Study on the shadowing effect for optical wave scattering from randomly rough surface

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Based on the Kirchhoff approximation for rough surface scattering and by calculating the shadowing function of the rough surface, the formula of the scattering cross section of the dielectric rough surface is presented with consideration of the shadowing effect for the optical wave incidence. It is obtained that in comparison with the conventional Kirchhoff solution, the shadowing effect should not be neglected for the optical wave scattering from the rough surface. The influence of the shadowing effect for different incidence angle, surface root mean square slope, and surface roughness on the scattering cross section is discussed in detail.

OCIS codes: 290.5880, 290.0290, 240.0240.

The study on optical wave scattering from rough surfaces has been the subject of intensive investigation over the past several decades for its application in a number of important research areas such as characterization of films and optical interface, the design of optical scanning instruments for use in the semiconductor industry[1−4]. Among the many surface scattering theories, the Kirchhoff approximation is widely used as the means for describing scattering from a surface. The Kirchhoff approximation contains the concept of shadowing in that any surface to which it is applied is divided into illuminated and shadowed regions[5−7]. On the illuminated region one assumes that the surface fields can be represented by the geometric optics fields, and in the shadowed region the fields are assumed to vanish. Even though the Kirchhoff approximation contains the concept of shadowing, this aspect of the approximation is often neglected in a statistical scattering analysis. In this letter, the monostatic and bistatic scattering from the two-dimensional (2D) rough surface are studied by applying the Kirchhoff approximation in conjunction with the shadowing effect.

Consider an incident plane wave $\hat{p}E_p^{(i)}\exp(ikr)$ impinging on a 2D rough surface as shown in Fig. 1. Here, we use the Kirchhoff solution for scattering from rough surface that takes the exact roughness profiles into account. This Kirchhoff solution treats the rough surface as locally flat with the assumption that the wavelength of the incident wave is small in comparison with the radius of curvature of the surface irregularities. Small angle interaction will be used to avoid the problem of shadowing (i.e. neglecting the edge effect).

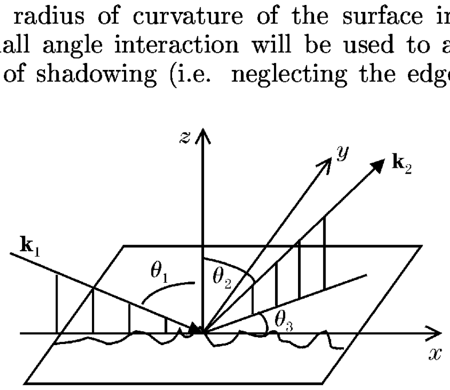

Fig. 1. Geometry of light scattering from 2D rough surface.

If the observation is performed at far field, the scattered field under the Kirchhoff approximation can be given by[8,9]

$$E_q^{(s)} = -\frac{ik\exp(ikR_0)}{4\pi R_0}E_p^{(i)}F_{pq}(\theta_1,\theta_2,\theta_3)\iint_A \exp(i\mathbf{v}\cdot\mathbf{r})\mathrm{d}S$$
$$= -\frac{ik\exp(ikR_0)}{4\pi R_0}E_p^{(i)}F_{pq}(\theta_1,\theta_2,\theta_3) \times\iint_A \exp[i(v_x x + v_y y + v_z z(x,y))]\mathrm{d}x\mathrm{d}y, \quad (1)$$

where $\mathbf{v} = \mathbf{k_1} - \mathbf{k_2}$, $\mathbf{k_1}$ and $\mathbf{k_2}$ are the propagation vectors of the incident and scattered waves, respectively, k is the wave number, A is the illuminating area, R_0 is the distance from the origin to a point of the scattered field, p and q denote the incident and the scattered polarization, respectively. $F_{pq}(\theta_1,\theta_2,\theta_3)$ is a dimensionless function depending on the incidence angle, scattering angle, the Fresnel reflection coefficient, and the polarization state, which can be given by[8]

$$F_{pq}(\theta_1,\theta_2,\theta_3) = R_{pq}(\theta_1) \times\frac{(1+\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2\cos\theta_3)}{(\cos\theta_1+\cos\theta_2)}, \quad (2)$$

where $R_{pq}(\theta_1)$ is the Fresnel reflection coefficient. The three components of $\mathbf{v}$ in Eq. (1) can be written as

$$\begin{cases} v_x = k(\sin\theta_1 - \sin\theta_2\sin\theta_3) \\ v_y = -k\sin\theta_2\sin\theta_3 \\ v_z = -k(\cos\theta_1+\cos\theta_2) \end{cases}. \quad (3)$$

