Correction of dispersion distortion of femtosecond pulses by using the non-planar surface of diffractive optical elements

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Received March 26, 2004

Peculiarities of propagation of femtosecond pulses through a focusing diffractive optical element (DOE) are considered. It is shown that the time delay between the pulse and phase wavefronts can be decreased by fabricating the DOE on the optimal curvilinear surface.

OCIS codes: 050.1380, 050.1940, 050.1970, 080.2740, 090.1970, 320.2250.

The difference between the phase and group velocity in propagation of femtosecond pulses through a focusing element (lens) results in a time delay between the pulse and phase wavefronts[1]. We note that while considering the problems concerned with propagation of these pulses, one should take into account their short duration and finiteness of the light velocity value. The pulse front means[1] a surface coinciding with the pulse peak at a fixed instant of time. The time delay of lenses is caused by material dispersion[2], whereas in diffractive optical elements (DOEs) it is a result of their chromatic properties. One of the methods for partial correction of dispersion distortion of femtosecond pulses is a combination of DOE and dielectric lens[3,4]. In this paper, we only consider a focusing DOE.

Using Fermat's principle in the paraxial approximation, the delay between the pulse and phase wavefronts for DOE (amplitude zone plate) can be calculated as

$$\Delta T(r) = \frac{r_0^2 - r^2}{2cf^2}\lambda\frac{\mathrm{d}f}{\mathrm{d}\lambda}, \quad (1)$$

where r is the radial coordinate in the plane of the zone plate, r_0 is the zone plate radius, f is the focal length, and $\mathrm{d}f/\mathrm{d}\lambda$ is the longitudinal chromatic aberration of the zone plate. In Eq. (1), the delay for the outer beam is equal to zero (where $r = r_0$).

In this paper, it is also shown that for typical values of the longitudinal chromatic aberration in optical systems the pulse front leads spatially the phase front by 2%−20% of the distance passed in the dispersion medium. These results in distortion of the pulse forming in the focusing region, and for femtosecond pulses, one cannot neglect these great delays[1].

The purpose of this paper is to show that by using a nonplanar DOE surface, one can reduce the value of dispersion distortion of femtosecond pulses compared with a planar DOE.

Let us consider propagation of femtosecond pulses through a planar DOE (zone plate).

The geometry of beam transmission through the DOE is shown in Fig. 1(a). For the axial beam passing the minimal length (equal to the focal length) the delay between the pulse and phase wavefronts is maximal, whereas for the outer beam passing the maximal length the time delay is zero.

The spatial dependence of the time delay between the pulse and phase wavefronts is the ratio of path difference between the outer and arbitrary beams passing through the zone plate to light velocity. From this, the delay between the pulse and phase wavefronts is defined as

$$\Delta T(r) = -\frac{L(r_0) - L(r)}{c} = -\frac{\sqrt{f^2 + r_0^2} - \sqrt{f^2 + r^2}}{c}, \quad (2)$$

where $L(r_0)$ and $L(r)$ are the optical lengths of beams from the point of geometrical focus to the edge of DOE aperture and from the point to the coordinate r, respectively.

In a paraxial approximation, for a plane incident wave and for $r_0 \ll f$, Eq. (2) is transformed to[1]

$$\Delta T(r) = -\frac{r_0^2 - r^2}{c \cdot (\sqrt{f^2 + r_0^2} + \sqrt{f^2 + r^2})} \approx -\frac{r_0^2 - r^2}{2 \cdot f \cdot c}. \quad (3)$$

Here it was taken into account that the Fresnel zone radius ρ_n is determined from $\rho_n \approx \sqrt{n\lambda f}$ (i.e. $\frac{\mathrm{d}f}{\mathrm{d}\lambda} \approx -\frac{f}{\lambda}$).

One of the methods for delay correction is the use of diffractive optics on nonplanar surfaces[4].

