The method to evaluate the position error in graphic positioning technology

Huiqing Lu (卢慧卿), Baoguang Wang (王宝光), Lishuang Liu (刘力双), and Yabiao Li (李亚标)

State Key Laboratory of Precision Measuring Technology and Instruments, Tianjin University, Tianjin 300072

Received March 23, 2004

In the measurement of automobile body-in-white, it has been widely studied to position the two-dimensional (2D) visual sensors with high precision. In this paper a graphic positioning method is proposed, a hollow tetrahedron is used for a positioning target to replace all the edges of a standard automobile body. A 2D visual sensor can be positioned through adjusting two triangles to be superposed on a screen of the computer, so it is very important to evaluate the superposition precision of the two triangles. Several methods are discussed and the least square method is adopted at last, it makes the adjustment more easy and intuitive with high precision.

OCIS codes: 080.2720, 110.1650, 040.1520, 130.6010.

Automobile body-in-white measurement is one of the most important subjects in automobile industry. It is difficult to measure the body of an automobile because its curve is so complicated and there are too many points, lines, and planes to be measured. Now the widely used method is to build a measuring station in the production line and put the two-dimensional (2D) visual sensors (which are called sensors for short hereinafter) in right places according to the important points, lines, and planes. There are more than one hundred sensors that will be positioned in one station, so how to position each sensor quickly and accurately becomes a key subject in automobile industry. "Graphic positioning technology" is proposed in such a background. During the graphic positioning, an ideal position and a real position of the sensor are drawn in a computer screen with two triangles. To adjust the sensor and make the real position be overlapped with the ideal position, we can complete the positioning with high efficiency[1].

The sensor used in graphic positioning is a visual measuring element and a figure sensor, the measuring equation of the sensor is

$$\begin{cases} X_g = 0 \\ Y_g = -\frac{f\sin\beta - D_{pc}}{X_i\cos\beta + f\sin\beta}\cdot Y_i \\ Z_g = \frac{f\sin\beta - D_{pc}}{\sin\beta\cdot(X_i\cos\beta + f\sin\beta)}\cdot X_i \end{cases}$$

A laser sheet emits to the measured area, the charge-coupled device (CCD) camera (MTV1881CB, pixels 795(H)×596(V)) is used for receiving the scattered light to obtain the profile or sectional shape of the measured object[2,3].

A hollow tetrahedron is used for a positioning target to replace all the edges of a standard automobile body, so it is possible to position sensors without a real body. In Fig. 1, the edge AB is used to replace an edge in standard automobile body. In practice, the light emitting diodes (LEDs) are put on the peaks of the tetrahedron, and two optical theodolites are used to attain the LEDs' locations, and then the tetrahedron is positioned according to the car drawing[4]. During the operation, we can previously suppose a triangle on the tetrahedron according to principal coordinates, just as triangle abc shown in Fig. 1. Then compute the corresponding triangle's coordinate values in the image plane $X_i - Y_i$ and draw it on the computer screen, just like the broken line triangle $a'b'c'$ shown in Fig. 2, which represents the ideal position of the sensor. Then we can adjust the sensor to form a real triangle in the three edges of the tetrahedron by the laser beam of the sensor, just as the triangle lmn shown in Fig. 1, this triangle represents the current position of the sensor. The coordinate values of this triangle's three apexes can be got by image processing, then we can draw the real triangle in the computer screen, just like the real line triangle $l'm'n'$ shown in Fig. 2. With the movement of the sensor, triangle lmn and its image triangle $l'm'n'$ will change accordingly. When the triangles are overlapped in the screen, the sensor is in the ideal position. Because there is the position error of the graphics in the computer screen, a proper method to evaluate the superposition precision is necessary.

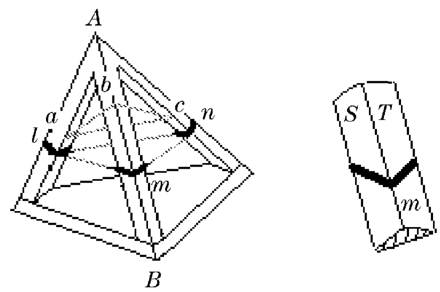

Fig. 1. Structure of the hollow tetrahedron and edge AB.

