Effect of higher order non-linearity in frequency variation of self-phase modulation in optical fiber communication

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Received May 28, 2004

In optical soliton propagation through a single mode optical fiber, it is established that self-phase modulation is maintained by the third order non-linearity of the silica-based glass material of the fiber. In this paper we show that the fifth order non-linearity has also some contribution in frequency variation of self-phase modulation.

OCIS codes: 060.4370, 060.4510, 060.5530, 190.5530.

In digital communication optical soliton has proved its strong candidature in signal communication because of the absence of chirping phenomenon and temporal broadening. In soliton communication the pulses broaden neither in time domain nor in frequency domain[1−4]. The broadening in time domain and the broadening in frequency domain are mutually compensated. It is well known that self-phase modulation (SPM) has a very strong contribution in such compensation. SPM is created because of the third order non-linearity of the silica-based glass material of the optical fiber[5]. As this material is non-crystalline because of its nature, the first order non-linearity is present, whereas the second order non-linearity is absent. In the same way only the third order and fifth order non-linearities are present because of the isotropic symmetry nature of the fiber material. We generally know that even to observe the effect of the third order non-linearity, we require a sizable power of the light input in the fiber and therefore the effect of the fifth order non-linearity is rarely seen. Ghatak *et al.* showed the effect of the fifth order non-linearity in refractive index on Gaussian pulse propagation through a lossy optical fiber and in some other specific cases[6]. They showed theoretically that within a certain limited range of the optical fiber pulse compression and decompression will happen in the case of soliton.

In this letter, we show that like the third order non-linearity, the other higher order (for example fifth order) non-linearities may also take a sizable role in frequency variation of self-phase modulation. The effect of the fifth order non-linearity can lead to a frequency variation, which may not be managed by dispersion for proper SPM.

In a linear medium, the electric polarization is assumed to be a linear function of the electric field. In high optical field intensity, the equation of polarization (P) in a nonlinear media becomes

$$P = \varepsilon_0\chi^{(1)}E + \varepsilon_0\chi^{(2)}E^2 + \varepsilon_0\chi^{(3)}E^3 + \varepsilon_0\chi^{(4)}E^4 + \varepsilon_0\chi^{(5)}E^5, \quad (1)$$

where $\chi^{(1)}$ is the linear dielectric susceptibility, $\chi^{(2)}$, $\chi^{(3)}$, $\chi^{(4)}$, and $\chi^{(5)}$ are the second, third, fourth, and fifth order nonlinear susceptibilities, respectively. In silica-based isotropic optical fiber waveguide $\chi^{(2)} = 0$ and $\chi^{(4)} = 0$.

Here the lowest order non-linearity is due to $\chi^{(3)}$.

Therefore, if the higher order non-linearities above $\chi^{(3)}$ are neglected, then

$$P = \varepsilon_0\chi^{(1)}E + \varepsilon_0\chi^{(3)}E^3. \quad (2)$$

Now we take a plane optical wave with an electric field variation of the form of

$$E = E_0\cos(\omega t - kz). \quad (3)$$

Then Eq. (2) becomes

$$P = \varepsilon_0\chi^{(1)}E_0\cos(\omega t - kz) + \varepsilon_0\chi^{(3)}E_0^3\cos^3(\omega t - kz). \quad (4)$$

By trigonometric simplification, the above expression can be written as

$$P = \varepsilon_0\left\{\chi^{(1)} + \frac{3}{4}\chi^{(3)}E_0^2\right\}E_0\cos(\omega t - kz) + \frac{1}{4}\varepsilon_0\chi^{(3)}E_0^3\cos 3(\omega t - kz). \quad (5)$$

Now neglecting the third harmonic generation of frequencies due to phase mismatching, we can get Eq. (5) as

$$P = \varepsilon_0\left\{\chi^{(1)} + \frac{3}{4}\chi^{(3)}E_0^2\right\}E_0\cos(\omega t - kz). \quad (6)$$

Here the intensity of the used optical wave is

$$I = \frac{1}{2}c\varepsilon_0 n_0 E_0^2. \quad (7)$$

Therefore the polarization of Eq. (6) can be written as

$$P = \varepsilon_0\left\{\chi^{(1)} + \frac{3}{2}\chi^{(3)}\frac{I}{(c\varepsilon_0 n_0)}\right\}E_0\cos(\omega t - kz), \quad (8)$$

where n_0 is the refractive index of the medium at low field. The general relationship between polarization and refractive index is given by

$$P = \varepsilon_0(n^2 - 1)E_0\cos(\omega t - kz). \quad (9)$$

Comparing Eq. (8) with Eq. (9), we get

$$n^2 = 1 + \chi^{(1)} + \frac{3}{2}\chi^{(3)}\frac{I}{(c\varepsilon_0 n_0)}, \tag{10a}$$

and therefore

$$n = n_0 + \frac{3}{4}\chi^{(3)}\frac{I}{(c\varepsilon_0 n_0^2)} = n_0 + n_2 I, \tag{10b}$$