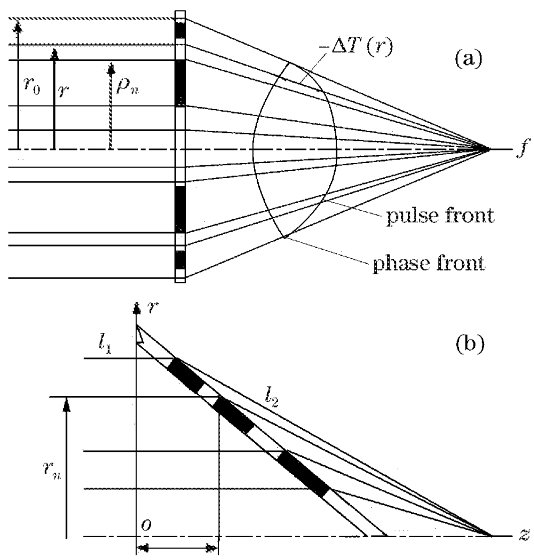

Fig. 1. Pulse transmission through an amplitude focusing DOE on a planar surface (a) and on a conic surface (b) (the reference radius is equal to zero).

Really, following the geometrical calculation of DOE fabricated on an arbitrary surface of rotation[5], we can write (Fig. 1(b))

$$l_1 + l_2 = l_3 + n\lambda/2, \tag{4}$$

where l_1 is the optical path of a beam parallel to the optic axis and passing through the boundary of the nth Fresnel zone, l_2 is the optical path of the same beam refracted on the DOE surface and directed to the focus, l_3 is the optical path of the reference beam (usually $l_3 = f$), n is the ordinal number of Fresnel zone, and λ is the radiation wavelength. Then

$$z_n(r) + \sqrt{r_n^2 + [f - z_n(r)]^2} = f + n\frac{\lambda}{2}. \tag{5}$$

From Eq. (5), the dependence of the focal length of the nonplanar DOE on the wavelength is[5]

$$f(\lambda) = z_n(r) + \frac{r_n^2 - \left(n\frac{\lambda}{2}\right)^2}{n\lambda}, \tag{6}$$

where $z_n(r)$ is the z-axis of the edge of the nth Fresnel zone on the DOE surface, and r_n is the projection of $z_n(r)$ onto the plane xoy.

Differentiating Eq. (6) with respect to λ, we define the chromatic aberration for the nonplanar DOE as

$$\frac{\mathrm{d}f}{\mathrm{d}\lambda} = -\left[\frac{f(\lambda) - z_n(r)}{\lambda} + \frac{n}{4}\right].$$

Equation (1), in a paraxial approximation, can be rewritten as

$$\Delta T(r) = -\frac{r_0^2 - r^2}{2cf^2}\left[f(\lambda) - z_n(r) + \frac{n\lambda}{4}\right]$$
$$\approx -\frac{r_0^2 - r^2}{2f(\lambda)c}\left[1 - \frac{z_n(r)}{f(\lambda)}\right]. \tag{7}$$

Comparing Eq. (7) with Eq. (3), we can see that, by choosing the shape of DOE surface and its space orientation, it is possible to correct the value of delay between the wave and pulse fronts.

Let us determinate the optimal shape of DOE surface. It is obvious that, for any kind of DOE surface, the minimal optical path corresponds to the central beam with the coordinate $r = 0$, $L(0) = f$; and the maximal optical path corresponds to the outer beam with the coordinate $r = r_0$ and $L(r_0) = z(r_0) + \sqrt{[f - z(r_0)]^2 + r_0^2}$.

The minimal optical path for the outer beam corresponds to the straight line: $L_{\min}(r_0) = \sqrt{f^2 + r_0^2}$. It follows from this that the DOE surface should be convex toward the focus. Hence, it is impossible to minimize the delay at the point $r = 0$ for any kind of DOE surface by less than $\Delta T(0)_{\min} = \frac{\sqrt{f^2+r_0^2} - f}{c}$.

At the same time, it follows from this that for DOE with a nontransparent central zone the minimal delay will be $\Delta T_{\mathrm{H}}(0)_{\min} = \frac{\sqrt{f^2+r_0^2} - \sqrt{f^2+\lambda f}}{c}$.

