Axial distribution of Gaussian beam limited by a hard-edged aperture

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In this letter, the axial distribution of Gaussian beam limited by a hard-edged aperture is studied. We theoretically analyze the axial diffraction of Gaussian beam limited by a hard-edged aperture, and give the simpler formulas of the axial diffraction intensities of Gaussian beam in Fresnel diffraction field and Fraunhofer diffraction field. The corresponding numerical calculation of axial diffraction intensity distribution of Gaussian beam with different wave waist is provided and the evolution of the diffraction distribution with the wave waist of Gaussian beam is explained. As the especial cases of the truncated Gaussian beam, the Gaussian beam in free space and the parallel light limited by the aperture are discussed too, and the system parameters of the truncated Gaussian beam which can cause it to equal to these cases are given. The theoretical results conform to the numerical analysis.

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The parallel light or the spherical wave as the simpler mathematical model of the laser beam makes the management of the diffraction of beam convenient[1−5]. However, the laser beam emitted from the source can be described approximately by the Gaussian beam, and such a form shows different characteristics from the parallel light and spherical wave. Moreover, during the propagation of the practical laser beam, there exists inevitably a hard boundary to truncate it, and the optical system with the limited caliber must result in the energy loss of Gaussian beam and the variation of the shape of its diffraction distribution. The quality of Gaussian beam during the propagation had attracted attentions in the past years. Wang *et al.*[6] analyzed the energy loss of Gaussian beam through a hard-edged aperture. Holmes and Li *et al.*[7,8] discussed the focal shift in focused truncated Gaussian beam, which was mainly located one point on propagation axis, to investigate the position of the main maximum. By using Bessel function, Schell *et al.*[9] evaluated the diffraction integral to obtain an analytical solution for the irradiance of the truncated Gaussian beam in the near zone as well as in the far zone. Ding *et al.*[9] derived the propagation of Bessel, Bessel-Gaussain, and Gaussian beams with a finite aperture on the basis of the fact that circ function can be expanded into an approximate sum of complex Gaussian function. These works emphasized the analytical solution of the three-dimensional diffraction distribution of the truncated Gaussian beam, but its axial distribution rule had not been discussed in detail. In this letter, we concentrate on studying the axial diffraction of Gaussian beam limited by a circular aperture. Recurring to the Kirchhoff's diffraction theory, we theoretically analyze the diffraction of Gaussian beam limited by a circular and give the simpler formulas of the axial diffraction distribution in Fresnel and Fraunhofer regions. The propagation behavior of the Gaussian beam along the axis is examined elaborately and the numerical calculation of the diffraction distribution is performed. The equivalent conditions of the diffraction of truncated Gaussian beam to that of Gaussian beam in free space, and that of the parallel light limited by the aperture, are given too.

It is supposed that the amplitude distribution of Gaussian beam on the aperture plane is $a(x_0, y_0)$, the pupil function of an aperture is $p(x_0, y_0)$, then the amplitude behind immediately the aperture diaphragm may be written as

$$b(x_0, y_0) = a(x_0, y_0)p(x_0, y_0). \tag{1}$$

According to Kirchhoff's diffraction theory[11], when the propagation distance z satisfies the relationship of $z^3 \gg \frac{\pi}{4\lambda}[(x_1-x_0)^2+(y_1-y_0)^2]^2_{\max}$, the diffraction distribution of the laser beam via the aperture in the propagation distance z has the following form

$$A_1(x_1, y_1, z) = \frac{\exp(ikz)}{i\lambda z} \times \int\int b(x_0, y_0)\exp\{i\frac{\pi[(x_1-x_0)^2+(y_1-y_0)^2]}{\lambda z}\}\mathrm{d}x_0\mathrm{d}y_0, \tag{2}$$

when the propagation distance z is large enough to satisfy the relationship $(x_0^2+y_0^2)/\lambda z \ll 1$, the second propagation factor $\exp\{i[\pi(x_0^2+y_0^2)]/(\lambda z)\}$ in the above integrant can be neglected, the diffraction is equal to Fraunhofer diffraction. The expression of the Fraunhofer diffraction can be written as

$$A_2(x_2, y_2, z) = \frac{\exp(ikz)\exp[i\pi(x_2^2+y_2^2)/(\lambda z)]}{i\lambda z} \times \int\int b(x_0, y_0)\exp(i\frac{2\pi}{\lambda z}(x_0x_2+y_0y_2))\mathrm{d}x_0\mathrm{d}y_0. \tag{3}$$

If both the aperture and the incident beam are axially symmetric, $a(x_0, y_0)$ and $p(x_0, y_0)$ can be expressed by the cylindrical coordinates $a(r_0)$ and $p(r_0)$, respectively, Eqs. (2) and (3) may be simplified in one-dimension in-

