2,k}$$
$$\left\{ Q\left(\Lambda^x\right)_{j+1/2,k}\alpha^x_{j+1/2,k} - \left[Q\left(\Lambda^x\right)_{j+1/2,k} - \lambda\left(\Lambda^x\right)^2_{j+1/2,k}\right] \bullet \left(d\left((\alpha^x_1)_{j-1/2,k}, (\alpha^x_1)_{j+1/2,k}, (\alpha^x_1)_{j+3/2,k}\right) \cdots d\left((\alpha^x_m)_{j-1/2,k}, (\alpha^x_m)_{j+1/2,k}, (\alpha^x_m)_{j+3/2}\right)\right)^T + \frac{\mu}{2}\left(\Lambda^x\right)_{j+1/2,k}\left(L^x\right)^T_{j+1/2,k} m^g_{j+1/2,k} \right\},$$

$$\vec{G}^n_{j,k+1/2} = \frac{1}{2}\left(\vec{G}^n_{j,k} + \vec{G}^n_{j,k+1}\right) - \frac{1}{2}R^y_{j,k1/2}$$
$$\left\{ Q\left(\Lambda^y\right)_{j,k+1/2}\alpha^y_{j,k+1/2} - \left[Q\left(\Lambda^y\right)_{j,k+1/2} - \lambda\left(\Lambda^y\right)^2_{j,k+1/2}\right] \bullet \left(d\left((\alpha^y_1)_{j,k-1/2}, (\alpha^y_1)_{j,k+1/2}, (\alpha^y_1)_{j.k+3/2}\right) \cdots d\left((\alpha^y_m)_{j,k-1/2}, (\alpha^y_m)_{j,k+1/2}, (\alpha^y_m)_{j,k+3/2}\right)\right)^T + \frac{\lambda}{2}\left(\Lambda^y\right)_{j,k+1/2}\left(L^y\right)^T_{j,k+1/2} m^f_{j,k+1/2} \right\},$$

here the lower indices $(j+1/2,k)$ and $(j,k+1/2)$ stand for some kind of average, for example Roe average, and

$$Q\left(\Lambda^x\right)_{j+1/2,k} = \text{diag}\left(Q\left((\lambda^x_1)_{j+1/2,k}\right)\cdots Q\left((\lambda^x_m)_{j+1/2,k}\right)\right),$$

$$Q\left(\Lambda^y\right)_{j,k+1/2} = \text{diag}\left(Q\left((\lambda^y_1)_{j,k+1/2}\right)\cdots Q\left((\lambda^y_m)_{j,k+1/2}\right)\right),$$

$$m^g_{j+1/2,k} = \min\ \text{mod}\left[\Delta_{j,k+1/2}\vec{G}, \Delta_{j+1}, k+1/2\vec{G}, \Delta_{j,k-1/2}\vec{G}, \Delta_{j+1,k-1/2}\vec{G}\right],$$

$$m^f_{j,k+1/2} = \min\ \text{mod}\left[\Delta_{j+1/2,k}\vec{G}, \Delta_{j+1/2}, k+1\vec{G}, \Delta_{j-1/2,k}\vec{G}, \Delta_{j-1/2,k+1}\vec{G}\right] .$$

The computing domain is a rectangular area $[-1,1] \times [0,3]$. All boundary conditions are out-flow conditions. The initial surface of plasma bubble of laser irradiation is a circle of the center $(0, 0.2)$ and radius 0.1 cm. At the beginning, variables in the bubble $\rho = 1$, $p = 100$, $u = v = 0$, and in the outside, $\rho = 1$, $p = 1$, $u = v = 0$. The spatial distribution of laser energy $E_{\rm L}$ is $\frac{e_{\rm L}}{9\ \Omega\ y\ \text{tg}\theta}\text{e}^{-2\frac{x^2}{\Omega^2 y^2}}$ where $E_{\rm L} = 78$ mJ is initial