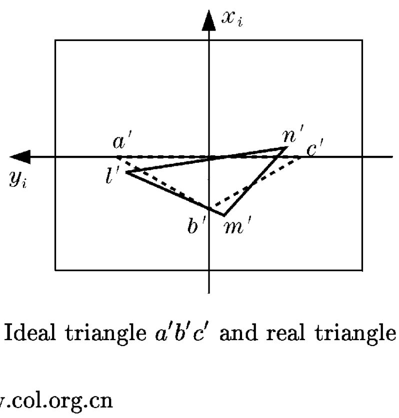

Fig. 2. Ideal triangle $a'b'c'$ and real triangle $l'm'n'$.

The superposition of two triangles is three edges' superposition in vision, virtually it is the superposition of the triangles' apexes. Only when three apexes l', m', and n' are all overlapped with three apexes a', b', and c' in screen, the triangles' figure, size, and position are coincident, and the two triangles are superposed. So, the coincidence error is not format error but position error[5].

For a point whose ideal position is known in the 2D plane, its allowable tolerance is a circle whose center lies in the ideal position. So in the positioning, the allowable tolerance of the real triangle's corresponding apex is a circle with a certain semidiameter (r pixels). When the distances of the corresponding apexes of the ideal triangle and the real triangle are less than r or equal to r, the real triangle's corresponding apexes are in the allowable tolerances, i.e. all the single factors in the triangle's superposition are in their allowable tolerances.

In order to evaluate the position error of triangle's superposition accurately, it is not enough only using the position error of the single factor, because it cannot show the holistic superposition. So a composite method for evaluating the superposition is necessary.

The area of the misaligned part can be used to evaluate the position error of two triangles' superposition, this method is so-called the least area method. The distance of a point in the real triangle to the ideal triangle's corresponding edge is the absolute value of declination of the point, when the point is in the ideal triangle, the sign of the declination is minus, else the sign of the declination is plus. Adjusting the 2D visual sensor, the status is best when the two misaligned areas of inside and outside the ideal triangle are equal and the sum of the areas reaches the minimum. The margin between the most declination and the least declination is used to evaluate the coincidence error. In Fig. 3, the deep gray part is the misaligned part inside the ideal triangle, and the light gray part is the outside one. Yet, the intersection of two triangles on same plane is very complicated, the number of the points of intersection is different, and the points perhaps lie in different sides. Thus the design of arithmetic becomes very complex.

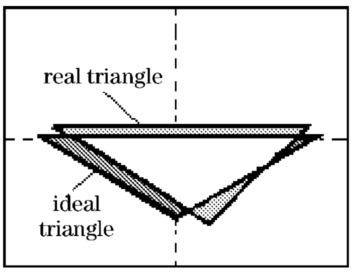

Fig. 3. The least area method.

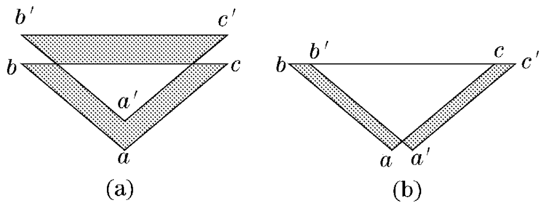

Fig. 4. The distance method cannot reflect the error.

The least distance between apexes can also be used to evaluate the position error of the triangles' superposition by the sum of the square of the distance between the ideal and real triangles' corresponding apexes. Adjusting the 2D visual sensor, the status is best when the sum of the square of the distance between corresponding apexes reaches the minimum. The maximal distance between the apexes is used to evaluate the coincidence error. The position error obtained by this means is

$$\begin{cases} aa'^2 + bb'^2 + cc'^2 = \min \\ E = \max(aa', bb', cc') \end{cases} \tag{1}$$

For the two forms shown in Fig. 4, the distances between the corresponding apexes are equal, but the areas of the misaligned parts are quite different. Evaluated by the distance method, the error of the form of Fig. 4(a) is much more than that of the form of Fig. 4(b), so this method cannot reflect the two triangles' coincidence error.

To avoid the disadvantages of the above methods, the least square method is put forward in this paper. Firstly, the word "residual" is defined as follows: every edge of the ideal triangle and real triangle is divided into n equal parts and then n points can be got from each edge, two points in the same position of the two triangles form a couple of points, for each couple of points, the distance between the two points is the absolute value of this couple of points' residual. For edge ab and edge ac of ideal triangle, if the point in ideal triangle is above the point in real triangle, the sign of this couple of points' residual is plus, else the sign is minus; For edge bc of ideal triangle, the definition of the sign is reverse. Defining the sign in this way can accord with the least area method. In Fig. 5, the couples of points' residuals are L_1, L_2, $\cdots$, L_n, M_1, M_2, $\cdots$, M_n, N_1, N_2, $\cdots$, N_n.

