Quasi-soliton generation in solid-state lasers with semiconductor saturable absorber

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Recent advances in ultrafast, ultra-short solid-state lasers have resulted in sub-6 fs pulses generated directly from the cavity of Ti:sapphire lasers. The generation of extremely short pulses is possible due to the formation of a quasi-Schrödinger soliton. Our investigation is directed to the peculiarities of the transition between femtosecond to picosecond generation. We found that the above transition is accompanied by the threshold and hysteresis phenomena. On the basis of soliton perturbation theory, the numerical simulation studying two different experimental situations has been performed, the first situation corresponds to the study of the lasers field's parameters under variation of control parameters (dispersion or pump power), the second one is for continuous variation of control parameter within a single generation session. Physically it corresponds to not repeated laser session but the variation of control parameter when the pulse has formed already.

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Recently, a considerable progress has been made in self-starting femtosecond lasers using semiconductor structures[1]. This allows the pulses as short as 6.5 fs to be generated directly from the resonator[2]. The laser systems with semiconductor absorbers combine the advantages of the Kerr lens mode-locked system, the self-starting ability, and the cavity alignment insensitivity[3]. The most striking feature of semiconductor absorbers used in the experiment is a long recovery time T (hundreds of femtoseconds) as compared with pulse duration. To explain an extremely short pulse generation, a soliton mode-locking mechanism was proposed[4]. This mechanism involves the stabilization of the Schrödinger soliton against laser continuum (noise) due to noise decaying within the positive net-gain window, which is much longer than the pulse duration. This decaying is the result of the dispersion spreading of the noise and the difference between the noise and pulse group velocities. However, as well as semiconductor loss saturation and slow change of the refractivity, other nonlinear effects, in particular as Stark effect, can contribute to mode-locking and produce a strong self-amplitude modulation (for example Ref. [5]). It was shown that the intracavity intensities in the real solid-state lasers are high enough to produce an ultrafast nonlinear response due to ac Stark effect at blow resonance excitation[6] and thus have to be taken into account.

Here we present a simple model for mode-locking mechanism in the presence of the quadratic blue Stark shift of the excitonic resonance at blow resonance excitation. In Ref. [7], an external signal driven anti-resonant Fabry-Perot modulation based on quantum-combined Stark effect in multi-quantum well semiconductor structures was used to actively mode-lock a diode-pumped Nd:YLF laser. However, in our case the mode-locking mechanism is purely due to the pulse self-action, which is power dependent and can self-start an ultra-short pulse operation.

The ac Stark shift which is due to the influences of non-resonant transitions on excitonic resonance ($\Delta\omega$) is proportional to the polarizibilty difference between ground and excited states Δa[8]: $\Delta\omega = |\Delta a| \times |E|/h$, where E is the field strength, and h is the planck constant. The typical values of $|\Delta a|$ for semiconductors are of order $(10^{-19} - 10^{-21})$ cm^3[9], corresponding to the Stark shift coefficient $\zeta = 8\pi|\Delta a|/(nch) = 1.26\times10^5 - 10^3/n$ cm$^2\cdot$J^{-1}, where n is the index of refractivity, and c is light velocity.

The evolution of the complex field envelope $a(t)$ in the laser system containing the gain medium, saturable absorber, frequency filter, and dispersive element obeys the nonlinear operator's equation $a^{k+1}(t) = AGDa^k(t)$, where k is the round trip number, t is the local time. The Lorenzian gain band is described as $G = \exp\left(\frac{gL_g}{1+L_g t_g \frac{\partial}{\partial t}}\right)$, where $L_g = \frac{1}{1+i(\omega_l-\omega_g)t_g}$, ω_l is the pulse carrier frequency, ω_g is the gain band center, t_g is the inverse gain bandwidth (we assumed t_g=20 fs for the Cr:forsterite laser with modulator based on PbS quantum-dot doped glass[3]), g is the saturated gain. $D = \exp\left(id\frac{\partial^2}{\partial t^2}\right)$ is the second-order group velocity dispersion operator, where d is the dispersion coefficient. Neglecting the population of higher energy levels of the semiconductor, for weak exciton-exciton and exciton-phonon bound approximation, one can describe the interaction between laser field and quantum-confined absorber by the generalized two-level model[8]. The evolutions of the off-diagonal element of the density matrix π and the population difference between ground and excited states η obey the differential equation