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tegral transformation

$$A_1(r_1,z) = 2\pi \frac{\exp(ikz)\exp(\frac{i\pi r_1^2}{\lambda z})}{i\lambda z} \times \int_0^\infty a(r_0)p(r_0)\exp(i\frac{\pi r_0^2}{\lambda z})J_0[i2\pi r_0 r_1/(\lambda z)]r_0 \mathrm{d}r_0, \quad (4)$$

and

$$A_2(r_2,z) = 2\pi \frac{\exp(ikz)\exp(\frac{i\pi r_2^2}{\lambda z})}{i\lambda z} \times \int_0^\infty a(r_0)p(r_0)J_0[i2\pi r_0 r_2/(\lambda z)]r_0 \mathrm{d}r_0, \quad (5)$$

where J_0 denotes the zero-order Bessel function. Let $r_1 = 0$, $r_2 = 0$ respectively in above two equations, we can easily obtain the formulas of axial diffraction of Gaussian beam truncated by an aperture

$$A_1(z) = 2\pi \frac{\exp(ikz)}{i\lambda z} \int_0^\infty a(r_0)p(r_0)\exp(i\frac{\pi r_0^2}{\lambda z})r_0 \mathrm{d}r_0, \quad (6)$$

and

$$A_2(z) = 2\pi \frac{\exp(ikz)}{i\lambda z} \int_0^\infty a(r_0)p(r_0)r_0 \mathrm{d}r_0. \quad (7)$$

Next, we take a circular hole as an example to discuss the axial diffraction of Gaussian beam in detail.

Suppose the spatial amplitude distribution of Gaussian beam on the aperture plane is $a(r_0) = L\exp(-r_0^2/\omega_0^2)$, where L is the peak value of the spatial amplitude distribution of Gaussian beam and ω_0 denotes the wave waist of Gaussian beam. The aperture diaphragm takes the circular hole with the radius R, and its pupil function can be expressed by $p(r_0) = \mathrm{circ}(r_0/R)$. Substituting these forms into Eqs. (6) and (7), and according to $I_1 = |A_1|^2$ and $I_2 = |A_2|^2$, the formulas of the axial intensity distributions in Fresnel diffraction region and Fraunhofer diffraction zone are easily obtained as

$$I_1(z) = \frac{L^2}{1+\frac{\lambda^2 z^2}{\pi^2\omega_0^4}} \times \left[1 - 2\exp\left(-\frac{R^2}{\omega_0^2}\right)\cos\left(\frac{\pi R^2}{\lambda z}\right) + \exp\left(-\frac{2R^2}{\omega_0^2}\right)\right], \quad (8)$$

$$I_2(z) = \frac{\pi^2 L^2 \omega_0^4}{\lambda^2 z^2}\left[1 + \exp\left(-\frac{R^2}{\omega_0^2}\right)\right]^2. \quad (9)$$

From Eq. (9), it is easy to see that in far field, the diffraction intensity of Gaussian beam limited by a circular aperture on the propagation axis is in inverse proportion to the square of the propagation distance. Equation (8) gives the general propagation rule of the truncated Gaussian beam in Fresnel zone.

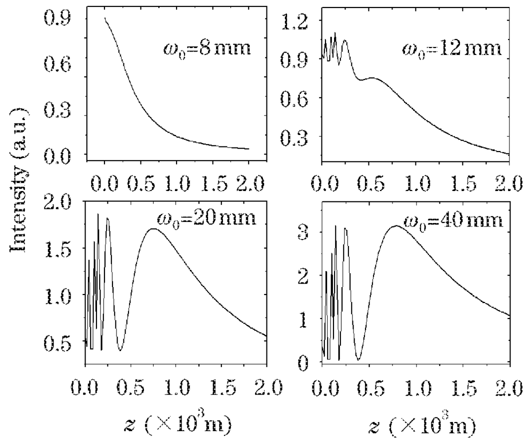

Fig. 1. Fresnel diffraction of Gaussian beam with different wave waist limited by a circular aperture.

For visualization, we perform the numerical calculation of the axial intensity diffractions, and the calculated results are shown in Fig. 1. The axial diffraction intensity distributions are provided when the wave waist of Gaussian beam takes different values, where the radius of the circular aperture takes 20 mm and the wavelength of the incident light is 0.5 μm. From Fig. 1, we can see that the axial intensity distribution is smooth decreasingly when the wave waist of Gaussian beam is smaller compared with the caliber of the circular aperture. Along with the increase of wave waist of Gaussian beam, the intensity profile appears the oscillation in Fresnel diffraction region. Moreover, when the propagation distance of laser beam is larger than 0.8×10^3 m, i.e., 4×10^5 multiples of the radius of circular aperture, the axial intensity shows the monotone decrease as the propagation distance increases.

