

Aperture-averaging effects for weak to strong scintillations in turbulent atmosphere

Yixin Zhang (张逸新)^{1,2}, Tuo Zhu (朱拓)¹, and Chunkan Tao (陶纯堪)²

¹School of Science, Southern Yangtze University, Wuxi 214036

²School of Electronic Engineering and Optoelectronic Techniques, Nanjing University of Science and Technology, Nanjing 210014

Received February 16, 2004

Under the approximations of (1) the received irradiance fluctuations of an optical wave caused by small scale turbulent eddies are multiplicatively modulated by the fluctuations caused by large scale turbulent eddies; (2) the scintillations caused by small- and large-scale eddies, respectively, are statistically independent; (3) the Rytov method for optical scintillation collected by the finite-diameter receiving aperture is valid for light wave propagation under weak to saturation fluctuation regime, we develop the applicable aperture-averaging analytic formulas in the weak-to-strong-fluctuation for the scintillations of plane and spherical waves, which include the outer- and inner-scale rules of turbulence.

OCIS codes: 010.1300, 010.1290, 010.1330, 030.7060.

An optical wave propagating through a turbulent medium, such as the atmosphere, experiences irradiance fluctuations (called scintillation), even over relatively short propagation paths. If the collecting aperture diameter of an optical receiver is larger than the spatial scale of the optical scintillations caused by atmospheric turbulence, the receiver can average fluctuations of the received waveform over the aperture area, leading to reduced signal fluctuations. This phenomenon, known as aperture averaging, had been recognized in early astronomical measurements^[1]. Fried *et al.*^[2-9], following a development by Tatarskii^[10], presented many formulas for the aperture-averaging factor in the weak turbulence case and strong-turbulence regime. However, no good theory exists of aperture-averaging which includes the factor of turbulent out-and inner-scale for the weak to strong irradiance fluctuations regime.

In this paper, starting with three approximations, we carry out the evaluation of aperture-averaging factor by the calculation of the variance of log-irradiance fluctuations for propagation over a horizontal path. Our calculations are based on the modified von Karman power spectrum of turbulence^[11] to derive expressions for the aperture-averaging factor under weak to strong irradiance fluctuations.

The aperture-averaging factor G is defined as the ratio of the irradiance flux variance obtained by a finite-size collecting lens to that obtained by a point aperture and is expressed as^[2]

$$G = \sigma_I^2(D) / \sigma_I^2(0), \quad (1)$$

where $\sigma_I^2(D)$ and $\sigma_I^2(0)$ are the variance collected by the finite diameter receiving aperture and the variance collected by a point aperture receiver, D is the diameter of a circular collecting aperture.

Under the Rytov approximation, the field of an optical wave propagating at distance L from the source is represented by^[10]

$$U(\rho, L) = U_a(\rho, L) \exp[\psi(\rho, L)], \quad (2)$$

where $\rho = (x, y)$ is the observation point in the transverse plane at propagation distance L , $U_a(\rho, L)$ is the

optical field in the absence of turbulence, and $\psi(\rho, L)$ is a complex phase perturbation caused by atmospheric turbulence along the propagation path. The validity of the Rytov method as described by Eq. (2) is generally limited to weak-fluctuation regimes because it does not take into account the roles of the inner- and outer-scale of turbulence.

In order to modify the Rytov approximation which can be applied in weak to strong fluctuation regimes, we use the following approximations: (1) the irradiance of an optical wave can be modeled as a modulation process in which the fluctuations caused by inner-scale eddies are multiplicatively modulated by the beam wanderings caused by outer-scale eddies^[8,12]; (2) the scintillations caused by inner- and outer-scale eddies, respectively, are statistically independent^[8,12]; (3) the Rytov method for optical scintillation collected by the finite-diameter receiving aperture is valid for light wave propagation over a horizontal path under weak to saturation regime^[6]. By above three approximations, we represent Eq. (2) as

$$U(\rho, L) = U_0(\rho, L) U_m(\rho, L) \exp[\psi_0(\rho, L) + \psi_m(\rho, L)], \quad (3)$$

where $\psi_0(\rho, L)$ and $\psi_m(\rho, L)$ are statistically independent complex phase perturbations that are only due to large-scale and small-scale eddies, respectively. $U_0(\rho, L)$ and $U_m(\rho, L)$ are amplitudes which are statistically independent modulated by the beam wander caused by the large-scale turbulent eddies and the scintillations caused by the small-scale turbulent eddies, respectively. $I(\rho, L) = |U(\rho, L)|^2 = I_0 \cdot I_m$, where $I_0 = |U_0|^2$ and $I_m = |U_m|^2$. By defining, the scintillation index is given by^[8]

$$\sigma_I^2(\rho) = \langle I_0^2 \rangle \langle I_m^2 \rangle - 1 = (1 + \sigma_{I_0}^2)(1 + \sigma_{I_m}^2) - 1, \quad (4)$$

where $\rho = |\rho|$, $\sigma_{I_0}^2$ and $\sigma_{I_m}^2$ are the normalized variances of the irradiance fluctuations for the large- and small-scale eddies, respectively, and $\langle \cdot \rangle$ denotes an ensemble average. We further express the normalized variances