where

$$n_0 = \left\{1 + \chi^{(1)}\right\}^{\frac{1}{2}}, \tag{10c}$$

and the nonlinear third order correction term is

$$n_2 = \frac{3}{4}\frac{\chi^{(3)}}{(c\varepsilon_0 n_0^2)}. \tag{10d}$$

If P_t is the power carried by a mode in the optical fiber, that is, the power carried by the used plane optical wave, and A_{eff} is the effective core area of the optical fiber, the intensity I becomes

$$I = \frac{P_t}{A_{\text{eff}}}. \tag{11}$$

By using Eq. (11) in Eq. (10b), we get

$$n = n_0 + n_2\frac{P_t}{A_{\text{eff}}}. \tag{12}$$

Now, propagation constant (β_0 is propagation constant for linear case) of the above wave may be approximately written as

$$\beta = \beta_0 + k_0 n_2\frac{P_t}{A_{\text{eff}}}. \tag{13}$$

Here an incident wave of the form $E(0,t) = A\exp(i\omega_0 t)$ would emerge as $E(z,t) = A\exp\{i(\omega_0 t - \beta z)\}$, which is equivalent to

$$E(z,t) = A\exp\left\{i\left(\omega_0 t - \beta_0 z - k_0 n_2 P_t\frac{z}{A_{\text{eff}}}\right)\right\}, \tag{14}$$

where z is the distance of the optical fiber from the input side where the frequency variation is observed. If the input wave is a pulse with a power variation given by $P_t(t)$, the phase part of the outlet wave would be $\exp\left[i\left\{\omega_0 t - \beta_0 z - k_0 n_2 P_t(t)\frac{z}{A_{\text{eff}}}\right\}\right]$. Then instantaneous frequency within the pulse at z is

$$\omega(t) = \omega_0 - \left(k_0 n_2\frac{z}{A_{\text{eff}}}\right)\frac{\mathrm{d}P_t}{\mathrm{d}t}. \tag{15}$$

Therefore frequency shift is

$$\Delta\omega = \left(k_0 n_2\frac{z}{A_{\text{eff}}}\right)\frac{\mathrm{d}P_t}{\mathrm{d}t}, \tag{16}$$

where ω is the angular frequency of the plane optical wave passing through optical fiber. At very high optical field intensity, some nonlinear media may adopt higher order non-linearity.

For such non-crystalline media $\chi^{(2)}$ and $\chi^{(4)}$ are considered zero, therefore the lowest order non-linearity arises due to $\chi^{(3)}$ as shown above. Although the value of $\chi^{(5)}$ is very smaller in comparison with $\chi^{(3)}$, still we can consider $\chi^{(5)}$ in some specific cases.

We can use Eq. (3) in Eq. (1) to get

$$P = \varepsilon_0\chi^{(1)}E_0\cos(\omega t - kz) + \varepsilon_0\chi^{(3)}E_0^3\cos^3(\omega t - kz) + \varepsilon_0\chi^{(5)}E_0^5\cos^5(\omega t - kz). \tag{17}$$

By trigonometric simplification, the expression of Eq. (17) must be written as

$$\begin{aligned} P = {} & \varepsilon_0\chi^{(1)}E_0\cos(\omega t - kz) \\ & + \varepsilon_0\chi^{(3)}E_0^3\left\{\frac{1}{4}\cos 3(\omega t - kz) + \frac{3}{4}\cos(\omega t - kz)\right\} \\ & + \varepsilon_0\chi^{(5)}E_0^5\left[\frac{1}{8}\cos 3(\omega t - kz) + \frac{3}{8}\cos(\omega t - kz)\right. \\ & + \frac{1}{8}\cos 3(\omega t - kz)\cos 2(\omega t - kz) \\ & \left. + \frac{3}{8}\cos(\omega t - kz)\cos 2(\omega t - kz)\right], \end{aligned} \tag{18}$$

or

$$\begin{aligned} P = {} & \varepsilon_0\chi^{(1)}E_0\cos(\omega t - kz) \\ & + \varepsilon_0\chi^{(3)}E_0^3\left\{\frac{1}{4}\cos 3(\omega t - kz) + \frac{3}{4}\cos(\omega t - kz)\right\} \\ & + \varepsilon_0\chi^{(5)}E_0^5\left[\frac{1}{8}\cos 3(\omega t - kz) + \frac{3}{8}\cos(\omega t - kz)\right. \\ & + \frac{1}{16}\cos 5(\omega t - kz) + \frac{1}{16}\cos(\omega t - kz) \\ & \left. + \frac{3}{16}\cos 3(\omega t - kz) + \frac{3}{16}\cos(\omega t - kz)\right]. \end{aligned} \tag{19}$$

Now neglecting the third and fifth harmonic generations of frequencies due to phase mismatching, we get Eq. (19) as

$$P = \varepsilon_0\chi^{(1)}E_0\cos(\omega t - kz) + \frac{3}{4}\varepsilon_0\chi^{(3)}E_0^3\cos(\omega t - kz) + \frac{5}{8}\varepsilon_0\chi^{(5)}E_0^5\cos(\omega t - kz). \tag{20}$$

