Average modulation transfer function of line-array fiber-optic image bundles

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The image quality evaluation in fiber-optic image bundles was addressed by the modulation transfer function (MTF). With the definition of the contrast transfer function (CTF), the MTF model of line-array fiber-optic image bundles was established and analyzed numerically. The average MTF was carefully evaluated by considering the influence of phase match on the MTF between input pattern and fiber-optic image bundles. In this paper, the average MTF is mean arithmetical value on the MTFs of eight different phases. The results show that, for certain fiber diameter and spatial frequency, the relationship between the core diameter and the average MTF is inverse proportion; for certain fiber cladding thickness, the relationship between the core diameter and the average MTF is also inverse proportion. And at Nyquist frequency, the MTF value is near 0.5.

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The modulation transfer function (MTF) of an imaging system is important to the initial specification and design of the system and any image analysis. While line-array fiber-optic image bundles are discrete systems and linear within an operating range[1], they are not space invariant because they have fixed sampling locations. So the average MTF means the average value of the MTFs for different sampling locations[2]. For the MTF evaluation of fiber-optic image bundles, Arefiev[3] measured a central irradiance in the image of variable slit on the bundles and calculated the average MTF by Fourier transformation of line-spread function. Barnard *et al.*[4] derived the average MTFs of both rectangular and hexagonal pixel shapes according as the MTF is defined as the ratio of the spectrum of the output to the spectrum of the input. Player[5] derived spread function and the average MTF with 'long-slit' approximation according as the MTF is Fourier transformation of line-spread function. Conde *et al.*[6] compared the influence of the spatial sampling and the cross-talk on the MTF. The influence of the core diameter on the MTF according to the definition of the contrast transfer function (CTF) is discussed in this paper.

According to the definition of CTF, suppose the pattern distribution projected upon line-array fiber-optic bundles is

$$I(u) = 1 + C_0(f)\cos(2\pi f u + \Phi). \quad (1)$$

After transmission through the fiber-optic bundles, the intensity distribution at the fiber-optic bundles output can be expressed as

$$I'(u) = 1 + C_i(f)\cos(2\pi f u + \Phi'), \quad (2)$$

and the MTF of the fiber-optic bundles is defined as

$$\mathrm{MTF}(f) = \frac{C_i(f)}{C_0(f)}, \quad (3)$$

where[7]

$$C_i(f) = \frac{\bar{I}_{\max} - \bar{I}_{\min}}{\bar{I}_{\max} + \bar{I}_{\min}}. \quad (4)$$

$\bar{I}_{\max}$ and $\bar{I}_{\min}$ are the maximum average and minimum average of fiber-optic sampling respectively. Supposing $C_0(f) = 1$, then we obtain

$$\mathrm{MTF}(f) = \frac{\bar{I}_{\max} - \bar{I}_{\min}}{\bar{I}_{\max} + \bar{I}_{\min}}. \quad (5)$$

As shown in Fig. 1, let Φ be the phase match parameter between the input pattern and the fiber bundles. Φ is the phase offset between the maximum of the input pattern and the bundles, and in the range of $0-2\pi$.

For certain frequency of the input pattern, several $I_{\max}$ and $I_{\min}$ occur by the fiber sampling. Then the averages of the maximum and the minimum values are

$$\bar{I}_{\max} = \frac{1}{n_1}\sum_{i=1}^{n_1} I_{\max i}, \quad \bar{I}_{\min} = \frac{1}{m_1}\sum_{i=1}^{m_1} I_{\min i}, \quad (6)$$

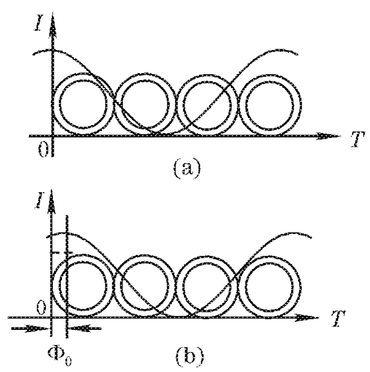

Fig. 1. Phase offset between the input pattern and the bundles for phase offset $\Phi = 0$ (a) and $\Phi = \Phi_0$ (b).

