Low-amplitude vector screening solitons

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Received April 2, 2004

We show self-coupled and cross-coupled vector beam evolution equations in the low-amplitude regime for screening solitons, which can exhibit the analytical solutions of bright-bright and dark-dark vector solitons. Our analysis indicates that these self-coupled vector solitons are obtained irrespective of the intensities of the two optical beams, whereas these cross-coupled vector solitons can be established when the intensities of the two optical beams are equal. Relevant examples are provided where the photorefractive crystal is lithium niobate ($LiNbO_3$). The stability properties of these vector solitons have been investigated numerically and it has been found that they are stable.

OCIS codes: 190.0190, 190.4420, 190.5530, 230.7370.

Spatial solitons in photorefractive (PR) materials have attracted much interest in the past few years[1-13]. It was found that self-trapping of optical beams takes place in both transverse dimensions and that these solitons can be observed at microwatt and lower power levels[5]. Thus, such PR solitons are the building blocks of all-optical switching devices where light itself guides and steers light without fabricated waveguides. Thus far, steady-state spatial solitons have been predicted and observed in photovoltaic (PV) materials[6], and in the screening configuration[7]. Very recently, vector solitons involving the two polarization components of an optical beam[8,9] were proposed for screening solitons, which obey a self-coupled or a cross-coupled system of nonlinear evolution equations. At present, bright-bright and dark-dark, self- or cross-coupled vector solitons, as well as bright-dark self-coupled vector solitons have been predicted for screening solitons. More recently, we have shown theoretically that the application of an external field enables steady-state solitons[10], soliton pairs[11,12], and vector solitons[13] in photorefractive-photovoltaic crystals. However, vector screening solitons have been obtained by numerical integration[8]. The analytical solutions of vector screening solitons also deserve special consideration in the low-amplitude regime.

In this paper, we show self-coupled and cross-coupled vector beam evolution equations in the low-amplitude regime for screening solitons, which can exhibit the analytical solutions of bright-bright and dark-dark vector solitons. Moreover, our analysis indicates that these self-coupled vector solitons are obtained irrespective of the intensities of the two optical beams, whereas these cross-coupled vector solitons can be established when the intensities of the two optical beams are equal. The stability properties of these vector solitons have been investigated numerically and we have found that they are stable. Relevant examples are provided where the PR crystal is assumed to be $LiNbO_3$.

In general, the nonlinear changes in the permeability are given by Ref. [8],

$$\Delta\tilde{\varepsilon} = -\left\{\tilde{\varepsilon}\cdot\left[(\tilde{r}\cdot\mathbf{E}_{\rm sc})\cdot\tilde{\varepsilon}\right]\right\}/\varepsilon_0, \tag{1}$$

where ε_0 is the permittivity of the vacuum, $\tilde{\varepsilon}$ is the permeability tensor, $\tilde{r}$ is the Pockels tensor, and $\mathbf{E}_{\rm sc}$ is the PR space-charge field. Let us consider an optical beam that propagates in a PR crystal along the z axis and is allowed to diffract only along the x direction. The slowly varying amplitudes $A_x(x,z)$ and $A_y(x,z)$ of the optical field $\mathbf{E}$ on the optical beam are defined as $\mathbf{E}(x,z,t) = A_x(x,z)\exp[i(k_x z-\omega t)]\hat{x} + A_y(x,z)\exp[i(k_y z-\omega t)]\hat{y} + c.c.$, where $k_x = kn_x$, $k_y = kn_y$, $k = 2\pi/\lambda$, n_x and n_y are the refractive indices for light polarized along x and y directions, respectively, and λ is the vacuum wavelength. Moreover, the external bias electric field is also applied in the x direction. Under these conditions, the components of $\Delta\tilde{\varepsilon}$ are $\Delta\varepsilon_{xx} = -\varepsilon_0 n_x^4 r_{xxx}E_{\rm sc}$, $\Delta\varepsilon_{yy} = -\varepsilon_0 n_y^4 r_{yyx}E_{\rm sc}$, and $\Delta\varepsilon_{xy} = \Delta\varepsilon_{yx} = -\varepsilon_0 n_x^2 n_y^2 r_{xyx}E_{\rm sc}$, and the space-charge field, $\mathbf{E}_{\rm sc} = E_{\rm sc}\hat{x}$, is given by Ref. [7,8],