From Eq. (1), taking the mean square value, we can get

$$\left\langle|E_q^{(s)}|^2\right\rangle = \frac{k^2|E_p^{(i)}|^2|F_{pq}|^2}{(4\pi R_0)^2} \times\int\int\int\int\exp[iv_x(x-x') + iv_y(y-y')] \times\langle\exp[iv_z(z-z')]\rangle\,\mathrm{d}x\mathrm{d}x'\mathrm{d}y\mathrm{d}y'. \quad (4)$$

The term $\langle\exp[iv_z(z-z')]\rangle$ in Eq. (4) is the characteristic function of the surface, which is usually denoted by $\chi(\tau_x,\tau_y)$, it can be expressed as the Fourier transform of surface probability density function (PDF) $p(z,z')$, i.e.

$$\chi(\tau_x,\tau_y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\exp[iv_z(z-z')]p(z,z')\mathrm{d}z\mathrm{d}z', \quad (5)$$

where $\tau_x=x-x'$, $\tau_y=y-y'$. For a Gaussian PDF with zero mean and the root mean square (rms) of the surface of δ, when the shadowing effect is not taken into account, we obtain[10,11]

$$\chi_1(\tau_x,\tau_y)=\exp\{-v_z^2\delta^2[1-R(\tau_x,\tau_y)]\}, \quad (6)$$

where $R(\tau_x,\tau_y)$ is the 2D rough surface height autocorrelation function. When the characteristic function is related to the shadowing function, it can be given as[11]

$$\chi_2[g(\tau_x,\tau_y)]=\left|\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}\exp(igZ)\exp(-Z^2)[1-0.5\mathrm{erfc}(Z)]^{\Lambda}\mathrm{d}Z\right|^2, \quad (7)$$

where

$$g(\tau_x,\tau_y)=\sqrt{2}v_z\delta[1-R(\tau_x,\tau_y)]^{1/2},$$

$$\Lambda(u_1)=\frac{\exp(-u_1^2)-u_1\sqrt{\pi}\mathrm{erfc}(u_1)}{2u_1\sqrt{\pi}}, \quad (8)$$

where $\mathrm{erfc}(u_1)=1-\mathrm{erf}(u_1)$ is the complementary error function, $u_1=\cot|\theta_1|/(\sqrt{2}s)$, $s=\sqrt{2}\delta/L$ is the surface rms slope, here L is the correlation length of the surface. If the shadowing effect is ignored corresponding to $\Lambda(u)=0$ and the incidence angle θ_1=0°, Eq. (7) will become to Eq. (6). Defining the normalized scattering radar cross section of the rough surface as[8]

$$\sigma_{pq}=\frac{4\pi R_0^2\left\langle|E_q^{(s)}|^2\right\rangle}{A\left\langle|E_p^{(i)}|^2\right\rangle}, \quad (9)$$

assuming the illuminated surface size is much larger than L, by using Eqs. (4) – (9) and performing the coordinate transformation $\tau_x=\tau\cos\psi$, $\tau_y=\tau\sin\psi$, Eq. (9) can be given by

$$\sigma_{pq}=\frac{k^2|F_{pq}|^2S(u_1,u_2)}{4\pi}\int_0^{\infty}\tau\mathrm{d}\tau \times\int_0^{2\pi}\exp[i\tau\sqrt{v_x^2+v_y^2}\cos(\psi-\phi)]\chi_{1,2}(\tau_x,\tau_y)\mathrm{d}\psi, \quad (10)$$

where $\phi=\tan^{-1}(v_y/v_x)$, $u_2=\cot|\theta_2|/(\sqrt{2}s)$, $S(u_1,u_2)$ is the modified Smith shadowing function[10], which is usually assumed to be independent of the scattering cross section calculation and can be easily obtained. When the shadowing effect is not considered, $S(u_1,u_2)=1$. With the variable transformation $u=\tau/L$, and performing the integration over ψ in Eq. (10), the incoherent scattering section of the one-dimensional (1D) surface with considering the shadowing effects can be written as

$$\sigma_{\mathrm{S}}=\frac{S(u_1,u_2)\left|v^2F_{pq}\right|^2L^2}{2v_z^2}\int_0^{\infty}uJ_0(uL\sqrt{v_x^2+v_y^2}) \times\{\chi_2(\sqrt{2}v_z\delta[1-R(u)]^{1/2})-\chi_2(\sqrt{2}v_z\delta)\}\mathrm{d}u, \quad (11)$$

where J_0 is Bessel function of order zero, $v^2=v_x^2+v_y^2+v_z^2$. If the shadowing effect is ignored, then $S(u_1,u_2)=1$, χ_1 can be obtained from Eq. (6) which leads to

$$\sigma_{\mathrm{US}}=\frac{\left|v^2F_{pq}\right|^2L^2}{2v_z^2}\int_0^{\infty}uJ_0(uL\sqrt{v_x^2+v_y^2}) \times\{\exp[-v_z^2\delta^2(1-R(u))]-\exp(-v_z^2\delta^2)\}\mathrm{d}u. \quad (12)$$