$$\frac{\mathrm{d}\pi}{\mathrm{d}t} + (t_a^{-1} - i(\omega_l - \omega_a - \Delta\omega))\pi = \frac{i}{h}\wp\eta,$$

$$\frac{\mathrm{d}\eta}{\mathrm{d}t} + \frac{\eta-\eta_0}{T} = -\frac{4}{h}\mathrm{Im}(\pi\wp^*),$$

where t_a is the inverse bandwidth of the absorption line, ω_a is the resonance frequency, $\wp$ is the matrix element of the interaction, η_0 is the equilibrium population difference. The Stark shift which is possible only in generalized two-level model, in quasi-monochromatic approximation, i.e. the cross modulation between different

pulse spectral components, is proportional to $\varsigma |a(t)|^2$. In the incoherent approximation, the combined effects of saturable absorption and Stark effect contribution are contained in the operator

$$A = \exp\left\{ \frac{-\gamma L_{\rm a} \exp\left[-\frac{{\rm Re}(L_{\rm a})}{U_{\rm a}} \int\limits_{t_0}^{t} |a(t')|^2 \, {\rm e}^{-\frac{t-t'}{T}} {\rm d}t' \right]}{1 + L_{\rm a} t_{\rm a} \frac{\partial}{\partial t}} - r \right\},$$

where $L_{\rm a} = \frac{1}{1+i[\omega_{\rm l}-(\omega_{\rm a}+\varsigma|a(t)|^2)]t_{\rm a}}$, $U_{\rm a}$ is the saturation energy, γ is the saturated loss at the time corresponding to the pulse peak, r is the unsaturated loss. The integral in the above equation accounts for the ordinary slow saturable absorption with recovery time T. We have assumed a Lorenzian profile for the excitonic resonance and neglected the two-photon absorption as well as other high-order intensity dependent effects. We normalized all times on $t_{\rm g}$, frequencies' dispersion coefficients on $(t_{\rm g})^2$, and intensities on $U_{\rm a}/t_{\rm g}$.

Using the expression for A one may obtain the instant intensity transmission

$$M(|a(t)|^2) = \exp\left[\frac{-2\Gamma}{1 + \omega^2\tau^2 - 2\omega\tau^2\chi |a(t)|^2} \right], \qquad (1)$$

where t is the local time, $\tau = t_{\rm a}/t_{\rm g}$, Γ is the nonsaturated loss, $\omega = \omega_{\rm t} - \omega_{\rm a}$, $\chi = U_{\rm a}\zeta/t_{\rm g}$. In our calculations $\chi = 13$, corresponding to $U_{\rm a} = 390$ μJ·cm^{-2}. It is seen that for the blow resonance excitation with $\omega < 0$ (e.g. for the red shift of the pulses carrier frequency from the excitation resonance), Eq. (1) describes the power-dependent loss saturation. The fast saturable absorption results from "pushing out" of the excitation resonance from the red shift pulses spectrum due to intensity-dependent blue Stark shift. The corresponding parameter of self-self amplitude modulation $\rho = \frac{\mu}{M} \frac{\partial M}{\partial |a|^2}\Big|_{|a|^2 \to 0}$ is plotted in Fig. 1 (laser mode cross-section in absorber $\mu = 30$ μm), depending on the mismatch between pulse carrier frequency and excitation resonance. One can see that ρ is

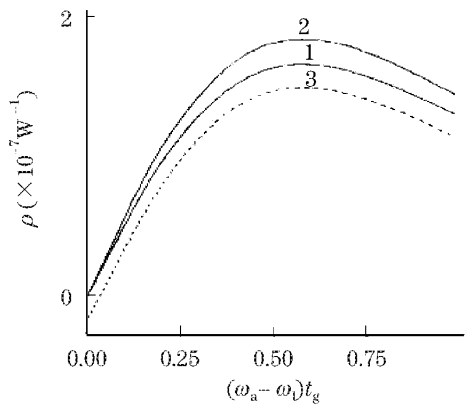

Fig. 1. Coefficient of self-amplitude modulation ρ versus normalized frequency mismatch from the excitonic resonance. Γ = 0.05 (curves 1 and 3), 0.1 (curve 2); coefficient of two-photon absorption is 23 cm·GW^{-1} (curve 3)[10].

of order 10^{-7} W^{-1} that is close to corresponding parameter typically for Kerr lens mode-locked system[11], but in contrast to the latter, the cavity is not restrained by the cavity geometrical parameters.

