A scheme for assisted cloning of a two-particle entangled state via GHZ class states

Youbang Zhan (詹佑邦)[1,2]

[1] *Department of Physics, Huaiyin Teachers College, Huaian 223001*

[2] *CCAST (World Laboratory), Beijing 100080*

We propose a protocol where one can realize quantum cloning of an unknown two-particle entangled state and its orthogonal complement state with assistance offered by a state preparer. The first stage of the protocol requires usual teleportation using a (or two) four-particle entangled state(s) as quantum channel(s). In the second stage of the protocol, with the assistance (through a two-particle projective measurement) of the preparer, the perfect copies and complement copies of an unknown state can be produced.

OCIS codes: 270.0270, 270.5570.

Quantum entanglement has generated much interest in the quantum information processing such as quantum teleportation[1], quantum dense coding[2], remote state preparation[3], and quantum key distribution[4]. In recent years, the possibility of cloning quantum states approximately has attracted much attention. A quantum state can not be cloned exactly due to the no-cloning theorem[5,6]. However, quantum cloning approximately is necessary in quantum information[7]. Though exact cloning is no possible, in the literature various cloning machines have been proposed[8−22] which operate either in a deterministic or probabilistic way. Universal quantum cloning machines was originally addressed by Bužek et al[10]. The deterministic state-dependent cloning machine, proposed firstly by Hillery *et al.*[11], is designed to generate approximate clones of states belonging to a finite set. Gisin and Bruβ *et al.*[12,13] constructed the universal qubit cloner that maximizes the local fidelity. The probabilistic cloning machine was first considered by Duan *et al.*[14] using a general unitary-reduction operation with a postelection of the measurement results. Murao *et al*[15] proposed the quantum telecloning process combining quantum teleportation and optimal quantum cloning from one input to M outputs. The other category of quantum cloning machines were developed[16−20].

Recently, Pati[16] proposed a scheme where one can produce perfect copies and orthogonal-complement copies of an arbitrary unknown state with minimal assistance from a state preparer. This scheme realizes perfect cloning and complementing of an unknown state using resources such as entangled state, Bell-state measurement, single-particle von Neumann measurement, and classical communication. Inspired by the scheme of Ref. [16], the purpose of this paper is to give a protocol which can produce perfect copies and orthogonal-complement copies of an unknown two-particle entangled state via a four-particle entangled state as the quantum channel. Different from the previous protocol using a single-particle von Nenmann orthogonal measurement[16], here we will realize the assisted cloning by using a two-particle projective measurement consisting of a set of nonmaximally entangled basis vectors. In addition, we also consider the assisted cloning via two four-particle entangled states as the quantum channels.

Suppose Alice has an input two-particle entangled state $|\phi\rangle_{12} = \alpha\,|00\rangle_{12} + \beta\,|11\rangle_{12}$ from a state preparer Victor, with α as a real number and β as a complex number, and $|\alpha|^2 + |\beta|^2 = 1$. Assume Alice and Bob share a four-particle entangled state of the type from Greenberger-Horne-Zeilinger (GHZ)[16,22] give by

$$|\phi\rangle_{3456} = \frac{1}{\sqrt{2}}\left(|0011\rangle_{3456} + |1100\rangle_{3456}\right). \tag{1}$$

Here, we assume that particles 3 and 4 belong to Alice while particles 5 and 6 belong to Bob. The input state $|\phi\rangle_{12}$ is unknown to both Alice and Bob. The initial state of the combined system is

$$|\Psi\rangle = |\phi\rangle_{12} \otimes |\phi\rangle_{3456} = \left|\Psi_{(1)}\right\rangle + \left|\Psi_{(2)}\right\rangle, \tag{2}$$

where

$$\left|\Psi_{(1)}\right\rangle = \frac{1}{2\sqrt{2}}\left|\Phi^{\pm}\right\rangle_{13}\left|\Phi^{\pm}\right\rangle_{24}\left(\alpha\,|11\rangle \pm \pm\beta\,|00\rangle\right)_{56},$$
$$\left|\Psi_{(2)}\right\rangle = \frac{1}{2\sqrt{2}}\left|\Psi^{\pm}\right\rangle_{13}\left|\Psi^{\pm}\right\rangle_{24}\left(\alpha\,|00\rangle \pm \pm\beta\,|11\rangle\right)_{56}, \tag{3}$$