$$\hat{E}_{\rm sc} = -\frac{\delta}{1+I/I_{\rm dark}}, \tag{2}$$

where $\hat{E}_{\rm sc} = E_{\rm sc}(qL_{\rm D}/k_{\rm B}T)$, $\delta = C\eta$, $C = VqL_{\rm D}/(k_{\rm B}TL_{\rm s})$, $\eta = 1\Big/\int_{-l/2L_{\rm s}}^{l/2L_{\rm s}}{\rm d}\xi/(1+I/I_{\rm dark})$, I is the power density profile of the optical beam, $L_{\rm D}=[k_{\rm B}T\varepsilon_{\rm s}/(q^2N_{\rm A})]^{1/2}$ is the Debye length, $L_{\rm s}$ is the soliton length scale, $N_{\rm A}$ is the number density of negatively charged acceptors, $I_{\rm dark}$ is the so-called dark irradiance, $k_{\rm B}$ is Boltzmann's constant, q is the charge on the electron, T is the absolute temperature, $\varepsilon_{\rm s}$ is the low-frequency dielectric constant, l is the width of the crystal between the electrodes, and V is the external voltage applied to the PR crystal. By expressing $A_x(x,z)$ and $A_y(x,z)$ in the following way, $A_x(x,z) = u(x)\exp(i\Gamma_x z)\sqrt{I_{\rm dark}}$ and $A_y(x,z) = v(x)\exp(i\Gamma_y z)\sqrt{I_{\rm dark}}$, where $v(x) = \sigma u(x)$, and σ is a constant, then one can quickly find the set of coupled equations,[8]

$$-2k_x\Gamma_x u + \frac{\mathrm{d}^2u}{\mathrm{d}x^2} = -k^2[\Delta\varepsilon_{xx}u + \Delta\varepsilon_{xy}\sigma u \exp(i\tau z)]/\varepsilon_0, \quad (3a)$$

$$-2k_y\Gamma_y u + \frac{\mathrm{d}^2u}{\mathrm{d}x^2} = -k^2[(\Delta\varepsilon_{xy}/\sigma)u \exp(-i\tau z) + \Delta\varepsilon_{yy}u]/\varepsilon_0, \quad (3b)$$

where $\tau = \Gamma_x - \Gamma_y + k_y - k_x$ is the phase mismatch that combines the material birefringence and the difference between the propagation constants.

Let the PR crystal be $LiNbO_3$ crystal, which is a good candidate for the observation of the self-coupled or cross-coupled vector solitons[8]. Moreover, let us assume that the (x, y) axes coincide with the extraordinary and ordinary components of the optical beam, and that the optical c axis of this crystal is oriented at an angle $\theta \approx 11.9°$ with respect to the z axis. In this case, $\Delta\varepsilon_{xx} = \Delta\varepsilon_{yy}$, and $\Delta\varepsilon_{xy} = \Delta\varepsilon_{yx} = 0$. Substituting $\Delta\varepsilon_{xx} = \Delta\varepsilon_{yy}$ and $\Delta\varepsilon_{xy} = \Delta\varepsilon_{yx} = 0$ into Eq. (3) leads to $k_x\Gamma_x = k_y\Gamma_y$. Thus Eq. (3) yields

$$k_x\Gamma_x u - (\mathrm{d}^2u/\mathrm{d}x^2)/2 = -(k^2 n_x^4 r_{xxx} E_{\mathrm{sc}}/2)u. \quad (4)$$

For the two orthogonal polarization beams, the total optical power density I can be obtained by summing the two Poynting fluxes, i.e. $I = I_{\mathrm{dark}}\left(|\sigma|^2+1\right)u^2$. For the convenience of the analysis, we transform Eq. (4) to dimensionless form by $\xi = x/L_{\mathrm{s}}$, where $L_{\mathrm{s}} = 1/(\pm 2k_x b)^{1/2}$, and $b = (k/2)n_x^3 r_{xxx} k_{\mathrm{B}}T/(qL_{\mathrm{D}})$ is the parameter that characterizes the strength and the sign of the optical nonlinearity. In the low-amplitude regime ($u^2 \ll 1$ and $v^2 \ll 1$), Eqs. (2) and (4) yield

$$u'' = \pm\left\{\frac{\Gamma_x}{b} - \delta + \delta\left(|\sigma|^2+1\right)u^2\right\}u, \quad (5)$$

where $u'' = \mathrm{d}^2u/\mathrm{d}\xi^2$. Using quadrature, we obtain the first integral of Eq. (5)

$$p^2 - p_0^2 = \pm\left[\left(\frac{\Gamma_x}{b} - \delta\right)(u^2 - u_0^2) + \frac{1}{2}\delta\left(|\sigma|^2+1\right)(u^4 - u_0^4)\right], \quad (6)$$

where $p = u'$, $p_0 = p(x=0)$, and $u_0 = u(0)$.