$\sigma_{I_0}^2$, $\sigma_{I_m}^2$, and σ_I^2 in the fashion $\sigma_{I_0}^2 = \exp(\sigma_{\ln I_0}^2) - 1$, $\sigma_{I_m}^2 = \exp(\sigma_{\ln I_m}^2) - 1$, and $\sigma_I^2 = \exp(\sigma_{\ln I}^2) - 1$, respectively, then Eq. (4) can be rewritten as

$$\sigma_I^2 = \exp(\sigma_{\ln I_0}^2 + \sigma_{\ln I_m}^2) - 1 = \exp(\sigma_{\ln I}^2) - 1, \quad (5)$$

where $\sigma_{\ln I_0}^2$, $\sigma_{\ln I_m}^2$, and $\sigma_{\ln I}^2$ are the log-irradiance variances of irradiance I_0 , I_m , and I , respectively. From Eq. (5), we have

$$\sigma_{\ln I}^2(\rho) = \sigma_{\ln I_0}^2(\rho) + \sigma_{\ln I_m}^2(\rho). \quad (6)$$

For a circular receiving aperture of diameter D , the scintillation indices associated with plane and spherical waves are given by^[7]

$$\sigma_{\ln I, \text{pl}}^2(D) = 8\pi^2 k^2 L \int_0^1 \int_0^\infty \kappa \Phi_n(\kappa, L_0, l_0) \exp(-\kappa^2 D^2/4) (1 - \cos L\kappa^2 \xi/k) d\kappa d\xi, \quad (7)$$

$$\sigma_{\ln I, \text{sph}}^2(D) = 8\pi^2 k^2 L \int_0^1 \int_0^\infty \kappa \Phi_n(\kappa, L_0, l_0) \exp(-\kappa^2 \xi^2 D^2/4) \{1 - \cos [L\kappa^2 \xi (1 - \xi)/k]\} d\kappa d\xi, \quad (8)$$

where $k = 2\pi/\lambda$ is the optical wave number, λ is the wavelength, $\Phi_n(\kappa, L_0, l_0)$ is the spectrum of refractive-index fluctuations, L_0 is the outer scale of turbulence, and l_0 is the inner scale of turbulence.

The commonly used spectral model is modified von Karman (MVK) spectrum of refractive-index fluctuations^[11]

$$\Phi_n(\kappa, L_0, l_0) = 0.033 C_n^2 \exp(-\kappa^2/\kappa_m^2) (\kappa^2 + \kappa_0^2)^{-11/6}, \quad (9)$$

where C_n^2 is the refractive index structure constant, $\kappa_m = 5.93/l_0$, and $\kappa_0 = 2\pi/L_0$. Substituting Eqs. (7)–(9) into Eq. (6), we have

$$\sigma_{\ln I, \text{pl}}^2(D) = 8\pi^2 k^2 L \int_0^1 \int_0^\infty \kappa [\Phi_n(\kappa, L_0) + \Phi_n(\kappa, l_0)] \exp(-\kappa^2 D^2/4) (1 - \cos L\kappa^2 \xi/k) d\kappa d\xi, \quad (10)$$

$$\sigma_{\ln I, \text{sph}}^2(D) = 8\pi^2 k^2 L \int_0^1 \int_0^\infty \kappa [\Phi_n(\kappa, L_0) + \Phi_n(\kappa, l_0)] \exp(-\kappa^2 \xi^2 D^2/4) \{1 - \cos [L\kappa^2 \xi (1 - \xi)/k]\} d\kappa d\xi, \quad (11)$$

where $\Phi_n(\kappa, L_0) \cong \Phi_n(\kappa, L_0, l_0 \rightarrow 0)$ and $\Phi_n(\kappa, l_0) \cong \Phi_n(\kappa, L_0 \rightarrow \infty, l_0)$.