As for the spectra of many natural rough surfaces taking the characteristic of Gaussian correlation and exponential correlation distribution, the optical scattering from these kinds of surfaces has been extensively investigated[1,12]. The Gaussian and exponential correlation functions can be respectively written as[8,9]

$$R(\tau)=\delta^2\exp(-\tau^2/L^2),\quad R(\tau)=\delta^2\exp(-|\tau|/L). \quad (13)$$

In order to examine the validity of the scattering formulation, we calculate the scattering cross section of a rough surface with/without shadowing effect. We consider the calculated behavior of $\lambda=0.6328$ μm light scattered by a silicon substrate with the dielectric constant 15.07+i0.15[13]. A Gaussian random rough surface of the silicon is given with $k\delta=0.3$, $kL=2.0$. The shadowing effect on the angular distribution of backscattering cross section σ_{HH}^0 of rough surface with Gaussian and exponential correlation functions is illustrated in Fig. 2. In performing the backscattering calculations of Eqs. (11) and (12), $\theta_1=\theta_2$, $\theta_3=\pi$. As depicted in Fig. 2, for both of the Gaussian and exponential surfaces, when the incidence angle exceeds 60°, the amplitude of σ_{HH}^0 decreases at a more rapid rate due to the shadowing effect, especially for low grazing angle incidence, this tendency shows good consistency with the experimental results[5].

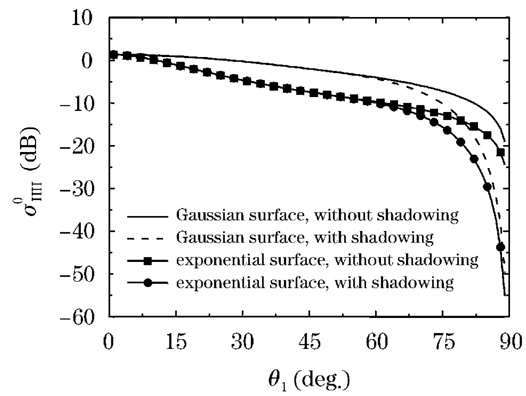

Fig. 2. The angular distribution of backscattering cross section for Gaussian and exponential rough surfaces.

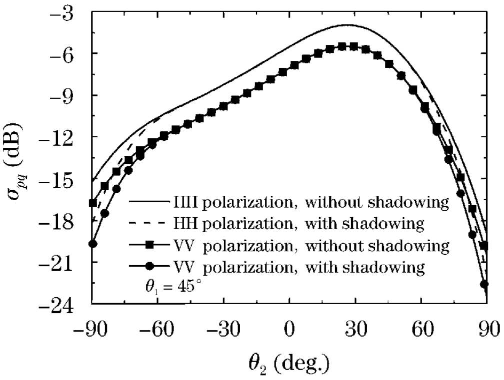

Fig. 3. The shadowing effect on the angular distribution of σ_{pq} for different polarizations.

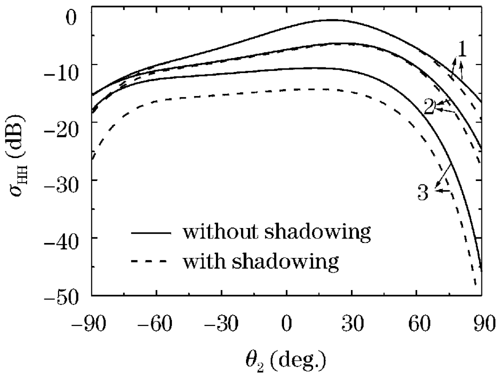

Fig. 4. The shadowing effect on the angular distribution of σ_{HH} with different incidence angle θ_1. Curves 1: $\theta_1 = 30°$; 2: $\theta_1 = 60°$; 3: $\theta_1 = 80°$.

To further explore the important scattering characteristic of the rough surface with shadowing, we consider the calculated behavior of bistatic scattering cross section in the specular plane ($\theta_3 = 0, \pi$). Since the correlation functions of many optical rough surfaces take the form of Gaussian, the angular distribution of bistatic scattering cross section σ_{HH} for above silicon Gaussian surface is presented in Fig. 3 with different polarization. Here the incidence angle θ_1 is set to be 45°. As illustrated in Fig. 3, for the same scattering angle θ_2, the amplitude of scattering cross section for HH polarization is larger than that of VV polarization. It is also found that when the scattering angle $\theta_2 > 60°$ or $\theta_2 < -54°$, the shadowing effect will appear for both polarizations. The amplitude of scattering cross section decreases at a more rapid rate due to the shadowing effect, especially for the large scattering angle $|\theta_2|$.