The boundary conditions for fundamental dark solitons are $u(0) = 0$, $u(+\infty) = u_\infty \neq 0$, and $u'(\infty) = u''(\infty) = 0$. Substituting $u_\infty \neq 0$, $u''(\infty) = 0$, and $x \to \infty$ into Eq. (5) leads to $\Gamma_x/b = \delta - \delta\left(|\sigma|^2+1\right)u_\infty^2$. By using boundary conditions $u(0) = 0$, $u_\infty \neq 0$, $u'(\infty) = 0$, and this latter form of Γ_x/b, and by integrating Eq. (6), one can get

$$u = u_\infty \tanh\left(\sqrt{\pm\delta(|\sigma|^2+1)/2}\,u_\infty\xi\right), \quad (7)$$

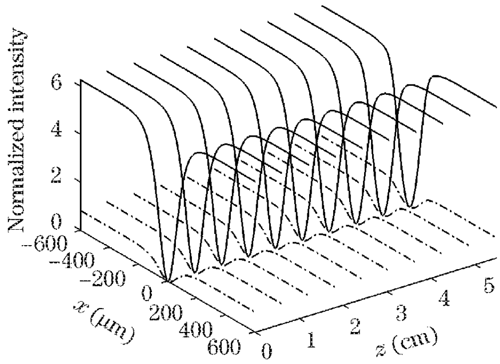

Fig. 1. Stable propagation of a dark-dark self-coupled vector screening soliton ($\sigma = 0.4$) when its x-polarized component (solid curve) is perturbed by 10% at the input.

from which this vector soliton component $v(\xi)$ is determined through $v(\xi) = \sigma u(\xi)$. As an example, let us assume that the $LiNbO_3$ crystal, which is oriented at $\theta = 11.9°$ with respect to the z axis, is biased by V/l = 10 kV/cm. In this case, $n_x = 2.200$, $n_y = 2.286$, and $\Delta\varepsilon = 235.85 \times 10^{-12}E_{\mathrm{sc}}/\varepsilon_0$ at $\lambda = 0.633$ μm. Figure 1 depicts the evolution of the dark-dark self-coupled vector soliton obtained at $I(x\to\infty) = 6I_{\mathrm{dark}}$ and $\sigma = 0.4$ under these conditions when its x-polarized input component (solid curve) is perturbed by 10% in its amplitude. The solitary behavior of this dark-dark self-coupled vector structure is of course evident in this figure since the pair does not break up.

The boundary conditions for fundamental bright solitons are $u(+\infty) = 0$, $p(\infty) = u''(\infty) = 0$, and $p(0) = 0$. Using conditions $u(\infty) = p(\infty) = 0$ and $p(0) = 0$, and substituting $x \to \infty$ into Eq. (6) yield $\Gamma_x/b = \delta - \delta\left(|\sigma|^2+1\right)u_0^2/2$. Further integration of Eq. (6) leads to

$$u = u_0 \mathrm{sech}\left(\sqrt{\pm(-\delta)(|\sigma|^2+1)/2}\,u_0\xi\right), \quad (8)$$

from which this vector soliton component $v(\xi)$ is simply obtained through a σ. Figure 2 shows the evolution of the bright-bright self-coupled vector screening soliton in $LiNbO_3$ at $I(x=0) = 5I_{\mathrm{dark}}$, $\sigma = 0.5$, $\lambda = 0.633$ μm, and $V/l = 15$ kV/cm when its x-polarized input component (solid curve) is perturbed by 10% in its amplitude.

