# Continuous-wave four-wave mixing with linear growth based on electromagnetically dual induced transparency

**Jiahua Li (李家华)[1], Wenxing Yang (杨文星)[2], and Jucun Peng (彭菊村)[3]**

[1] *Department of Physics, Huazhong University of Science and Technology, Wuhan 430074*

[2] *Centre for Cold Atom Physics, Wuhan Institute of Physics and Mathematics, Chinese Academy of Sciences, Wuhan 430071*

[3] *Department of Physics, Xiaogan Normal University, Xiaogan 432100*

Received March 8, 2004

Using Schrödinger-Maxwell formalism, we propose and analyze a continuous-wave four-wave mixing (FWM) scheme for the generation of coherent light in a six-level atomic system based on electromagnetically dual induced transparency. We derive the corresponding explicit analytical expressions for the generated mixing field. We find that the scheme greatly enhances FWM production efficiency and is also capable of inhibiting and delaying the onset of the detrimental three-photon destructive interference by choosing the proper decay rate in the second electromagnetically induced transparency (EIT) process. In addition, such an optical process also provides possibilities for producing short-wave-length coherent radiation at low pump intensities.

*OCIS codes:* 190.4380, 190.4180, 270.1670.

Electromagnetically induced transparency (EIT) is an optical transparency of three-level medium at a resonant transition induced by application of a coherent electromagnetic field at an adjacent transition. The accompanying reduction of the group velocity of light and large enhancement of the nonlinearity of the coherent atomic medium at the few-photon level by many orders of magnitude in the EIT transparency windows have been intensively and actively investigated in connection with many potential applications, especially low-intensity nonlinear optics (i.e., it can reach a level of few photons), optical multi-wave mixing, quantum information processing, and quantum information storage[1−15]. Yan *et al.* demonstrated that EIT in a standard Λ-type configuration can be used to suppress both single-photon and two-photon absorptions simultaneously[3]. Harris *et al.* proposed using EIT to suppress absorption of the short-wavelength light generated in a four-wave mixing (FWM) scheme and showed that the FWM efficiency can be greatly enhanced[5]. Recently, Deng *et al.* reported the first theoretical investigation of optical coherent four-wave mixing (OCFWM) with a weak probe based on EIT and showed that such a scheme can lead to many orders of magnitude enhancement in the amplitude of the generated wave in a typical four-level atomic system[9]. Later on, Wu *et al.* analyzed and discussed a FWM scheme in a five-level atomic system and hyper-Raman scattering (HRS) in resonant coherent media by the use of EIT, which lead to suppress single-photon, two-photon, and three-photon absorptions in both FWM and HRS schemes and enable the FWM to proceed through real and resonant intermediate states without absorption loss[11,12]. The recent studies showed when the generated field becomes sufficiently intense in such an efficient FWM process, however, it can be absorbed dramatically. Consequently, a robust three-photon destructive interference between the two different excitation channels occurs, resulting in a saturated FWM production. Quite recently, a new scheme proposed by Deng *et al.* retains the significantly enhanced conversion efficiency enabled by ultra-slow propagation of pump waves, yet is also capable of inhibiting and delaying the onset of detriment three-photon destructive interference which limits the further growth of FWM[14,15]. In this letter, we propose and analyze a continuous-wave (CW) FWM scheme that is modified from the Deng's scheme[15]. Specifically, there are two differences: 1) we use the real intermediate level to replace the virtual intermediate level due to the fact that EIT may suppress single-photon and multiphoton absorptions simultaneously, which can lead to significant enhancement of the FWM production efficiency[11]; 2) we use CW lasers to replace pulsed lasers, which facilitates the experimental implementation and the following analysis of the steady state[4].

Consider the FWM scheme depicted in Fig. 1. Three fields (frequencies are $\omega_p$, $\omega_1$, and $\omega_2$, respectively) induce the FWM processes $|0\rangle \to |2\rangle \to |3\rangle \to |5\rangle \to |0\rangle$ and generate the coherent radiation field (amplitude is $E_m(\omega_m, z)$) at a frequency $\omega_m = \omega_p + \omega_1 + \omega_2$. The control fields with frequencies $\omega_{c1}$ and $\omega_{c2}$ couple the transitions $|1\rangle \leftrightarrow |2\rangle$ and $|4\rangle \leftrightarrow |5\rangle$ to create EIT effect, respectively.