Let us define the optimal profile of the DOE surface[6]. The optical path of the beam with the arbitrary coordinate r (see Fig. 1) to the point f can be written as

$$L(r) = z(r) + \sqrt{[f - z(r)]^2 + r_0^2},$$

or

$$L(r) = f - \frac{r}{\tan[\varphi(r)]} + \frac{r}{\sin[\varphi(r)]} = f + r\frac{1 - \cos[\varphi(r)]}{\sin[\varphi(r)]}, \tag{8}$$

where $z(r)$ is the arbitrary surface of rotation and $\phi(r)$ is the angle at which the beam arrives at the focus. The propagation time delay is determined from

$$\Delta T(r) = \frac{L(r_0) - L(r)}{c}. \tag{9}$$

Upon differentiating Eq. (9) with respect to r, in view of Eq. (8), we will find extremes of function from the condition $\frac{\mathrm{d}[\Delta T(r)]}{\mathrm{d}r} = 0$. After transformations, we obtain

$$1 + \frac{r \cdot \varphi'(r)}{\sin[\varphi(r)]} = 0. \tag{10}$$

Dividing the variables $\frac{\mathrm{d}\varphi}{\sin(\varphi)} = -\frac{\mathrm{d}r}{r}$ and integrating the obtained expression, in view of $\cos(\varphi) + 1 \geq 0$ and $\cos(\varphi) - 1 \leq 0$ for any ϕ, we obtain

$$\varphi(r) = \arccos\left(\frac{C \cdot r^2 - 1}{C \cdot r^2 + 1}\right), \tag{11}$$

where $C = \frac{1}{r_0^2}\frac{1+\cos(\varphi_0)}{1-\cos(\varphi_0)}$ is the constant and $\varphi_0 = \mathrm{arctg}\left(\frac{r_0}{f}\right)$.

Accordingly, the shape of the optimal DOE surface $z(r)$ is defined by

$$z(r) = f\{1 - \tan[\varphi(r)]\}. \tag{12}$$

It is easily seen that Eq. (12) imposes constraint on the relative DOE aperture, in other words, in the general

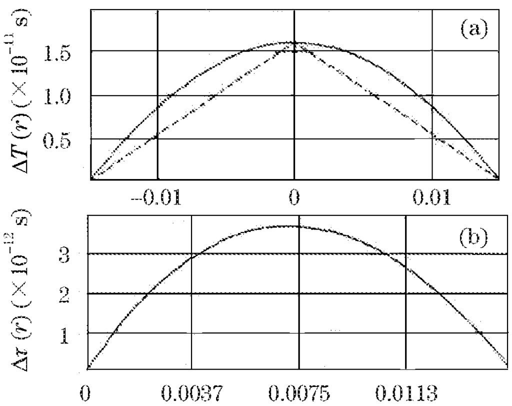

Fig. 2. Delay between the phase and pulse wavefronts along r. (a): $\Delta T(r)$ is the time delay for a planar DOE (solid curve) and a conic element (dashed curve). (b): The difference $\Delta\tau(r)$ between the delays $\Delta T(r)$ for the planar and conic DOEs against radial coordinate (a half of the plot is shown due to its symmetry).

case, we are not able to attain full correction of the delay between the wavefront and the pulse front. However, it is possible to decrease or correct the delay value. Transform Eq. (12) to $x^2 + y^2 = \frac{(z-f)^2}{(f/r_0)^2}$[4], which corresponds to the canonical equation of cone of the second order with the vertex at the point (x_0, y_0, z_0): $\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} - \frac{(z-z_0)^2}{c^2} = 0$, where $x_0 = y_0 = 0$, $a = b = 1$, $z_0 = f$, and $c = f/r_0$.

Therefore, the DOE surface for different values of the relative aperture is a cone with the vertex at the focal point.

To demonstrate the partial correction of the effect of delay between the pulse front and the wavefront, Fig. 2 shows values of delay for planar and conic DOEs plotted against radial coordinate. The parameters of the DOE are radiation wavelength of 3 mm, diameter of 30 mm, and focal length of $0.7D$.

From analysis of the dependencies, we see that, by fabricating DOE on a conic surface, one can reduce the delay between the pulse front and the wavefront by 20%−40%.

This paper has shown that, by using a nonplanar DOE surface, one can reduce the value of dispersion distortion of femtosecond pulses compared with DOE on a plane surface. So the optimal shape of DOE surface is a conic one.

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