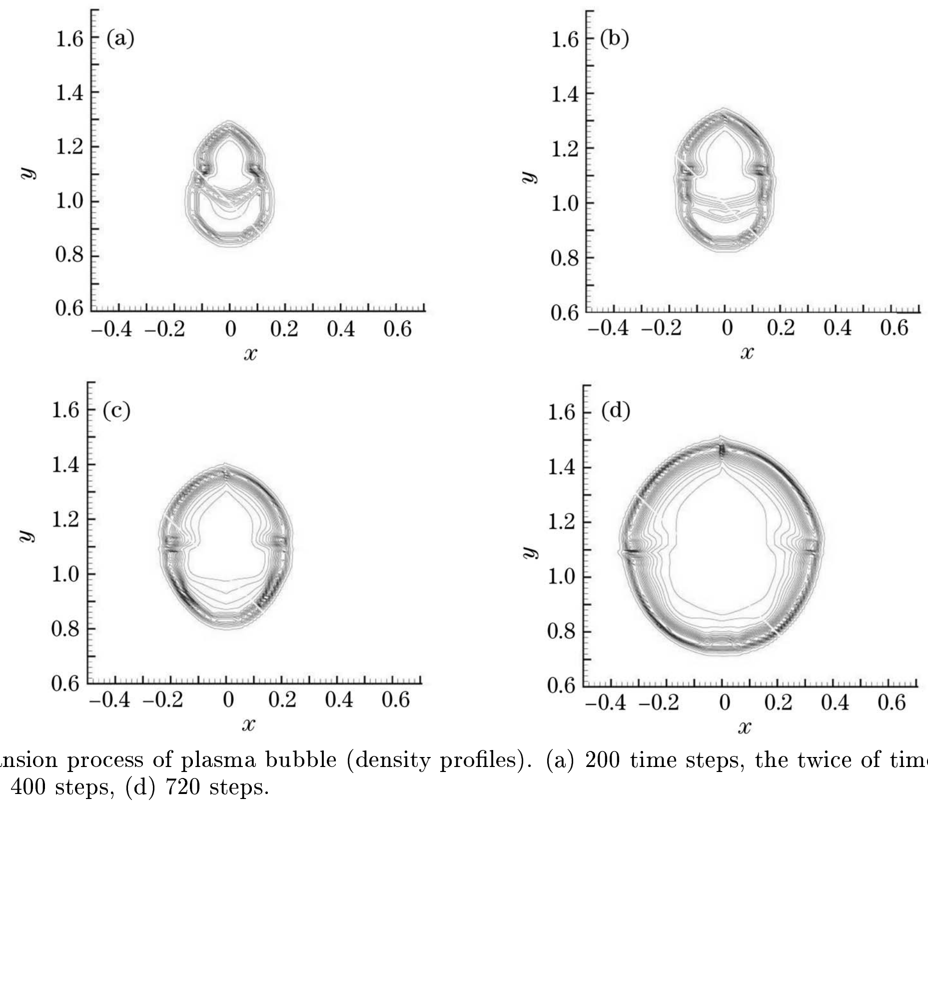

Fig. 1. The expansion process of plasma bubble (density profiles). (a) 200 time steps, the twice of time laser energy supplied, (b) 280 steps, (c) 400 steps, (d) 720 steps.

laser energy parameter, θ is the convergent angle of a converging lens and equals 60°. The direction of laser propagating is along negative y direction and the time of laser energy supplied is 15 ns in this computation. Ω is the waist radius of Gaussion beam and equals 0.1 cm at the focus of beam (focus coordinate: x=0.0, y=1.0).

The numerical results are shown in Fig. 1. From the figure we can see that the expansion speed of plasma is larger in the direction of inverse laser propagating than other directions when the laser energy is supplied. The plasma bubble is elongated in the directions of inverse laser propagating. After laser energy is stopped, the expansion speeds along every direction tend to identical and the plasma bubble tends to plump gradually. The results agree with the experiments qualitatively.

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