For plane wave, using the approximation $1 - \cos x \cong x^2/2$ for calculating the variance of log-irradiance fluctuations caused by small-scale turbulent eddies, we have

$$\sigma_{\ln I_m, \text{pl}}^2(d, l_0) \cong \frac{0.08\sigma_1^2 \eta_m^{7/6}}{(1 + d^2 \eta_m/4)^{7/6}} = \frac{0.08\sigma_1^2 (2\pi\lambda L/l_0)^{7/6}}{(1 + 0.5d^2 \pi\lambda L/l_0)^{7/6}}, \quad (12)$$

where $\sigma_1^2 = 1.24 C_n^2 k^{7/6} L^{11/6}$, $d = \sqrt{kD^2/4L}$, and $\eta_m = 2\pi\lambda L/l_0$. The approximation $1 - \cos x \sim 1$ in calculating the variance of log-irradiance fluctuations caused by large-scale turbulent eddies, we have

$$\sigma_{\ln I_0, \text{pl}}^2(d, L_0) \cong \frac{0.636\sigma_1^2 \eta_0^{-5/6}}{1 + 0.3d^2 \eta_0} = \frac{0.636\sigma_1^2 (2\pi\lambda L/L_0)^{-5/6}}{1 + 0.6d^2 \pi\lambda L/L_0}, \quad (13)$$

where $\eta_0 = 2\pi\lambda L/L_0$. Combining the results of Eqs. (12), (13), and (6), we have the aperture-averaging factor

$$G_{\text{pl}}(D) = \left\{ \exp[(0.08\sigma_1^2 \eta_m^{7/6})/(1 + d^2 \eta_m/4)^{7/6}] + (0.636\sigma_1^2 \eta_0^{-5/6})/(1 + 0.3d^2 \eta_0) - 1 \right\} / \left\{ \exp[0.08\sigma_1^2 \eta_m^{7/6} + 0.636\sigma_1^2 \eta_0^{-5/6}] - 1 \right\}. \quad (14)$$

Similarly, the aperture-averaging factor of the spherical wave can be calculated by Eq. (11) and be given by

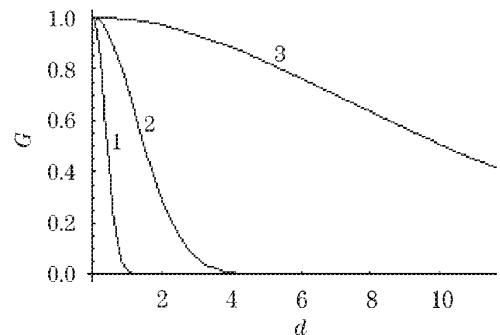


Fig. 1. Aperture-averaging factor versus aperture radius scaled by the Fresnel zone size associated with an unbounded plane wave under various irradiance variances. 1: $\sigma_1^2 = 50$; 2: $\sigma_1^2 = 2$; 3: $\sigma_1^2 = 0.01$. $L = 1000$ m, $L_0 = 1$ m, and $l_0 = 0.01$ m.

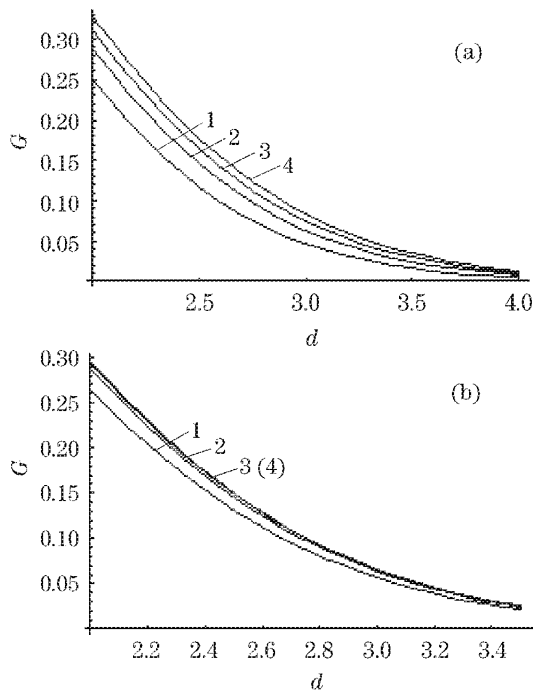


Fig. 2. Aperture-averaging factor versus aperture radius scaled by the Fresnel zone size associated with an unbounded plane wave under various outer- and inner-scale. (a) 1: $L_0 = 0.5$ m; 2: $L_0 = 1$ m; 3: $L_0 = 1.5$ m; 4: $L_0 = 2$ m. $\sigma_1^2 = 4$, $L = 1000$ m, and $l_0 = 0.01$ m; (b) 1: $l_0 = 0.005$ m; 2: $l_0 = 0.01$ m; 3: $l_0 = 0.015$ m; 4: $l_0 = 0.02$ m. $\sigma_1^2 = 4$, $L = 1000$ m, and $L_0 = 1$ m.

