## Slow and superluminal pulse propagation in temporal nonlocal response Kerr media with a CW pump

Qiguang Yang (杨启光)<sup>1,2</sup>, Seong Min Ma<sup>1</sup>, Jae Tae Seo<sup>1</sup>, Bagher Tabibi<sup>1</sup>, Huitian Wang (王慧田)<sup>2</sup>, and S. S. Jung<sup>3</sup>

<sup>1</sup>Department of Physics, Hampton University, Hampton, VA 23668, USA
<sup>2</sup>National Laboratory of Solid State Microstructures and Department of Physics, Nanjing University, Nanjing 210093
<sup>3</sup>Korea Research Institute of Standards and Science, Daejeon, 305-600, South Korea

Slow and superluminal light propagation in temporal nonlocal response Kerr media has been investigated. The group velocity, as low as tens of few centimeters per second, can be easily obtained by tuning the wavelength of a continuous-wave (CW) laser in a typical Kerr material, such as in alexandrite.

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Controllable slow and fast propagation of a pulse light has attracted a great deal of attention recently because of its possible applications in many novel devices such as all-optical buffers, variable time delay, quantum communication and computation. Slow group velocity and superluminal propagation of light wave have been demonstrated in some specific materials recently. The origin of these effects is the highly dispersive linear and/or nonlinear optical properties of the materials. Regions of sharp normal dispersion give rise to very slow group velocity while regions of strong anomalous dispersion give rise to the superluminal effects.

The anomalous dispersion near an absorption resonance implies that the group velocity can be superluminal. The group velocities of pulses could be greater than the speed of light in vacuum, negative, or infinite in an absorber<sup>[1]</sup>. The normal dispersion in the wings of the resonance leads to slow propagation of a laser pulse. Small group velocity can be obtained if the central frequency of a pulse is well removed from the absorption resonance. However, the pulses suffer significant changes in amplitude due to the strong absorption. Recently, several other techniques have been proposed to demonstrate slow light and superluminality effects. The most popular technique is electromagnetically induced transparency (EIT). EIT effect causes greatly reduced absorption for the probe pulse in a very narrow wavelength range due to the quantum interference effects. This rapid change in absorption suggests a rapid variation of the refractive index with frequency, and thus a large group index and a very small group velocity. Using the EIT technique, the group velocity of light as low as meters per second or even completely stopped light was demonstrated in ultracold  ${\rm gases}^{[2,3]},$  hot  ${\rm gases}^{[4]},$  and in a very cold  ${\rm solid}^{[5]}.$ M. Bigelow et al. observed superluminal and slow light propagation in room-temperature solids $^{[6,7]}$ . In their experiments, the population difference of a two level system has components that oscillate at the beat frequency between the pump and probe. This coherent population oscillation (CPO) induces a very narrow spectral hole or antihole in the homogeneously broadened absorption profile. The narrow hole or antihole gives rise to high dispersion and leads to slow or superluminal light propagation. In an off-resonance Raman medium, a steep dispersion in the vicinity of the two-photon resonance may be caused by a strong coupling field, and leads to

slow light propagation<sup>[8,9]</sup>. Slow light propagation has also been observed in photorefractive material by two-wave mixing (PR-TWM)<sup>[10,11]</sup>. The PR-TWM process is a highly dispersive process due to the slow nonlocal response of the PR effect.

Recently we showed that a cosinoidal amplitude modulated pulse can propagate with both positive and negative group velocity in a temporal-nonlocal response Kerr material (TNLRKM)<sup>[12]</sup>. Our results indicate that the group velocity of the light is determined by the nonlinear refractive index, the relaxation time of the nonlinear refractive index grating, the difference of the angular frequency of the two incident frequency elements, and the thickness of the Kerr material. In this letter, we consider the propagation of a Gaussian pulse in TNLRKMs pumped by a strong (continuous-wave)CW laser.

Two-wave coupling of in a TNLRKM has been investigated for many years<sup>[13]</sup>. A phase shift between the moving index grating and the moving laser intensity pattern is obtained due to the finite material response time in TNLRKMs. This phase shift makes the energy transfer between the incident beams possible. Two-wave coupling is a highly dispersive process; the energy transfer efficiency decreases quickly when the difference of the angular frequency between the two light beams deviates from the inverse lifetime of the index grating<sup>[14]</sup>.