where $|\Phi^{\pm}\rangle_{ij}$ and $|\Psi^{\pm}\rangle_{ij}$ are the Bell states of particles i and j

$$\left|\Phi^{\pm}\right\rangle_{ij} = \frac{1}{\sqrt{2}}\left(|00\rangle \pm |11\rangle\right)_{ij},$$
$$\left|\Psi^{\pm}\right\rangle_{ij} = \frac{1}{\sqrt{2}}\left(|01\rangle \pm |10\rangle\right)_{ij}. \tag{4}$$

In the Eq. (3), the notes "$\pm$" in the column from right to left correspond to the Bell state of particles (1, 3) and (2, 4), respectively. Assume Alice performs Bell-state measurements on particles (1, 3) and (2, 4), respectively, and if the measurement outcome of Alice is $|\Psi^{+}\rangle_{13}\,|\Psi^{-}\rangle_{24}$ (the probability of this result is only 1/8), then the resulting six-particle state can be written as

$$\left|\Psi^{-}\right\rangle_{24}\left\langle\Psi^{-}|\Psi^{+}\right\rangle_{13}\left\langle\Psi^{+}|\Psi\right\rangle = \frac{1}{2\sqrt{2}}\left|\Psi^{+}\right\rangle_{13}\left|\Psi^{-}\right\rangle_{24}\left(\alpha\,|00\rangle - \beta\,|11\rangle\right)_{56}. \tag{5}$$

After these measurements, Alice sends the measurement result to Bob through a classical channel. According to the measure outcome of Alice, Bob will operate a unitary transformation $I_5 \otimes (\sigma_z)_6$ on Eq. (5), and to get the original state from particles 5 and 6.

To create either a copy or an orthogonal-complement copy of the unknown two-particle state $|\phi\rangle$, Alice needs assistance of Victor. According to the projection postulate of quantum mechanics, if Alice applies projectors $|\Psi^-\rangle_{24}\langle\Psi^-|\Psi^+\rangle_{13}\langle\Psi^+|$ into the combined state $|\Psi\rangle$, the state of particles 1, 2, 3, and 4 will collapse in the entangled state $|\Psi^+\rangle_{13}|\Psi^-\rangle_{24}$ (see Eq. (5)). Alice sends particles 1 and 2 to Victor and keeps particles 3 and 4 in her possession. Since Victor knows the parameters α and β of original state $|\phi\rangle_{12}$ completely, he carries out a two-particle projective measurement on the particles 1 and 2 in a set of mutually orthogonal basis vectors $\{|\varphi\rangle, |\varphi_\perp\rangle, |\psi\rangle, |\psi_\perp\rangle\}$, which is given by

$$\begin{aligned}
|\varphi\rangle_{12} &= \alpha|00\rangle_{12} + \beta|11\rangle_{12}, \\
|\varphi_\perp\rangle_{12} &= \beta^*|00\rangle_{12} - \alpha|11\rangle_{12}, \\
|\psi\rangle_{12} &= \alpha|01\rangle_{12} + \beta|10\rangle_{12}, \\
|\psi_\perp\rangle_{12} &= \beta^*|01\rangle_{12} - \alpha|10\rangle_{12}.
\end{aligned} \qquad (6)$$