$$G_{\text{sph}}(D) = \left\{ \exp[0.008\sigma_1^2\eta_m^{7/6}/(1 + 0.021d^2\eta_m)^{7/6} + 0.636\sigma_1^2\eta_0^{-5/6}/(1 + 0.1d^2\eta_0)] - 1 \right\} / \left\{ \exp[0.008\sigma_1^2\eta_m^{7/6} + 0.636\sigma_1^2\eta_0^{-5/6}] - 1 \right\}. \quad (15)$$

Figure 1 shows the aperture-averaging factor in Eq. (14) as a function of the circular aperture radius $D/2$ scaled by the Fresnel zone size $\sqrt{L/k}$ and Rytov variance $\sigma_1^2 = 50$, 2, and 0.01. The result agrees with Ref. [5]. Figure 2 (partial curves) shows the aperture-averaging factor in Eq. (14) as a function of the aperture radius $D/2$ scaled by the Fresnel zone size and the outer scale $L_0 = 0.5$, 1, 1.5, and 2 m (Fig. 2(a)), and inner scale $l_0 = 0.005$, 0.01, 0.015, and 0.02 m (Fig. 2(b)). The results show that the finite outer- and inner-scale are factors which have the modulation action on the aperture averaging of optical scintillations. Figure 3 shows the aperture-averaging factor in Eq. (14) as a function of the circular aperture radius $D/2$ scaled by the Fresnel zone size $\sqrt{L/k}$ under various values of the wave source wavelengths. The curves in Fig. 3 show that the aperture-averaging factor is dependent on the wave source wavelengths, this result agrees with the experimental observation^[13].

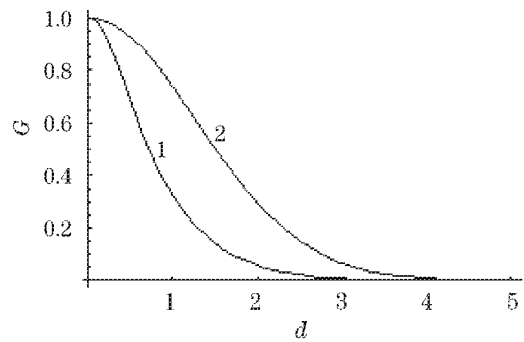


Fig. 3. Aperture-averaging factor versus aperture radius scaled by the Fresnel zone size associated with an unbounded plane wave under various wave source wavelengths. 1: $L = 1000$ m, $l_0 = 0.02$ m, $\sigma_1^2 = 4$, $L_0 = 1$ m, $\lambda = 0.6328$ μm ; 2: $L = 1000$ m, $l_0 = 0.02$ m, $\sigma_1^2 = 4$, $L_0 = 1$ m, $\lambda = 10.6$ μm .

In conclusion, our results show that the size of aperture, the inner- and outer-scale, and the wavelength of wave source are factors which should be considered in the measurement for theoretic analysis and the experimental measurement for optical scintillations.

This work was supported by the Key Project of Chinese Ministry of Education under Grant No. 01091. Y. Zhang's e-mail address is zyxlxy30@hotmail.com.

References

1. A. H. Mikesell, A. A. Hosg, and J. S. Hall, *J. Opt. Soc. Am.* **41**, 689 (1951).
2. D. L. Fried, *J. Opt. Soc. Am.* **57**, 169 (1967).
3. R. F. Lutomirski and H. T. Yura, *J. Opt. Soc. Am.* **59**, 1247 (1969).
4. H. T. Yura and W. G. McKinley, *Appl. Opt.* **22**, 1608 (1983).
5. J. H. Churnside, *Appl. Opt.* **30**, 1982 (1991).
6. Y. X. Zhang, *J. Infrared and Millimeter Waves* (in Chinese) **11**, 301 (1992).
7. E. L. Bass, B. D. Lackovic, and L. C. Andrews, *Opt. Eng.* **34**, 26 (1995).
8. L. C. Andrews, R. L. Phillips, and C. Y. Hopen, *Wave in Random Media* **10**, 53 (2000).
9. Y. X. Zhang and J. H. Qian, *Laser Technology* (in Chinese) **25**, 81 (2001).
10. V. I. Tatarskii, *Wave Propagation in a Turbulent Medium* (in Chinese) J. S. Wen *et al.* (trans.) (Science in China Press, Beijing, 1978) pp. 25 – 35.
11. Y. X. Zhang and Z. Y. Chi, *Light Wave Propagating and Imaging in the Atmosphere* (in Chinese) (National Defense Industry Press, Beijing 1997) p. 14.
12. Y. X. Zhang, Z. F. Song, and Z. B. Gong, *Chin. Sci. Bulletin (Kexue Tongbao)* **33**, 1909 (1988).
13. Z. Azar, H. M. Loebenstein, G. Appelbaum, E. Azoulay, U. Halavee, M. Tamir, and M. Tur, *Appl. Opt.* **24**, 2401 (1985).