Let us consider a biased $LiNbO_3$ crystal with its optical c axis oriented in the z direction. An optical beam propagates in the crystal along the z axis, and is polarized at 45° between x and y directions. Thus $\Delta\varepsilon_{\mathrm{ee}} = \Delta\varepsilon_{\mathrm{oo}} = 0$, $\Delta\varepsilon_{\mathrm{eo}} = \Delta\varepsilon_{\mathrm{oe}}$, $|\sigma|^2 = 1$, and $n_x = n_y$, which implies that $\Gamma_x = \Gamma_y$. Under these conditions, Eq. (3) can be simplified to

$$\Gamma_x u - u''/2k_x = -(\pm k/2)n_x n_y^2 r_{xyx} E_{\mathrm{sc}} u. \quad (9)$$

We now transform the equation to dimensionless variables by substitutions $\hat{E}_{\mathrm{sc}} = E_{\mathrm{sc}}(qL_{\mathrm{D}}/k_{\mathrm{B}}T)$ and $\zeta = x/d$, where $d = 1/(\pm 2k_x g)^{1/2}$ and $g = (k/2)n_x n_y^2 r_{xyx} k_{\mathrm{B}}T/(qL_{\mathrm{D}})$. In the low-amplitude regime, the dimensionless equation is

$$\ddot{u} = \pm\left(\frac{\Gamma_x}{g} - \delta + 2\delta u^2\right)u, \quad (10)$$

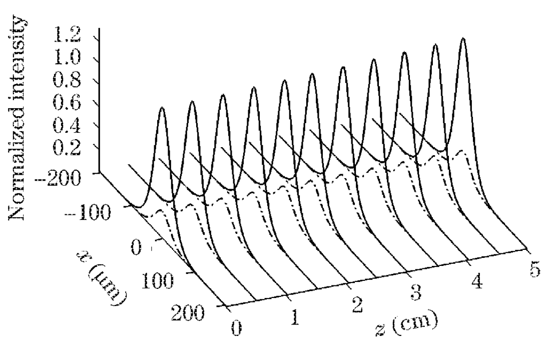

Fig. 2. Stable propagation of a bright-bright self-coupled vector screening soliton ($\sigma = 0.5$) when its x-polarized component (solid curve) is perturbed by 10% at the input.

where $\ddot{u} = \mathrm{d}^2u/\mathrm{d}\zeta^2$. We integrate Eq. (10), using quadrature, and obtain

$$(\dot{u})^2 - (\dot{u}_0)^2 = \pm\left\{\left(\frac{\Gamma_x}{b} - \delta\right)\left(u^2 - u_0^2\right) + \delta\left(u^4 - u_0^4\right)\right\}, \tag{11}$$

where $\dot{u} = \mathrm{d}u/\mathrm{d}\zeta$ and $\dot{u}_0 = \dot{u}\,(\zeta = 0)$.

For fundamental dark solitons boundary conditions $u_\infty \neq 0$ and $\ddot{u}\,(\infty) = 0$, Eq. (10) gives $\Gamma_x/g = \delta - 2\delta u_\infty^2$. By using boundary conditions $u\,(0) = 0$, $u_\infty \neq 0$, and $\dot{u}\,(\infty) = 0$, and this latter form of Γ_x/g, and by integrating Eq. (11), we find

$$u = u_\infty \tanh\left(\sqrt{\pm\delta} u_\infty \zeta\right). \tag{12}$$

For fundamental bright solitons, substituting boundary conditions $u\,(\infty) = \dot{u}_\infty = 0$ and $\dot{u}_0 = 0$, and $\zeta \to \infty$ into Eq. (11) leads to $\Gamma_x/g = \delta - \delta u_0^2$. The integral in Eq. (11) yields

$$u = u_0 \mathrm{sech}\left[\sqrt{\pm(-\delta)} u_0 \zeta\right]. \tag{13}$$

Similarly, a stability study of bright-bright and dark-dark cross-coupled vector screening solitons reveals that they are stable.

We have shown that self-coupled and cross-coupled vector beam evolution equations can exhibit the analytical solutions of bright-bright and dark-dark vector solitons in the low-amplitude regime for screening solitons. Our analysis indicates that these self-coupled vector solitons are obtained irrespective of the intensities of the two optical beams and that these cross-coupled vector solitons can be established when the intensities of the two optical beams are equal. The stability properties of these vector solitons have been investigated numerically and we have found that they are stable.

This work was supported by the National Natural Science Foundation of China (No. 60078004), and by the Director Foundation of Xi'an Institute of Optics and Precision Mechanics, Chinese Academy of Sciences. K. Lu's e-mail address is kq.lu@263.net.

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