Adjusting the 2D visual sensor until the sum of all the couples of points' residuals is zero and the sum of the square of the residuals reaches the minimum, then we can think that the real triangle $l'm'n'$ is overlapped with the ideal triangle $a'b'c'$. The formula is

$$\begin{cases} \sum\limits_{i=1}^{n} (L_i + M_i + N_n) = 0 \\ \sum\limits_{i=1}^{n} (L_i^2 + M_i^2 + N_i^2) = \min \end{cases}. \tag{2}$$

The margin between the most residual and the least residual of all the couples of points is used to evaluate

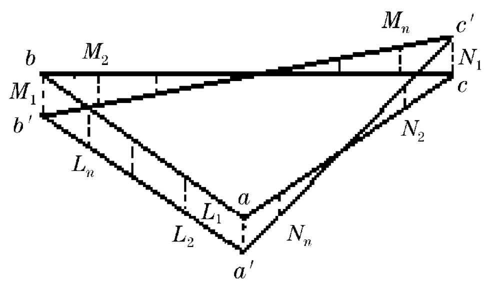

Fig. 5. The least square method.

Table 1. Experimental data

	First Location	Second Location
Axis of Containing Cylinder	$\begin{cases} x = 0.088t - 44.984 \\ y = -0.996t + 280.0 \\ z = -0.010t - 355.944 \end{cases}$	$\begin{cases} x = 0.089t - 39.841 \\ y = -0.996t + 260.0 \\ z = -0.009t - 353.244 \end{cases}$
Radius of Containing Cylinder	0.0495 mm	0.0486 mm
Error	0.0990 mm	0.0972 mm

the coincidence error, the formula is

$$\begin{aligned} E = &\max(L_1, L_2, \cdots, L_n, M_1, M_2, \cdots, M_n, \\ &N_1, N_2, \cdots, N_n) \\ &-\min(L_1, L_2, \cdots, L_n, M_1, M_2, \cdots, M_n, \\ &N_1, N_2, \cdots N_n). \end{aligned} \quad (3)$$

Compared with the above methods, the calculation by the least square method is much complex, but it reflects the triangles' superposition effectively through getting points in the triangle's sides. Furthermore, a certain formula can be transferred, which is very convenient to process by computer. Therefore, among the three methods discussed in this paper, the least square method is the fittest to evaluate the position error.

In fact, in order to simplify the process, the single factor and the general factor can be used together. Let all the triangle's apexes be in their allowable tolerances (two triangles are near superposition), and then use the least residual method to evaluate the superposition. This will be useful in practice.

To test the precision of the method, we put both the tetrahedron and the sensor in a coordinate measuring machine (CMM) and keep the position of sensor fixed, and then adjust the tetrahedron and make the real triangle be overlapped with the ideal triangle in screen. After superposition, the two planes of a gauge pasted on the tetrahedron are measured in a CMM and their equations are worked out. The line equation of the edge which is formed by the intersection of those two planes is deduced. Keeping the sensor fixed and repeat the above processes for several times, we can get several equations of the line of the intersection. The diameter of the cylinder which can contain all the lines is the precision of the virtual positioning technology. Placing the sensor in two different positions and repeat the above experiment, taking the maximin error of these two experiment, we can get the result of the test. The experimental data are listed in Table 1, from which we can see that the precision of the method is 0.1 mm.

H. Lu's e-mail address is luhuiqing4532@163.com.

References

1. D. H. Zou, *Research on 3D Vision Inspection and Its Application in ADC station* (in Chinese) Ph.D dissertation (Tianjin University, 1992) p. 123.
2. B. G. Wang, Z. H. He, L. C. Chen, and Y. Ni, Acta Optica Sinica (in Chinese) **22,** 481 (2002).
3. B. G. Wang, Z. H. He, Y. B. Liao, and L. C. Chen, Chinese J. Lasers **B11,** 198 (2002).
4. B. G. Wang and M. Z. Zhu, Chin. J. Mechanical Engineering **10,** 136 (1997).
5. Z. E. Liu, *The Examination of the Shape and Location Error* (in Chinese) (Publishing House of the University of Science and Technology of Beijing, Beijing, 1988) pp. 1 – 50.