The above four nonmaximally entangled basis states $\{|\varphi\rangle, |\varphi_\perp\rangle, |\psi\rangle, |\psi_\perp\rangle\}$ are related to the computation basis vectors $\{\langle 00|, |01\rangle, |10\rangle, |11\rangle\}$, and form a complete orthogonal basis in a four-dimensional Hilbert space. We find that the $|\varphi\rangle_{12}$ is equal to $|\phi\rangle_{12}$ and the basis $|\varphi_\perp\rangle_{12}$ is equal to $|\phi_\perp\rangle_{12}$, where $|\phi_\perp\rangle_{12} = \beta^*|00\rangle_{12} - \alpha|11\rangle_{12}$ is the orthogonal-complement state to $|\phi\rangle_{12}$. Moreover, $|\psi\rangle_{12} = (I_1 \otimes (\sigma_x)_2)|\phi\rangle_{12}$ and $|\psi_\perp\rangle_{12} = (I_1 \otimes (\sigma_x)_2)|\phi_\perp\rangle_{12}$ is the orthogonal-complement state to $|\psi\rangle_{12}$ (the above σ_z and σ_x are Pauli operators). Thus, the entangled state $|\Psi^+\rangle_{13}|\Psi^-\rangle_{24}$ in the basis $\{|\varphi\rangle, |\varphi_\perp\rangle, |\psi\rangle, |\psi_\perp\rangle\}$ can be rewritten as

$$\begin{aligned}
|\Psi^+\rangle_{13}|\Psi^-\rangle_{24} = \frac{1}{2}[&|\varphi\rangle_{12}(\alpha|11\rangle_{34} - \beta^*|00\rangle_{34}) \\
&+ |\varphi_\perp\rangle_{12}(\alpha|00\rangle_{34} + \beta|11\rangle_{34})|\psi\rangle_{12}(-\alpha|10\rangle_{34} \\
&+ \beta^*|01\rangle_{34}) + |\psi_\perp\rangle_{12}(\alpha|01\rangle_{34} + \beta|10\rangle_{34})].
\end{aligned} \qquad (7)$$

If the result of Victor's measurement on the two particle 1 and 2 is $|\varphi_\perp\rangle_{12}$, Eq. (5) can be written as

$$\begin{aligned}
&|\varphi_\perp\rangle_{12}\langle\varphi_\perp|\Psi^-\rangle_{24}\langle\Psi^-|\Psi^+\rangle_{13}\langle\Psi^+|\Psi\rangle \\
&= \frac{1}{4\sqrt{2}}|\varphi_\perp\rangle_{12} \otimes |\phi\rangle_{34} \otimes (I_5 \otimes (\sigma_z)_6)|\phi\rangle_{56}.
\end{aligned} \qquad (8)$$

Victor sends the measurement outcome to Alice through a classical channel with two classical bits, then Alice knows that the state of her particles 3 and 4 has been found in the original state $|\phi\rangle_{34} = \alpha|00\rangle_{34} + \beta|11\rangle_{35}$, which is just a copy of the state $|\phi\rangle_{12}$. If the result of Victor is $|\varphi\rangle_{12}$, then two cbits from Victor to Alice would yield a complement state given by

$$\begin{aligned}
&|\varphi\rangle_{12}\langle\varphi|\Psi^-\rangle_{24}\langle\Psi^-|\Psi^+\rangle_{13}\langle\Psi^+|\Psi\rangle \\
&= \frac{1}{4\sqrt{2}}|\varphi\rangle_{12} \otimes |\phi_\perp\rangle_{34} \otimes (I_5 \otimes (\sigma_z)_6)|\phi\rangle_{56}.
\end{aligned} \qquad (9)$$

It is clear from Eq. (9) that Alice gets a complement copy of the unknown state. From Eq. (7), if the results of Victor are $|\psi\rangle_{12}$ and $|\psi_\perp\rangle_{12}$, Eq. (5) can be written respectively as

$$\begin{aligned}
|\psi\rangle_{12}\langle\psi|\Psi^-\rangle_{24}\langle\Psi^-|\Psi^+\rangle_{13}\langle\Psi^+|\Psi\rangle &= \frac{1}{4\sqrt{2}}|\psi\rangle_{12} \\
&\otimes (I_3 \otimes (\sigma_x)_4)|\phi_\perp\rangle_{34} \otimes (I_5 \otimes (\sigma_z)_6)|\phi\rangle_{56},
\end{aligned} \qquad (10)$$

$$\begin{aligned}
|\psi_\perp\rangle_{12}\langle\psi_\perp|\Psi^-\rangle_{24}\langle\Psi^-|\Psi^+\rangle_{13}\langle\Psi^+|\Psi\rangle &= -\frac{1}{4\sqrt{2}}|\psi_\perp\rangle_{12} \\
&\otimes (I_3 \otimes (\sigma_x)_4)|\phi\rangle_{34} \otimes (I_5 \otimes (\sigma_z)_6)|\phi\rangle_{56},
\end{aligned} \qquad (11)$$

From Eqs.(10) and (11), one can see that Alice gets a copy and a complement copy (all are up to doing a rotation operation) of the unknown state, respectively.