where n_1 is the number of the maximum output and m_1 the number of the minimum output. Substituting Eq. (6) into Eq. (5), we obtain

$$\mathrm{MTF}(\Phi) = \frac{\frac{1}{n_1}\sum_{i=1}^{n_1} I_{\max i} - \frac{1}{m_1}\sum_{i=1}^{m_1} I_{\min i}}{\frac{1}{n_1}\sum_{i=1}^{n_1} I_{\max i} + \frac{1}{m_1}\sum_{i=1}^{m_1} I_{\min i}}. \quad (7)$$

When simulating the MTF of the line-array fiber-optic image bundles as shown in Fig. 2, for a common one-dimensional spatial coordinate system u, the input is the sine intensity distribution $g(u)$, the output is the scanning image $I(u')$ passing through the fiber-optic bundles.

The sine intensity distribution can be written as

$$g(u) = 1 + \cos 2\pi f u, \quad (8)$$

where f is spatial frequency.

With the core diameter $d = 2r$, the cladding thickness b, and the fiber diameter D, there is $D = 2R = 2r + 2b$. With the sampling interval Δu (a invariant value) and the original phase u_0, there is

$$u_i = u_0 + \Delta u \cdot i, \quad (9)$$

u_i is spatial coordinate of the ith Δu and ranges from 0 to $u_0 + n \times D$, n is the number of fiber.

In Fig. 3, the area for the ith Δu is

$$m_i = \int_{(i-1)(\Delta u)}^{i(\Delta u)} \sqrt{R^2 - u^2}\mathrm{d}u, \quad (10)$$

m_i is the shadow area shown in Fig. 3. It is needed to calculate each parallel stripes like Δu, with $a = \frac{R}{\Delta u}$ the stripes number. In intervening space of fiber core, $m_i = 0$. The area of Δu in fiber cladding is zero. $S = \pi r^2$ is the area of fiber and $S = 2\sum_{i=1}^{a} m_i$. M_i is the ratio of each Δu area to the whole cross section of the fiber, and

$$M_i = \frac{2m_i}{S}. \quad (11)$$

g_i is the input sine intensity distribution of the ith Δu, and

$$g_i = 1 + \cos 2\pi f(u_0 + \Delta u \cdot i). \quad (12)$$

Fig. 2. Sketch map of line-array fiber-optic image bundles.

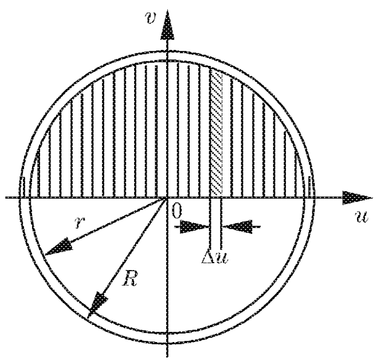

Fig. 3. Sampling sketch map of the fiber-optic.

I_j is the scanning image output of the jth fiber, and

$$I_j = \sum_{i=1+2a(j-1),\ k=1}^{i=2aj,\ k=2a} g_i \cdot M_k. \quad (13)$$

Then the extremum I_m can be determined as

$$I_m = \begin{cases} I_{\min i} = I_j & (I_{j+1} - I_j)(I_j - I_{j-1}) < 0 \text{ and } I_j < I_{j-1} \\ I_{\max i} = I_j & (I_{j+1} - I_j)(I_j - I_{j-1}) < 0 \text{ and } I_j > I_{j-1} \end{cases}. \quad (14)$$

According to Eqs. (6) and (7), MTF value corresponding to the original phase u_0 and the spatial frequency f can be obtained. Taking different original phases, the different MTF is gotten. The average of the tth different MTF is MTF_a, and

$$\mathrm{MTF}_a = \frac{\sum_{i=1}^{t} \mathrm{MTF}_i}{t}. \quad (15)$$

Taking different spatial frequency, the MTF is gotten, and the MTF versus spatial frequency relationship can be fitted with polynomial regression.