As expansion of the laser operator equation in series in t, local field energy and intensity, provided that the pulse duration is much shorter that T, give the following laser dynamical equation similar to the generalized Landau-Ginzburg equation (a similar equation for the physical situation in Ref. [12])

$$\frac{\partial a(k,t)}{\partial k} = \Big[c_1 + ic_2 \frac{\partial}{\partial t} + (c_3 + ic_4) \frac{\partial^2}{\partial t^2} + (c_5 + ic_6) |a(k,t)|^2 + (c_7 + ic_8)\varepsilon + (c_9 + ic_{10}) \frac{\varepsilon^2}{2} + (c_{11} + c_{12})\varepsilon \frac{\partial}{\partial t} \Big] a(k,t), \qquad (2)$$

where, $\varepsilon = \int\limits_{t_0}^{t} |a(k,t')|^2 \, {\rm d}t'$, $c_1 = gJ_{\rm g} - \gamma J_{\rm a} - r$, $c_2 = 2g\Omega J_{\rm g}^2 - 2\gamma\omega\tau^2 J_{\rm a}^2$, $c_3 = (1-3\Omega^2)J_{\rm g}^3 - \gamma(1-3\omega^2\tau^2)J_{\rm a}^3\tau^2$, $c_4 = g(\Omega^3 - 3\Omega)J_{\rm g}^3 - \tau(\omega^3\tau^3 - 3\omega\tau)J_{\rm a}^3\tau^2$, $c_5 = -2\gamma\omega\tau\chi J_{\rm a}^2$, $c_6 = -\gamma\chi(1-\omega^2\tau^2)J_{\rm a}^2$, $c_7 = \gamma J_{\rm a}^2$, $c_8 = -\gamma\omega\tau J_{\rm a}^2$, $c_9 = -\gamma J_{\rm a}^3$, $c_{10} = \gamma\omega\tau J_{\rm a}^3$, $c_{11} = -\gamma(1-\tau^2\omega^2)J_{\rm a}^3\tau$, $c_{12} = 2\omega\gamma\tau^2 J_{\rm a}^3$, $J_{\rm a} = \frac{1}{1+\omega^2\tau^2}$, $J_{\rm g} = \frac{1}{1+\Omega^2}$, $\Omega = \omega_{\rm l} - \omega_{\rm g}$. Here, the term $-2\chi\gamma J_{\rm a}^2\omega\tau |a(k,t)|^2 a(k,t)$ is responsible for the fast saturable absorption.

Equation (2) has quasi-soliton $a(k,t) = a_0 \sec h^{1+i\psi}[(t-k\delta)/t_{\rm p}] {\rm e}^{i\phi Z}$, where $t_{\rm p}$ is the pulse duration, a_0 is the amplitude, ψ is the chirp, δ and ϕ are the time and phase delay after the full round-trip, respectively. Pulse duration and frequency shift for chirp-free solution and group velocity dispersion are presented in Fig. 2. It is seen that the minimal pulse duration (close to the limit defined by $t_{\rm g}$) are provided by normalized absorption-gain lines mismatch $(\omega_{\rm a} - \omega_{\rm g})t_{\rm g} \approx 0.3 - 0.7$. The increase of nonsaturated loss Γ shortens the pulse (curve 3 compared with 1 in Fig. 2(a)), that corresponds to growth of ρ (curve 2 in Fig. 1). The minimum of the pulse duration on mismatch $\omega_{\rm a} - \omega_{\rm g}$ corresponds approximately to the maximum of ρ in Fig. 1. Using the absorber with smaller $U_{\rm a}$ increases $t_{\rm p}$ (curve 2) because the absorber operates in the regime of strong saturation which introduces an additional blue shift from the gain band center (curve 2 in Fig. 2(c), for the mechanism of this shift see Ref. [13]). Thus, the stronger saturation does not favor the Stark-induced mode-locking. Strong self-phase modulation due to ac Stark effect, which is described by coefficient c_6, requires the negative dispersion for chirp compensation (Fig. 2(b)). When the absorber band width is narrower than the gain band width (curve 4), the region of $\omega_{\rm a} - \omega_{\rm g}$ reduces, where chirp-free pulses exist, which can be explained by band overlapping of the pulse spectrum and excitonic line. However, for $t_{\rm a} > t_{\rm g}$, the generation of chirped pulses with duration close to $t_{\rm g}$ is possible (Fig. 3). The large τ requires positive dispersion d for pulse existence (curve 2 compared with 1), which may be explained by change of the sign of self-phase modulation coefficient in c_6. However, for large τ, i.e. for the absorption line much narrower than the gain