In the process of teleportation, if the Alice's measurement outcomes are other seven entangled states $|\Phi^\pm\rangle_{13}|\Phi^\pm\rangle_{24}$, $|\Psi^\pm\rangle_{13}|\Psi^+\rangle_{24}$ and $|\Psi^-\rangle_{13}|\Psi^-\rangle_{24}$, applying the same analysis method as above, Alice will obtain a copy (or complement copy) of the unknown state at her place.

Now, we will generalize the above scheme for producing more copies or complement copies using two four-particle GHZ states. Suppose that the unknown input state of Alice from Victor is still the two-particle entangled state $|\phi\rangle_{12} = \alpha|00\rangle_{12} + \beta|11\rangle_{12}$. The two four-particle GHZ states, as the quantum channels, are given by

$$\begin{aligned}
|\phi\rangle_{3456} &= \frac{1}{\sqrt{2}}(|0011\rangle + |1100\rangle)_{3456}, \\
|\phi\rangle_{789,10} &= \frac{1}{\sqrt{2}}(|0011\rangle + |1100\rangle)_{789,10}.
\end{aligned} \qquad (12)$$

Here Alice possesses particles 3 and 4, Bob possesses particles 5, 7, 8 and 9, and Carla possesses particles 6 and 10. The input state $|\phi\rangle_{12}$ is unknown to Alice, Bob and Carla. The combined ten-particle state is expressed as

$$\begin{aligned}
|\Phi\rangle &= |\phi\rangle_{12} \otimes |\phi\rangle_{3456} \otimes |\phi\rangle_{789,10} \\
&= |\Phi_{(1)}\rangle + |\Phi_{(2)}\rangle,
\end{aligned} \qquad (13)$$

where

$$\begin{aligned}
|\Phi_{(1)}\rangle = \frac{1}{4}|\Phi^\pm\rangle_{13}|\Phi^\pm\rangle_{24}\cdot(&\alpha|110011\rangle + \alpha|111100\rangle \pm\pm \\
&\beta|000011\rangle \pm\pm\beta|001100\rangle)_{56789,10},
\end{aligned} \qquad (14)$$

$$\begin{aligned}
|\Phi_{(2)}\rangle = \frac{1}{4}|\Psi^\pm\rangle_{13}|\Psi^\pm\rangle_{24}\cdot(&\alpha|000011\rangle + \alpha|001100\rangle \pm\pm \\
&\beta|110011\rangle \pm\pm\beta|111100\rangle)_{56789,10}.
\end{aligned} \qquad (15)$$

In the Eqs. (14) and (15), the notes "$\pm$" in the column from right to left correspond to the Bell state of particles (1, 3) and (2, 4), respectively. Now let Alice carries out Bell-state measurements on her particles (1, 3) and (2, 4), respectively. If the result of Alice's measurement is $|\Psi^+\rangle_{13}|\Psi^-\rangle_{24}$ (the probability of this result is only 1/8), then the resulting state will be

$$\begin{aligned}
&|\Psi^-\rangle_{24}\langle\Psi^-|\Psi^+\rangle_{13}\langle\Psi^+|\Phi\rangle \\
&= \frac{1}{4}|\Psi^+\rangle_{13}|\Psi^-\rangle_{24}(\alpha|000011\rangle + \alpha|001100\rangle \\
&\quad -\beta|110011\rangle - \beta|111100\rangle)_{56789,10} \\
&= |\Phi'_{(1)}\rangle + |\Phi'_{(2)}\rangle,
\end{aligned} \qquad (16)$$

where

$$\left|\Phi'_{(1)}\right\rangle = \frac{1}{8}\left|\Psi^+\right\rangle_{13}\left|\Psi^-\right\rangle_{24}\left|\Phi^\pm\right\rangle_{57}\left|\Psi^\pm\right\rangle_{89}(\alpha\left|01\right\rangle - \pm\pm\beta\left|10\right\rangle)_{6,10}\,, \quad (17)$$

$$\left|\Phi'_{(2)}\right\rangle = \frac{1}{8}\left|\Psi^+\right\rangle_{13}\left|\Psi^-\right\rangle_{24}\left|\Psi^\pm\right\rangle_{57}\left|\Psi^\pm\right\rangle_{89}(\pm\alpha\left|00\right\rangle - \pm\beta\left|11\right\rangle)_{6,10}\,. \quad (18)$$