When the fiber diameter is identical and the core diameter is different, the average MTF of the bundles is calculated. The simulation results are shown in Figs. 4 and 5. Here we adopted sampling interval $\Delta u = 0.1$ μm, $n = 1000$, and $t = 8$. In Fig. 4, two groups of data were given, they are: $d = 28$ μm, $b = 1$ μm, $D = 30$ μm (the first group); $d = 29$ μm, $b = 0.5$ μm, $D = 30$ μm (the second group). In Fig. 5, also two groups of data were given, they are: $d = 16$ μm, $b = 1$ μm, $D = 18$ μm (the first group); $d = 17$ μm, $b = 0.5$ μm, $D = 18$ μm (the second group). When the cladding thickness is identical and the core diameter is different, the average MTF versus spatial frequency relationship is shown in Fig. 6. Here we adopted $\Delta u = 0.1$ μm, $n = 1000$, and $t = 8$, three groups of data were given, they are: $d = 15$ μm, $b = 0.5$ μm, $D = 16$ μm (the first group); $d = 20$ μm, $b = 0.5$ μm, $D = 21$ μm (the second group); $d = 30$ μm, $b = 0.5$ μm, $D = 31$ μm (the third group).

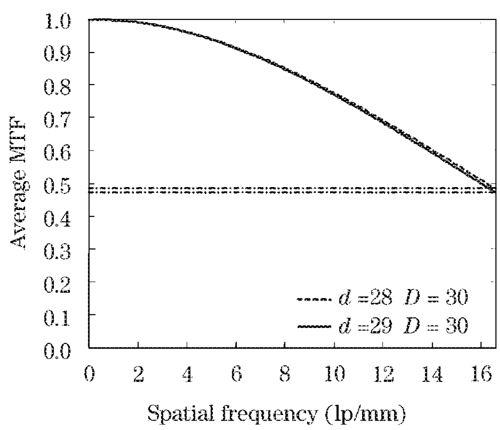

Fig. 4. Relationship of average MTF to spatial frequency.

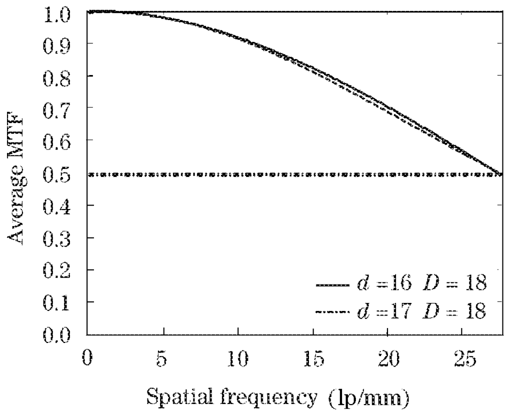

Fig. 5. Relationship of average MTF to spatial frequency.

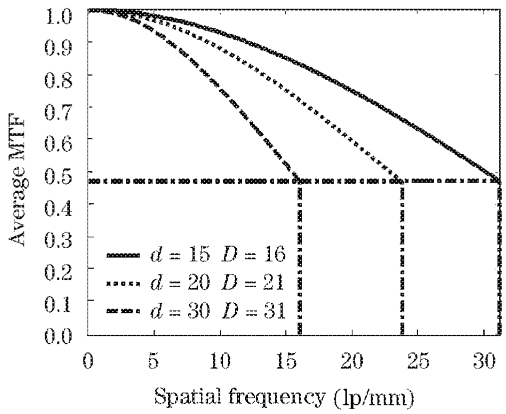

Fig. 6. Relationship of average MTF to spatial frequency.

The results shown in Figs. 4 and 5 indicate that, for identical fiber diameter and different core diameter, when the frequency of the input pattern is less than the Nyquist frequency, with frequency increasing, the MTF value reduces, and at Nyquist frequency, the MTF value is near 0.5. The relationship between the core diameter and the average MTF is inverse proportion for the certain fiber diameter and spatial frequency. For identical fiber cladding thickness and spatial frequency, when the frequency of the input pattern is less than the Nyquist frequency, with fiber core diameter increasing, the MTF value reduces.

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