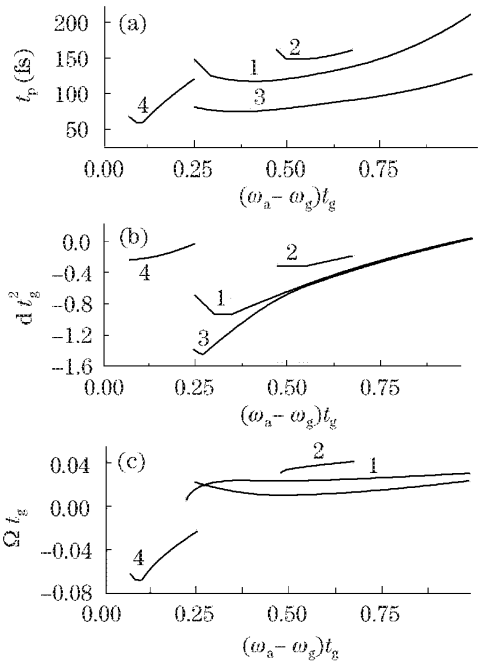

Fig. 2. Duration t_p (a), normalized second order group velocity dispersion dt_g^2 (b), and normalized frequency mismatch from the excitonic resonance Ωt_g (c) for chirp-free solution versus normalized mismatch between gain and loss resonances. $\chi = 13$ (curves 1, 3, 4), 8 (curve 2); Γ =0.05 (curves 1, 2), 0.1 (curves 3, 4); τ =1 (curves 1, 2, 3), 3 (curve 4); $g - r = 0.01$.

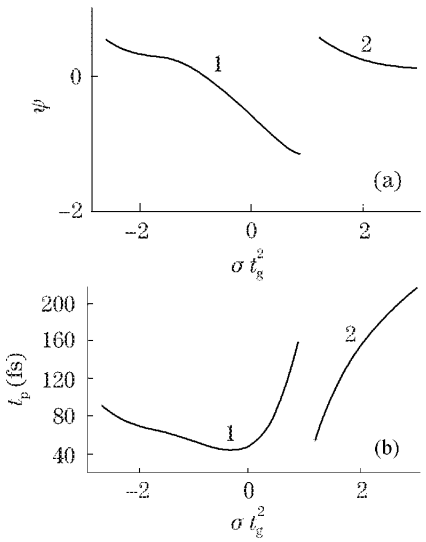

Fig. 3. Chirp ψ (a) and duration t_p (b) versus normalized group velocity dispersion. τ = 1 (curve 1), 30 (curve 2); $(\omega_a - \omega_g)t_g$ = 0.5 (curve 1), 0.2 (cuve 2); χ =13; Γ = 0.1; $g - r$ =0.001.

line, it would be necessary to account for the coherent nature of pulse-semiconductor interaction, which may cause new effects, for example self-induced transparency[14], and may transform mode-locking dynamics essentially.

The described dependence of the pulse parameters on the frequency shift from excitonic resonance was confirmed by Knox *et al.*[5]. It was shown that the resonance interaction with quantum-confined semiconductor structure does not favor an ultra-short pulse formation (Fig. 2(a)). Relatively, small Stark shift of the laser frequency from the excitonic resonance causes femtosecond pulse operating, while the increase of the Stark shift brings the pulse duration into picosecond region.