In the Eqs. (17) and (18), the notes " ± " in the column from right to left correspond to the Bell state of particles (5, 7; 8, 9) and (8, 9; 5, 7), respectively. After the above measurements, Alice sends her result via a classical channel with four bits of information to both Bob and Carla. In the next step, Bob performs another Bell-state measurements on his particles (5, 7) and (8, 9), respectively. If the result of Bob's measurement is $|\Psi^-\rangle_{57}|\Psi^+\rangle_{89}$, the resulting state (the probability of this result is only 1/64) will be written as

$$\left|\Psi^+\right\rangle_{89}\left\langle\Psi^+|\Psi^-\right\rangle_{57}\left\langle\Psi^-|\Psi^-\right\rangle_{24}\left\langle\Psi^-|\Psi^+\right\rangle_{13}\left\langle\Psi^+|\Phi\right\rangle = \frac{1}{8}\left|\Psi^+\right\rangle_{13}\left|\Psi^-\right\rangle_{24}\left|\Psi^-\right\rangle_{57}\left|\Psi^+\right\rangle_{89}\left|\phi\right\rangle_{6,10}\,. \quad (19)$$

From it, one can see that the state of particles 6 and 10 of Carla is found to be in the original state. In the second stage of our protocol, Alice and Bob send particles (1, 2) and (5, 8) to Victor, respectively. When Victor gets the particles (1, 2) and (5, 8), he chooses to measure the states in the basis $\left\{|\varphi\rangle_{ij}, |\varphi_\perp\rangle_{ij}, |\psi\rangle_{ij}, |\psi_\perp\rangle_{ij}\right\}$, which is given by

$$\begin{aligned}
|\varphi\rangle_{ij} &= \alpha\,|00\rangle_{ij} + \beta\,|11\rangle_{ij}\,,\\
|\varphi_\perp\rangle_{ij} &= \beta^*\,|00\rangle_{ij} - \alpha\,|11\rangle_{ij}\,,\\
|\psi\rangle_{ij} &= \alpha\,|01\rangle_{ij} + \beta\,|10\rangle_{ij}\,,\\
|\psi_\perp\rangle_{ij} &= \beta^*\,|01\rangle_{ij} - \alpha\,|10\rangle_{ij}\,.
\end{aligned} \quad (20)$$

where $i, j = 1, 2$ or $5, 8$. In the new basis, the total state can be written as

$$\begin{aligned}
&\left|\Psi^+\right\rangle_{89}\left\langle\Psi^+|\Psi^-\right\rangle_{57}\left\langle\Psi^-|\Psi^-\right\rangle_{24}\left\langle\Psi^-|\Psi^+\right\rangle_{13}\left\langle\Psi^+|\Phi\right\rangle\\
&= \frac{1}{32}[|\varphi\rangle_{12}(\alpha\,|11\rangle_{34} - \beta^*\,|00\rangle_{34}) + |\varphi_\perp\rangle_{12}(\alpha\,|00\rangle_{34}\\
&\quad +\beta\,|11\rangle_{34}) + |\psi\rangle_{12}(-\alpha\,|10\rangle_{34} + \beta^*\,|01\rangle_{34}) - |\psi_\perp\rangle_{12}\\
&\quad (\alpha\,|01\rangle_{34} + \beta\,|10\rangle_{34})]\cdot[|\varphi\rangle_{58}(\alpha\,|11\rangle_{79} - \beta^*\,|00\rangle_{79})\\
&\quad + |\varphi_\perp\rangle_{58}(\alpha\,|00\rangle_{79} + \beta\,|11\rangle_{79}) + |\psi\rangle_{58}(\alpha\,|10\rangle_{79}\\
&\quad -\beta^*\,|01\rangle_{79}) + |\psi_\perp\rangle_{58}(\alpha\,|01\rangle_{79} + \beta\,|10\rangle_{79})]\cdot|\phi\rangle_{6,10}\,.
\end{aligned} \quad (21)$$