Using Eq. (7) in Eq. (20), the polarization can be written as

$$P = \varepsilon_0\left[\chi^{(1)} + \frac{3}{2}\chi^{(3)}\frac{I}{(c\varepsilon_0 n_0)} + \frac{5}{2}\chi^{(5)}\frac{I^2}{(c^2\varepsilon_0^2 n_0^2)}\right] \times E_0\cos(\omega t - kz). \tag{21}$$

Comparing general relationship of refractive index with polarization, that is, Eq. (9) with Eq. (21), we get

$$n^2 = 1 + \chi^{(1)} + \frac{3}{2}\chi^{(3)}\frac{I}{(c\varepsilon_0 n_0)} + \frac{5}{2}\chi^{(5)}\frac{I^2}{(c^2\varepsilon_0^2 n_0^2)}, \quad (22)$$

and therefore

$$n = n_0 + \frac{3}{4}\chi^{(3)}\frac{I}{(c\varepsilon_0 n_0^2)} + \frac{5}{4}\chi^{(5)}\frac{I^2}{(c^2\varepsilon_0^2 n_0^3)}. \quad (23)$$

So finally the refractive index in this case can be written as

$$n = n_0 + n_2 I + n_3 I^2, \quad (24)$$

where, $n_0 = \{1 + \chi^{(1)}\}^{\frac{1}{2}}$, $n_2 = \frac{3}{4}\frac{\chi^{(3)}}{(c\varepsilon_0 n_0^2)}$, and $n_3 = \frac{5}{4}\frac{\chi^{(5)}}{(c^2\varepsilon_0^2 n_0^3)}$, n_2 and n_3 are the third and fifth order nonlinear correction terms, respectively.

Now using Eq. (11) in Eq. (24), one may get

$$n = n_0 + n_2\frac{P_t}{A_{\rm eff}} + n_3\frac{P_t^2}{A_{\rm eff}^2}. \quad (25)$$

So the ultimate propagation constant for the wave in the media may be approximately written as

$$\beta = \beta_0 + k_0 n_2\frac{P_t}{A_{\rm eff}} + k_0 n_3\frac{P_t^2}{A_{\rm eff}^2}. \quad (26)$$

Hence the electric field of the incident wave of the form of Eq. (14) becomes

$$E(z,t) = A\exp\left[i\left\{\omega_0 t - \beta_0 z - k_0 n_2 P_t(t)\frac{z}{A_{\rm eff}} - k_0 n_3 P_t^2\frac{z}{A_{\rm eff}^2}\right\}\right], \quad (27)$$

where z and t signify distance and time scales, respectively.

If the input wave is a pulse with a power shape function $P_t(t)$, its phase-dependent part at z distance is $\exp\left[i\left\{\omega_0 t - \beta_0 z - k_0 n_2 P_t(t)\frac{z}{A_{\rm eff}} - k_0 n_3 P_t^2\frac{z}{A_{\rm eff}^2}\right\}\right]$, and the angular frequency shift in the pulse at z is

$$\Delta\omega = (k_0 n_2\frac{z}{A_{\rm eff}})\frac{{\rm d}P_t}{{\rm d}t} + (k_0 n_3\frac{z}{A_{\rm eff}^2})2P_t\frac{{\rm d}P_t}{{\rm d}t}. \quad (28)$$

Here it is interesting to note that if the fifth order nonlinearity is considered, then in the frequency variation the additional term $(k_0 n_3\frac{z}{A_{\rm eff}^2})2P_t\frac{{\rm d}P_t}{{\rm d}t}$ is found with a very low value of n_3. Undoubtedly at long distance (z), considerable power (P_t), and its considerable variation of power $\frac{{\rm d}P_t}{{\rm d}t}$, the composite term $(k_0 n_3\frac{z}{A_{\rm eff}^2})2P_t\frac{{\rm d}P_t}{{\rm d}t}$ is necessarily significant. Therefore we can see that the role of the fifth order non-linearity cannot be ignored if $\frac{{\rm d}P_t}{{\rm d}t}$ is high, which is for a very fast variation of power with time. It can be again concluded that even seventh order non-linearity may affect the frequency variation if the power variation is significantly large in the power shape function of the wave envelope in the nonlinear medium. Therefore the SPM should be managed at a different way if the fifth order non-linearity is included.

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References

1. M. Romagnoli, I. Cristiani, and P. Franco, IEICE Trans. Electron. **E81,** 208 (1998).
2. I. Cristiani, P. Franco, M. Midrio, and M. Romagnoli, Opt. Commun. **146,** 241 (1998).
3. S. K. Turitsyn and E. G. Shapiro, Opt. Lett. **23,** 682 (1998).
4. K. Prozean, Proc. Indian National Science Academy (PINSA) **A67,** 609 (2001).
5. G. P. Agarwal, *Applications of Nonlinear Fibre Optics* (Academic Press, New York, 2001).
6. A. Kumar, S. N. Sarkar, and A. K. Ghatak, Opt. Lett. **11,** 321 (1986).