Consider the situation where a strong CW beam and a weak pulse beam interact in a TNLRKM. The pulse can be viewed as a linear superposition of many monochromatic plane waves, each with a definite frequency  $\omega$  and wave vector  $\mathbf{k}$ . The interference of these plane waves makes an overall waveform, which can be represented as

$$\mathbf{E}_{\text{pulse}}(\mathbf{r}, t) = \sum_{j} \left( \frac{1}{2} \mathbf{A}_{j}(\mathbf{r}) e^{i(\mathbf{k}_{j} \cdot \mathbf{r} - \omega_{j} t)} + c.c. \right), \quad (1)$$

where  $\mathbf{A}_{j}(\mathbf{r})$  is the slowly varying amplitude of component j of the pulse,  $\omega_{j}$  is the corresponding frequency, and  $k_{j}$  is the linear part of the wave vector.

Assuming slowly varying envelope approximation and neglect the interaction among the different frequency components of the weak pulse, the Maxwell's equations reduce to the following wave equations:

$$i2k_{c}\frac{dA_{c}}{dz} = -\frac{n_{0}^{2}n_{2}\omega_{c}^{2}}{\pi c}(|A_{c}|^{2} + |A_{j}|^{2})A_{c} -\frac{n_{0}^{2}n_{2}\omega_{j}^{2}}{\pi c}\frac{|A_{j}|^{2}A_{c}}{1 - i\delta\tau},$$
(2)

$$i2k_{j}\frac{\mathrm{d}A_{j}}{\mathrm{d}z} = -\frac{n_{0}^{2}n_{2}\omega_{j}^{2}}{\pi c}(|A_{c}|^{2} + |A_{j}|^{2})A_{j} -\frac{n_{0}^{2}n_{2}\omega_{c}^{2}}{\pi c}\frac{|A_{c}|^{2}A_{j}}{1 + i\delta\tau},$$
(3)

where  $n_0$  is the linear refractive index coefficient, c is the light velocity in vacuum,  $n_2$  is the nonlinear refractive index coefficient. The symbols with subscript 'c' represent parameters of the CW beam.

The time response function of the nonlinear refractive index is assumed to obey a Debye relaxation equation of the form:

$$\frac{\mathrm{d}n_{\mathrm{NL}}}{\mathrm{d}t} = n_2 I \tau - \frac{n_{\mathrm{NL}}}{\tau},\tag{4}$$

where I is the intensity of the optical field,  $\tau$  is the relaxation time of the nonlinear refractive index.

Since it is the phase which plays the most important role for slowing light down, we kept all the phase modulation contributions in Eqs. (2) and (3), even though they do not contribute to energy transfer directly.

The nonlinear phase changes due to the two-beam coupling are given by

$$\varphi_{c}^{NL}(z) = \frac{n_{2r}}{2n_{2i}} \ln(1 + \Delta\alpha \cdot L_{eff}) 
+ \frac{(n_{2r} - n_{2i}\delta\tau)}{2(n_{2r}\delta\tau - n_{2i})} \ln\left(1 - \frac{I_{j0}}{I_{0}} + \frac{I_{j0}}{I_{0}}(1 + \Delta\alpha \cdot L_{eff})^{\zeta}\right),$$
(5)

$$\varphi_{j}^{\text{NL}}(z) = \frac{n_{2r}}{2n_{2i}} \ln(1 + \Delta\alpha \cdot L_{\text{eff}}) - \frac{(n_{2r} + n_{2i}\delta\tau)}{2(n_{2r}\delta\tau - n_{2i})} \ln\left(1 - \frac{I_{\text{c0}}}{I_{0}} + \frac{I_{\text{c0}}}{I_{0}}(1 + \Delta\alpha \cdot L_{\text{eff}})^{-\zeta}\right),$$
(6)

where  $\alpha$  is the linear absorption coefficient,  $n_2 = n_{2r} + i n_{2i}$ ,  $\Delta \alpha = \frac{2n_{2i}I_0\omega}{c}$ ,  $L_{\rm eff} = \frac{1-e^{-\alpha z}}{\alpha}$ ,  $\zeta = \frac{n_{2r}\delta\tau}{n_{2i}(1+\delta^2\tau^2)}$ ,  $\delta = \omega_{\rm c} - \omega_j$  and  $I_0 = I_{c0} + I_{j0}$ . The total phase of jth component of the pulse beam is given by  $\varphi_j(z,t) = \frac{\omega_j n_0 z}{c} - \omega_j t + \varphi_j^{\rm NL}(z)$ . In order to form a wave packet at space point z and the