In order to perform the stability analysis we calculate the net-gain Σ behind the pulse table. Σ is the sum of saturated gain in active medium, loss in absorber, and unsaturable loss. Evidently, the pulse would be unstable, if $\Sigma > 0$ for some frequency ω_n, within the noise spectrum. Figure 4 shows Σ calculated for five selected values of ω_n (curves $1-5$). It is seen that the decrease of Γ (Fig. 4(a) compared with (c)), or, equally, decrease of U_a (Fig. 4(a), U_a = 390 μJ·cm^{-2} in comparison with U_a = 240 μJ·cm^{-2} in Fig. 4(b)) or growth of τ (Fig. 4(d)) destabilizes the chirp-free pulse, but the suitable detuning between absorption and gain lines $\omega_a - \omega_g$ and suitable absorber parameters τ and U_a allow for the generation of stable pulses with short duration, which is much shorter than the absorption recovery time.

To investigate the self-starting ability of the laser, we analyzed the stability of continuous wave (CW) regime. As the self-starting criterion the condition of modulation destabilization of CW operation and setting up pulsations with the repetition rate correlated with the laser repetition rate[15] was chosen. A CW-solution of Eq. (2) is $N = N_0 \exp(ik\xi)$, where real amplitude N_0 and phase ξ satisfy

$$\frac{a_{\max} J_g}{1 + \sigma N_0^2 J_g} - \frac{\Gamma J_a}{1 + N_0^2 J_a} = 2\Gamma\chi J_a^2 \tau\omega N_0^2,$$

$$\xi = \phi + (1 - \omega^2\tau^2)\gamma\chi J_a^2 N_0^2, \qquad (3)$$

here, all parameters have the same meaning as in Eq. (2), σ is the ratio of the loss saturation energy to the gain saturation energy ($\sigma = 10^{-3}$). The CW power is normalized to the loss saturation power, $a_{\max} = 1$. We

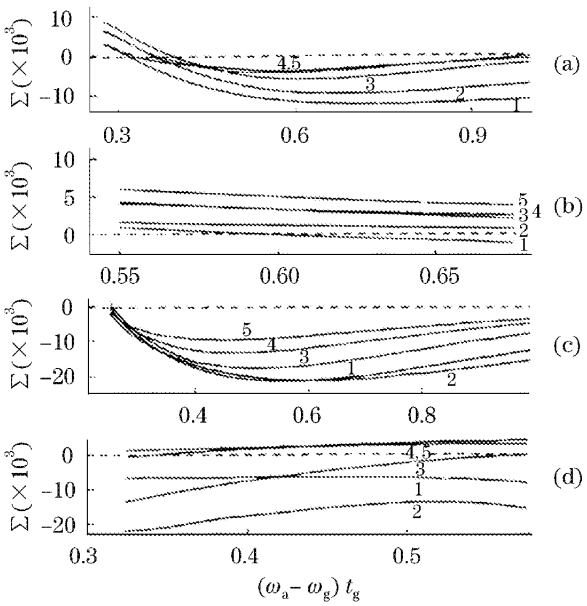

Fig. 4. Net-gain Σ behind the pulse tail for five selected noise frequencies ω_n. ω_n = 1 (curve 1), 0.5 (curve 2), 0 (curve 3), -0.5 (curve 4), -1 (curve 5); χ = 13 (a, c, d), 8 (b); Γ = 0.05 (a, b), 0.1 (c, d); τ = 1 (a, b, c), 3 (d).