In Fig. 4, the angular distributions of bistatic scattering cross section σ_{HH} for the above silicon surface are plotted for different incidence angles (θ_1=30°, 60°, 80°). It is easily observed that as the incidence angle increases, the range of the scattering angle θ_2 corresponding to the shadowing effect appearing will increase. Therefore, for both larger incidence and scattering angles, the shadowing effect on the surface scattering will be more obvious. It should be noted that as the incidence angle θ_1=80° (grazing incidence, corresponding to curves 3 in Fig. 4), the shadowing effect will appear in the whole scattering region of $|\theta_2| \leq 90°$. We can also find that for the same scattering angle, the larger the incidence angle is, the smaller the value of σ_{HH} will be.

Figure 5 also shows the angular distribution of bistatic scattering cross section for exponential surface but with different surface rms slope s, and the incidence angle θ_1=45°, $k\delta$ =0.4. Compared to the relationship between the shadowing function and rms slope s[10,11], it indicates that with increasing s, the shadowing function will decrease for the same scattering angle. Hence, similar to the analysis of shadowing effect on the scattering pattern with different incidence angle, we can also obtain the result that with larger value of s, the range of the scattering angle θ_2 for the shadowing effect appearing will increase. The above results obtained from Fig. 5 are also valid for the case of Gaussian surface.

The effect of surface roughness parameters $k\delta$ on the bistatic scattering cross section of aforementioned silicon rough surface with optical wave incidence is also investigated. Figure 6 shows the HH polarization σ_{HH} as a function of θ_2 with/without shadowing effect considered for different $k\delta$, and the incidence angle θ_1=45°, s=0.25. It is shown that the region of the scattering angle $|\theta_2|$ for the shadowing effect appearing almost does not change with different surface roughness within the Kirchhoff approximation, which is different from the influence of incidence angle and rms slope on the shadowing effect for the scattering pattern. Another point worth noting is that for small $k\delta$, the amplitude if σ_{HH} is also small. Hence, we can conclude that the amplitude of scattering cross

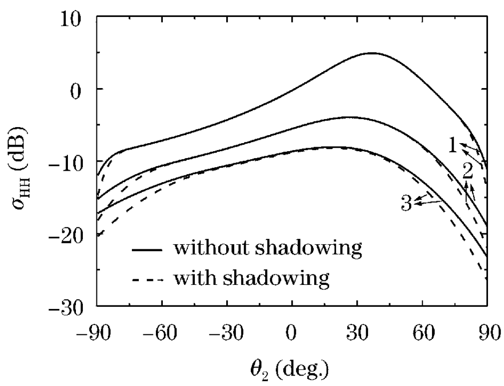

Fig. 5. The shadowing effect on the angular distribution of σ_{HH} with different s. Curves 1: s = 0.1; 2: s = 0.25; 3: s = 0.5.

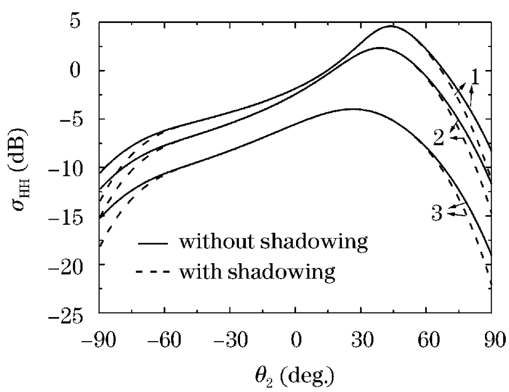

Fig. 6. The shadowing effect on the angular distribution of σ_{HH} with different $k\delta$. Curves 1: $k\delta$ = 1.0; 2: $k\delta$=0.7; $k\delta$=0.3.

section is mainly determined by the surface roughness. The above results obtained in Figs. 4 – 6 are also valid for the case of VV polarization.

In conclusion, the formula of the scattering cross section for the rough surface with/without shadowing effect is considered and presented by the Kirchhoff approximation. It is obtained that compared with the conventional Kirchhoff solution, the shadowing effect should not be neglected for the optical wave scattering from the rough surface. Calculations were carried out to compare the backscattering results for the Gaussian surface and exponential surface with optical wave incidence. Numerical results also show that the angular distribution of the bistatic scattering cross section from the random surface of optical material is heavily influenced by the shadowing effect. With the larger rms slope and incidence angle, the shadowing effect will be more obvious with increasing the scattering angle. Future investigation will include the shadowing effect on the dielectric rough surface for the case of the beam incidence. In addition, the accuracy of the related numerical results with shadowing effect also needs further verification by the experimental research.

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