The total phase of jth component of the pulse beam is given by  $\varphi_j(z,t) = \frac{\omega_j n_0 z}{c} - \omega_j t + \varphi_j^{\rm NL}(z)$ . In order to form a wave packet at space point z and time point t, the phases of all the frequency components of the pulse should be the same, thus they interfere with each other constructively. Mathematically this is expressed as  $\partial \varphi_j(z,t)/\partial \omega_j = 0$ . Therefore the group velocity of the pulse is given by

$$v_{\rm g} = \frac{c}{n_0 + \omega \frac{\mathrm{d}n_0}{\mathrm{d}\omega_j} \Big|_{\omega_0} + \frac{c}{z} \frac{\mathrm{d}\varphi_j^{\rm NL}(z)}{\mathrm{d}\omega_j} \Big|_{\omega_0}},\tag{7}$$

where  $\omega_0$  is the carrier frequency of the pulse beam. Usually the dispersion of the linear refractive index is very small in the off-resonance case. Thus one can neglect the second term in the denominator.

Using the parameters for a typical alexandrite $^{[15]}$ , we calculated the spectral profiles of a 1-ms (FWHM) Gaussian pulse beam through a 5-mm alexandrite with and without a 500-W/cm $^2$  CW pump beam at 514.5 nm. Figure 1 shows an example for the case of  $\omega_0 = \omega_c$ . Clearly the lower frequency part of the pulse has been amplified while the higher frequency part losses its energy. It is worthy to notice that two-wave coupling is a highly dispersive process, the two-wave coupling efficiency decreases very fast when  $|\omega_{\rm c} - \omega_j| \tau$  deviates from 1. We noticed that the pulse has been distorted significantly in the current example. This will be improved in the case of  $\omega_0 \neq \omega_c$ . It is the phase which relates to the group velocity directly, we therefore show the nonlinear phase changes of the pulse beam in Fig. 2. The most interesting result showed here is that the nonlinear phase change is highly dispersive. Therefore it is possible to use this property to make slow and fast light propagation in TNLRKMs.

Figure 3 illustrates the group velocity of the pulse as a function of the detuning parameter  $(\omega_c - \omega_0)\tau$ . As one may expect that the group velocity is only determined by the linear dispersion of the material when the detuning parameter is large. When the detuning parameter is positive, as shown in Fig. 4, the pulse experiences amplification, and the higher frequency part of the pulse

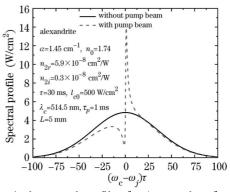


Fig. 1. Typical spectral profile of a 1-ms pulse after through a 5-mm alexandrite sample. Without a strong CW pump, the pulse experiences absorption only. With a strong CW, the lower frequency part has been amplified by the CW beam through two-wave coupling effects.

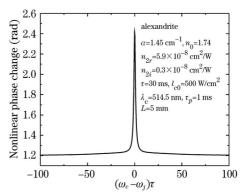


Fig. 2. Two-wave coupling induced nonlinear phase change of the different frequency components of the weak pulse beam.

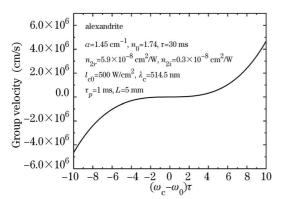


Fig. 3. Group velocity of the weak pulse versus the detuning parameter.

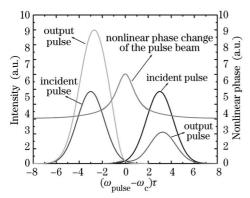


Fig. 4. Light-induced dispersion in the Kerr material.

experiences larger phase change, thus  $\frac{c}{z} \frac{\mathrm{d} \varphi_j^{\mathrm{NL}}(z)}{\mathrm{d} \omega_j} \Big|_{\omega_0}$  is positive and the group velocity will be decreased by the coupling effect between the CW pump and the weak pulse. Small group velocity, which is as low as tens of few centimeters per second, may be easily obtained by decreasing the detuning parameter. When the detuning parameter is negative, the pulse losses energy and the lower frequency part experiences larger nonlinear phase changes, the negative  $\frac{c}{z} \frac{\mathrm{d} \varphi_j^{\mathrm{NL}}(z)}{\mathrm{d} \omega_j} \Big|_{\omega_0}$  makes negative group velocity possible, as shown in Fig. 3. The negative group velocity indicates superluminal propagation of the light pulse.

In conclusion, we investigated the propagation properties of a laser pulse in TNLRKMs. Using a CW pump

beam, the group velocity of the weak pulse beam can be easily controlled. Both slow and superluminal light propagation can be obtained by simply tuning the CW wavelength. The experiments are being carried out in our Lab.

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