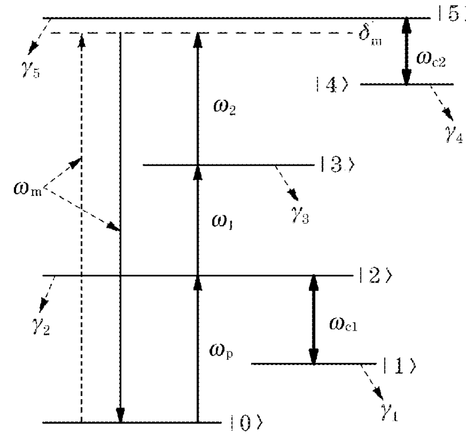


Fig. 1. Schematic diagram of the six-level atomic system.

In Fig. 1, a CW control field $\Omega_{c1}$ with frequency $\omega_{c1}$ creates a transparency window to reduce the absorption of another CW pump field $\Omega_p$. Another CW control field $\Omega_{c2}$ with frequency $\omega_{c2}$ drives the states $|5\rangle$ transparent for the three-photon excitation. The dashed upward arrows represent the absorptions of the generated waves, resulting in a second excitation channel to the same FWM state $|5\rangle$.

In the Schrödinger picture, using the electro-dipole and rotating-wave approximations, the semiclassical Hamiltonian describing the atom-field interaction for the system under study can be written as

$$H = \sum_{j=0}^{5} \varepsilon_j |j\rangle\langle j| - \left(\Omega_{c1} e^{i\theta_{c1}} |2\rangle\langle 1| + h.c.\right) - \left(\Omega_p e^{i\theta_p} |2\rangle\langle 0| + h.c.\right) - \left(\Omega_1 e^{i\theta_1} |3\rangle\langle 2| + h.c.\right) - \left(\Omega_2 e^{i\theta_2} |5\rangle\langle 3| + h.c.\right) - \left(\Omega_{c2} e^{i\theta_{c2}} |5\rangle\langle 4| + h.c.\right) - \left(\Omega_m e^{i\theta_m} |5\rangle\langle 0| + h.c.\right), \quad (1)$$

where $\theta_n = \vec{k}_n \cdot \vec{r} - \omega_n t \, (n = p, 1, 2, \mathrm{m}, \mathrm{c1}, \mathrm{c2})$ are phase factors of the positive frequency fields. $\Omega_n$ denote one-half Rabi frequencies for the relevant laser-driven transitions, i.e., $\Omega_p = D_{20}E_p/(2\hbar)$, $\Omega_{c1} = D_{21}E_{c1}/(2\hbar)$, $\Omega_{c2} = D_{54}E_{c2}/(2\hbar)$, $\Omega_1 = D_{32}E_1/(2\hbar)$, $\Omega_2 = D_{53}E_2/(2\hbar)$, $\Omega_m = D_{50}E_m/(2\hbar)$, with $D_{ab}$ standing for the dipole moment elements for the transition between levels $|a\rangle$ and $|b\rangle$. $\varepsilon_j \, (j = 0 - 5)$ is the energy of the atomic state $|j\rangle$. For simplicity, in what follows we take $\varepsilon_0 = 0$ for the ground state $|0\rangle$ as the energy origin. Turning to the interaction picture, the Hamiltonian can be rewritten as

$$H_{\text{int}} = -\Delta\omega_1 |1\rangle\langle 1| - \Delta\omega_2 |2\rangle\langle 2| - \Delta\omega_3 |3\rangle\langle 3| - \Delta\omega_4 |4\rangle\langle 4| - \Delta\omega_5 |5\rangle\langle 5| - (\Omega_p e^{i\vec{k}_p\cdot\vec{r}} |2\rangle\langle 0| + \Omega_{c1} e^{i\vec{k}_{c1}\cdot\vec{r}} |2\rangle\langle 1| + \Omega_1 e^{i\vec{k}_1\cdot\vec{r}} |3\rangle\langle 2| + \Omega_2 e^{i\vec{k}_2\cdot\vec{r}} |5\rangle\langle 3| + \Omega_{c2} e^{i\vec{k}_{c2}\cdot\vec{r}} |5\rangle\langle 4| + \Omega_m e^{i\vec{k}_m\cdot\vec{r}} |5\rangle\langle 0| + h.c.), \quad (2)$$

where $\Delta\omega_1 = \omega_p - \omega_{c1} - \varepsilon_1$ and $\Delta\omega_3 = \omega_p + \omega_1 - \varepsilon_3$ are two separate two-photon detunings. $\Delta\omega_2 = \omega_p - \varepsilon_2$, $\Delta\omega_4 = \omega_p + \omega_1 + \omega_2 - \omega_{c2} - \varepsilon_4$, and $\delta_m = \Delta\omega_5 = \omega_p + \omega_1 + \omega_2 - \varepsilon_5$ are single-photon, four-photon, and three-photon detunings, respectively.