investigate the stability of CW solution against complex perturbation $v = v_0 \exp(\lambda k)$. The self-starting occurs when $\mathrm{Re}(\lambda) > 0$ and $\mathrm{Im}(\lambda) = \pm l$, l is the natural number (nondecaying oscillations will be set up with repetition rate which is equal to be of inverse cavity round-trip). The linearization of Eq. (2) and summing up with complex conjugated give the equation for eigenvalues of perturbation[10]

$$\lambda = gJ_{\mathrm{g}} - \gamma J_{\mathrm{a}} + \left[\frac{\varepsilon_{\mathrm{a}} N_0^2 J_{\mathrm{a}}}{1 - i\omega T_{\mathrm{c^a}}} - \frac{\varepsilon_{\mathrm{g}} \sigma N_0^2 J_{\mathrm{g}}}{1 - i\omega T_{\mathrm{cg}}}\right][1 + \Re]$$

$$-\omega c_2 - \omega^2 c_3 + i\omega^2 c_4 + (c_5 - ic_6) N_0^2 [2 + \Re],$$

where

$$\Re = [\lambda + \gamma J_{\mathrm{a}} - gJ_{\mathrm{g}} + \frac{\varepsilon_{\mathrm{g}} \sigma N_0^2 J_{\mathrm{g}}}{1 - i\omega T_{\mathrm{cg}}} - \frac{\varepsilon_{\mathrm{a}} N_0^2 J_{\mathrm{a}}}{1 - i\omega T_{\mathrm{c^a}}}$$

$$+\omega c_2 + \omega^2 c_3 - i\omega^2 c_4 - 2(c_5 - ic_6) N_0^2] /$$

$$\left[\frac{\varepsilon_{\mathrm{a}} N_0^2 J_{\mathrm{a}}}{1 - i\omega T_{\mathrm{c^a}}} - \frac{\varepsilon_{\mathrm{g}} \sigma N_0^2 J_{\mathrm{g}}}{1 - i\omega T_{\mathrm{cg}}} + (c_5 - ic_6) N_0^2\right],$$

$T_{\mathrm{cg}} = \frac{T_{\mathrm{g}}}{1 + \sigma N_0^2 J_{\mathrm{g}}}$, T_{g} is the gain-medium upper state lifetime, $\varepsilon_{\mathrm{g}} = \frac{\sigma_{\max}}{(1 + \sigma N_0^2 J_{\mathrm{g}})^2}$, $\varepsilon_{\mathrm{a}} = \frac{\Gamma}{(1 + N_0^2 J_{\mathrm{a}})^2}$.

The dependence of $\mathrm{Im}(\lambda)$ versus $\mathrm{Re}(\lambda)$ is shown in Fig. 5 for three different mismatches between gain and loss lines. One can see that the CW regime is unstable for all three cases, but self mode-locking is possible only for the case 1, where the condition $\mathrm{Im}(\lambda) = 1$ is satisfied and self-amplitude modulation coefficient ρ (see Fig. 1) is close to its maximum.

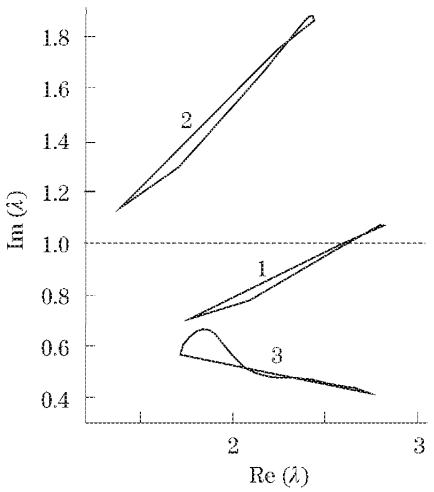

Fig. 5. Complex increment λ of CW perturbation to CW-solution. $\chi = 13$; $\Gamma = 0.1$; $\tau = 1$; $g - r = 0.01$; $(\omega_{\mathrm{a}} - \omega_{\mathrm{g}}) t_{\mathrm{g}} = 0.7$ (curve 1), 0.5 (curve 2), 1 (curve 3).

Now we will check the validity of approximation that neglected the contribution of two-photon absorption in comparison with one-photon absorption made in the beginning of the article. Comparing curves 3 and 2 in Fig. 1, plotted for the situation with and without two-photon absorption, one may conclude that in our case the two-photon absorption contribution is small and we may neglect it.

In conclusion, we have analyzed the mode-locking mechanism in CW solid-state laser with semiconductor saturable absorber in the presence of the Stark shift of the excitonic resonance. Calculations show that it is strong enough to self-start and maintain an ultra-short pulse generation.

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