Assume Victor first carries out a two-particle projective measurement on the particles (1, 2) and then on particles (5, 8) and in both cases let the results be $|\varphi_\perp\rangle_{12}$ and $|\varphi_\perp\rangle_{58}$. Then Victor sends the classical information (two bits) to Alice and (two bits) to Bob. According to the information of Victor, Alice and Bob can find that their particles (3, 4) and (7, 9) are in the unknown state, respectively. Thus, the final state after two-particle projective measurements is given by

$$\begin{aligned}
&|\varphi_\perp\rangle_{12}\langle\varphi_\perp|\varphi_\perp\rangle_{58}\left\langle\varphi_\perp|\Psi^+\right\rangle_{89}\left\langle\Psi^+|\Psi^-\right\rangle_{57}\\
&\left\langle\Psi^-|\Psi^-\right\rangle_{24}\left\langle\Psi^-|\Psi^+\right\rangle_{13}\left\langle\Psi^+|\Phi\right\rangle\\
&= \frac{1}{32}|\varphi_\perp\rangle_{12}\otimes|\phi\rangle_{34}\otimes|\varphi_\perp\rangle_{58}\otimes|\phi\rangle_{79}\otimes|\,phi\rangle_{6,10}\,.
\end{aligned} \quad (22)$$

It is clear that Alice, Bob and Carla each get a perfect copy of the unknown state. If Victor's outcome for particles (1, 2) and (5, 8) are $|\psi\rangle_{12}$ and $|\psi\rangle_{58}$, the final state is given by

$$\begin{aligned}
&|\psi\rangle_{12}\langle\psi|\psi\rangle_{58}\left\langle\psi|\Psi^+\right\rangle_{89}\left\langle\Psi^+|\Psi^-\right\rangle_{57}\\
&\left\langle\Psi^-|\Psi^-\right\rangle_{24}\left\langle\Psi^-|\Psi^+\right\rangle_{13}\left\langle\Psi^+|\Phi\right\rangle\\
&= -\frac{1}{32}|\psi\rangle_{12}\otimes(I_3\otimes(\sigma_x)_4)\,|\phi_\perp\rangle_{34}\otimes|\psi\rangle_{58}\\
&\quad\otimes(I_7\otimes(\sigma_x)_9)\,|\phi_\perp\rangle_{79}\otimes|\phi\rangle_{6,10}\,.
\end{aligned} \quad (23)$$

After sending two classical bits to Alice and two to Bob from Victor, Alice knows that her state of particles 3 and 4 has been found in the complement copy of the unknown state (up to a rotation operator), and Bob gets a complement copy for his particles 7 and 9 (up to a rotation operator, too), and Carla gets the copy of the unknown state. If what Victor measure is another outcome for particles (1, 2) and (5, 8), with Eq. (21) Alice and Bob can acquire a perfect copy or a complement copy (up to a rotation operator) of the unknown state.

In conclusion, we have proposed a protocol which can produce perfect copies or orthogonal-complement copies of an arbitrary unknown two-particle entangled state, via quantum and classical channel, with assistance. Our protocol requires resources such as a four-particle GHZ state (or two four-particle GHZ states) as quantum channel(s), Bell-state measurement, classical communication, and two-particle projective measurement. This protocol includes two stages. The first stage of the protocol requires usual teleportation. In the second stage, Victor (a preparer of state) will perform two-particle projective measurements on particles which from Alice and Bob. According to information from Victor, Alice and Bob can acquire either a perfect copy or an orthogonal-complement copy of unknown state.

This work was supported by Naturel Science Foundation of Education Bureau of Jiangsu Province of China under Grant No. 04KJB140014. Y. Zhan's email is ybzhan@hytc.edu.cn.

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