In Gaussian optics, the spatial intensity distribution of Gaussian beam in free space can be expressed as $I_G(r,z) = I_0|\omega_0 \exp[-r^2/\omega^2(z)]/\omega(z)|^2$, where $\omega(z) = \omega_0[1 + (\lambda z/\pi\omega_0^2)^2]^{1/2}$, thus, the axial intensity can be expressed as $I_G(0,z) = \frac{I_0}{1+[\lambda z/(\pi\omega_0^2)]^2}$. When the waist of Gaussian beam is much smaller than the caliber of the aperture, R/ω_0 is very big, $\exp(-R^2/\omega_0^2)$ and $\exp(-2R^2/\omega_0^2)$ are very small in Eq. (8), the axial intensity distribution follows the relationship of $I_1(z) = L^2/[1 + (\lambda^2 z^2)/(\pi^2\omega_0^4)]$, which is the same as the propagation of Gaussian beam in free space[12]. Therefore, the intensity is monotone decreasing as the propagation distance increases. When the propagation distance is far enough, $(\lambda z/\pi\omega_0^2)^2 \gg 1$ (this is just the condition of the far field), the axial intensity can be simplified into $I_G(0,z) = (\pi^2\omega_0^4 I_0)/)\lambda^2 z^2)$, just like Eq. (9). Here, the diffraction intensity I_G is in inverse proportion to the square of propagation distance z.

From Figs. 1(a)−(d), the wave waist of Gaussian beam increases gradually, in another word, the caliber of the hard-edged aperture decreases relatively, the axial diffraction intensity takes on the oscillation in Fresnel diffraction field. The reason is that R/ω_0 gets smaller with the waist increasing, and $\exp(-R^2/\omega_0^2)$ becomes bigger gradually, $-2\exp(-R^2/\omega_0^2)\cos((\pi R^2)/(\lambda z))$ in Eq. (8) cannot be neglected which is just the origin of the oscillation. The position of each extremum point is approximately estimated. Furthermore, the more the wave waist ω_0 increases, the bigger the amplitude of the fluctuation $2\exp(-R^2/\omega_0^2)$ becomes, the more severe the oscillation of the diffraction intensity is.

When the wave waist is infinite, the axial intensity can be written as $I_1(z) = L^2[2-2\cos(\pi R^2/\lambda z)]$, which is the diffraction of circular aperture illuminated by the parallel light[13]. Since the diffraction distribution is only related to the caliber of the aperture and the positions of the extremum points are located accurately ($z = R^2/k\lambda$ ($k = 0, 1, 2\cdots$)), it will simplify many problems about the propagation of Gaussian beam. However, in the propagation of the practical beam, the waist cannot take infinite value. When does the truncated Gaussian beam equal to the parallel light? We perform the numerical calculation of the axial diffractions of Gaussian beam with a big wave waist limited by circular aperture and that of the circular hole illuminated by parallel light. The corresponding results are shown in Fig. 2, where the radius of circular aperture still takes 20 mm, the wavelength of incident light is 0.5 μm, and the wave waist of Gaussian beam is 100 mm. It can be seen that when $R/\omega_0 = 0.2$, the axial intensity diffraction of Gaussian beam (shown as the solid line) conforms very well to the diffraction of the circular aperture (shown as the scatter squares).

Therefore, the axial diffraction comes from the contributions of two parts, one is the free diffraction of a part of Gaussian beam passing through the aperture, and the other is the boundary diffraction effect of the hard-edged aperture. When the wave waist of Gaussian beam is very small, the diffraction field on the propagation axis presents mainly the characteristics of Gaussian beam. In contrast, when the wave waist of Gaussian beam is very big, the diffraction field on the propagation axis takes on the effect of the boundary.

In Fraunhofer diffraction field, the axial diffraction intensity is in inverse proportion to the square of the propagation distance according to Eq. (9). From Figs. 1 and 2, it can be seen that if the propagation distance z is bigger than 800 m, that is, z/R is bigger than 4×10^4, this rule is satisfied very well.

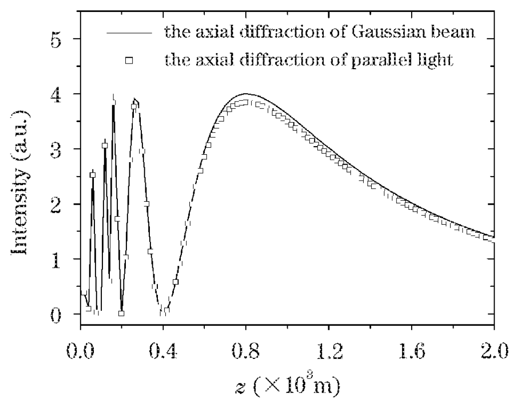

Fig. 2. Axial intensity diffractions of Gaussian beam limited by a circular aperture (shown as solid line) and that of the parallel light through a circular aperture (shown as scatter squares).

In conclusion, in this letter, we perform the theoretic study of the axial diffraction of Gaussian bean limited by a hard-edged aperture, and provide the simpler formulas of the axial diffraction intensity distribution in Fresnel and Fraunhofer diffraction region. Moreover, we discuss in detail the characteristics of the axial diffraction of Gaussian beam limited by an aperture by calculating numerically and analyzing theoretically, and interpret some interesting phenomena, such as the monotone decrease of the axial diffraction intensity of Gaussian beam with a small wave waist, the oscillation of the axial diffraction distribution of Gaussian beam with a big wave waist in Fresnel zone, the smooth in Fraunhofer region, the equivalence of the truncated Gaussian beam and Gaussian beam in free space, and the parallel light, and so on. These conclusions are helpful for the practice application of laser beam propagation in military and civil domain.

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