Let us assume that the wave function has the form

$$|\Psi\rangle = A_0 |0\rangle + A_1 e^{i(\vec{k}_p - \vec{k}_{c1})\cdot\vec{r}} |1\rangle + A_2 e^{i\vec{k}_p\cdot\vec{r}} |2\rangle + A_3 e^{i(\vec{k}_p + \vec{k}_1)\cdot\vec{r}} |3\rangle + A_4 e^{i(\vec{k}_p + \vec{k}_1 + \vec{k}_2 - \vec{k}_{c2})\cdot\vec{r}} |4\rangle + A_5 e^{i\vec{k}_m\cdot\vec{r}} |5\rangle, \quad (3)$$

where $A_j \, (j = 0 - 5)$ denote the probability amplitudes of the atomic states.

Making use of the Schrödinger equation in the interaction picture $i\partial_t |\Psi\rangle = H_{\text{int}} |\Psi\rangle$, the equations of motion for the probability amplitudes of the atomic wave functions can be readily obtained as

$$(\partial_t + \tilde{\gamma}_1) A_1 = i\Omega_{c1}^* A_2 \quad (4a)$$

$$(\partial_t + \tilde{\gamma}_2) A_2 = i\Omega_p A_0 + i\Omega_{c1} A_1 + i\Omega_1^* A_3 \quad (4b)$$

$$(\partial_t + \tilde{\gamma}_3) A_3 = i\Omega_1 A_2 + i\Omega_2^* A_5 e^{i\delta\vec{k}\cdot\vec{r}} \quad (4c)$$

$$(\partial_t + \tilde{\gamma}_4) A_4 = i\Omega_{c2}^* A_5 e^{i\delta\vec{k}\cdot\vec{r}} \quad (4d)$$

$$(\partial_t + \tilde{\gamma}_5) A_5 = i\Omega_{c2} A_4 e^{-i\delta\vec{k}\cdot\vec{r}} + i\Omega_m A_0 + i\Omega_2 A_3 e^{-i\delta\vec{k}\cdot\vec{r}}, \quad (4e)$$

where we have introduced the new definitions $\tilde{\gamma}_n = \gamma_n - i\Delta\omega_n \, (n = 1 - 5)$. The notation $\delta\vec{k} = \vec{k}_m - \left(\vec{k}_p + \vec{k}_1 + \vec{k}_2\right)$ represents the phase matching condition and $\gamma_n \, (n = 1 - 5)$ are the decay rates of state $|j\rangle$. In the following, we consider all laser fields to be continuous and analyze the FWM process in the steady state. Following the standard method described before in Refs. [4] and [12], the pump laser fields are assumed to be sufficiently weak ($|\Omega_{p,1,2}| \ll |\Omega_{c1,c2}|$), so that in essence all of the atomic population remains in the ground state $|0\rangle$, i.e., $A_0 \approx 1$. Therefore, we can obtain straightforwardly the steady-state solution of Eqs. (4) and the slowly varying part of the polarization of the generated FWM field, i.e., $P = ND_{05}A_5A_0^*$. In the limit of plane waves and slowly varying amplitude approximations, the generated FWM field $\Omega_m = \Omega_m(\omega_m, z)$ obeys Maxwell's equation

$$(\partial_t + V_g \partial_z)\Omega_m = V_g \cdot iM\left(R\Omega_m - Le^{-i\delta kz}\right), \quad (5)$$

with

$$M = \frac{\omega_m N |D_{05}|^2}{4\hbar\varepsilon_0 c}$$

$$R = i\tilde{\gamma}_4\left(\tilde{\gamma}_1\tilde{\gamma}_2\tilde{\gamma}_3 + \tilde{\gamma}_3|\Omega_{c1}|^2 + \tilde{\gamma}_1|\Omega_1|^2\right) / [\left(\tilde{\gamma}_1\tilde{\gamma}_2\tilde{\gamma}_3 + \tilde{\gamma}_3|\Omega_{c1}|^2 + \tilde{\gamma}_1|\Omega_1|^2\right)\left(\tilde{\gamma}_4\tilde{\gamma}_5 + |\Omega_{c2}|^2\right) + \tilde{\gamma}_4|\Omega_2|^2\left(\tilde{\gamma}_1\tilde{\gamma}_2 + |\Omega_{c1}|^2\right)]$$

$$L = i\tilde{\gamma}_1\tilde{\gamma}_4\Omega_1\Omega_2\Omega_p / [\left(\tilde{\gamma}_1\tilde{\gamma}_2\tilde{\gamma}_3 + \tilde{\gamma}_3|\Omega_{c1}|^2 + \tilde{\gamma}_1|\Omega_1|^2\right)\left(\tilde{\gamma}_4\tilde{\gamma}_5 + |\Omega_{c2}|^2\right) + \tilde{\gamma}_4|\Omega_2|^2\left(\tilde{\gamma}_1\tilde{\gamma}_2 + |\Omega_{c1}|^2\right)],$$

where $V_g$ is the group velocity of the generated FWM field. Neglecting pump depletion, we can readily solve Eq. (5) for the slowly varying field $\Omega_m$. Applying the boundary condition, $\Omega_m|_{z=0} = 0$ yields explicit analytical expression of the generated FWM field

$$\Omega_m(\omega_m, z) = \frac{ML}{MR + \delta k}\left(e^{-i\delta kz} - e^{iMRz}\right). \quad (6)$$

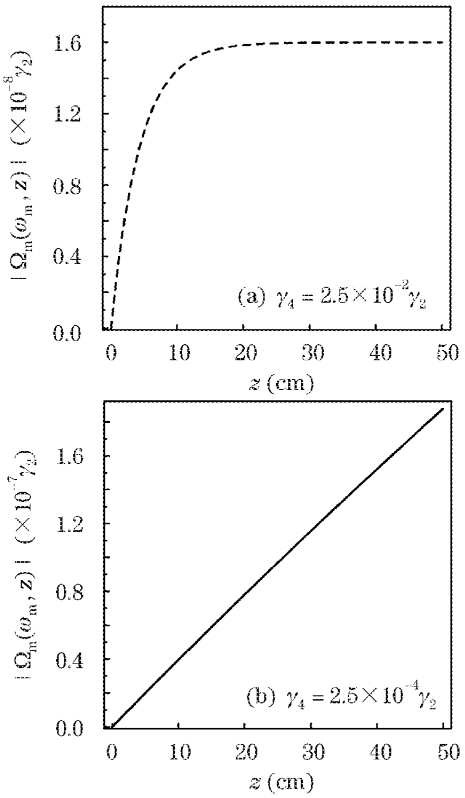


Fig. 2. Plots of the generated FWM field $|\Omega_m(\omega_m, z)|$ as a function of propagation distance $z$. The fitting parameters: $\Omega_{c1} = 0.5\gamma_2$, $\Omega_{c2} = 0.1\gamma_2$, $\Omega_p = \Omega_1 = \Omega_2 = 0.02\gamma_2$, $\gamma_1 = 10^{-5}\gamma_2$, $\gamma_3 = 2 \times 10^{-2}\gamma_2$, $\gamma_5 = 10^{-2}\gamma_2$, $\frac{\omega_m N |D_{05}|^2}{4\hbar\varepsilon_0 c} = 0.1\gamma_2$ cm$^{-1}$.

In the following analysis, for the CW control and pump fields we will only consider the case of resonant excitations, i.e., $\Delta\omega_j = 0\,(j = 1 - 5)$, in which the EIT effects are the most prominent. Under the conditions of perfect phase matching $\delta k = 0$ and weak pump field $\Omega_{p,1,2}$, we have analyzed the influence of the decay rate $\gamma_4$ of the metastable state $|4\rangle$ on the generated FWM field $\Omega_m$, as shown in Figs. 2(a) and (b). Note that, in this paper, parameters $\Omega_{p,1,2,c1,c2}$, $\gamma_j$ and $M$ are scaled with $\gamma_2$. From Fig. 2(a), we find that the amplitude of the generated FWM field increases monotonously with the increase of the propagation distance $z$ firstly, then the amplitude of the FWM field reaches the saturation value and is independent of the propagation distance $z$. This is the behavior of a quantum destructive interference between the two different excitation channels: $|0\rangle \rightarrow |5\rangle$ (the dotted line in Fig. 1) and $|0\rangle \rightarrow |2\rangle \rightarrow |3\rangle \rightarrow |5\rangle$[14,15]. From Fig. 2(b), we can observe that the amplitude of the generated FWM field increases linearly with the propagation distance $z$ increasing in the same region of propagation depth. This indicates the absence of the quantum destructive interference. Note that the amplitude of the generated FWM field in this case is an order of magnitude higher than that achievable when the destructive interference is present.

In conclusion, in the present work we analyze a CW FWM scheme in the context of dual-EIT. The results show that the proper choice of the decay rate of the metastable state can reduce the absorption of the generated FWM field and increase the propagation distance under perfect phase matching condition. At the same time, it is proved that the dual-EIT scheme is capable of substantially delaying the onset of the three-photon quantum destructive interference. It may have important technical implications in ultra-slow propagation enhanced nonlinear optical schemes.

The authors would like to thank Dr. Ying Wu for many stimulating discussions. This work was supported in part by the National Natural Science Foundation of China under Grant No. 90103026 and 10125419. J. Li's e-mail address is huajia_li@163.com.

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