Influence of the net gain on characteristic of stochastic resonance in a single-mode laser system

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The phenomenon of stochastic resonance (SR) is found in a single-mode laser system driven by the colored pump noise with signal modulation and the quantum noise with cross-correlation between the real and imaginary parts. When the net gain a_0 changes, it is found that, 1) the shape of the curve of the signal-to-noise ratio (SNR) versus the pump noise self-correlation time τ exhibits a changing process of multiform SR, from single-peak SR to simultaneous existence of resonances and suppressions; 2) the curve of SNR versus signal frequency Ω experiences a complicated changing process from the monotonous descending to the simultaneous appearances of a maximum and a minimum, and finally to monotonous descending; 3) the curve of SNR versus cross-correlation coefficient between the real and imaginary parts of the quantum noise λ_q appears an acute single-peak SR. Therefore, the net gain a_0 greatly influences the characteristic of SR of laser system.

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The phenomenon of stochastic resonance (SR) has attracted many researchers' attention since it was discovered by Benzi *et al.*[1−13], and also been proved by experiments. The SR phenomenon was firstly detected in the two-mode ring laser[14], then was discovered in a single-mode laser linear system driven by correlative colored quantum noise and colored pump noise[15]. Recently, the SR phenomenon described by the curve of the signal-to-noise ratio (SNR) versus the cross-correlation coefficient between the real and imaginary parts of the quantum noise λ_q has also been found in a single-mode laser linear system. Contrary to the deterministic description, the noise has essential and decisive impact on the SR. In laser system, the net gain a_0 not only determines the critical value ($a_0 = 0$, the threshold) of output laser, but also acts as an important parameter in the description of the statistical property of the system. In this paper, we focus on the influence of a_0 on the SR in the single-mode laser system, and attempt to provide a theoretic foundation for the design of laser system optimized by the application of SR phenomenon.

Adopting the linear approximation method, we have studied a single-mode laser system driven by the colored pump noise with signal modulation and the quantum noise with cross-correlation between the real and imaginary parts. And we have found that when a_0 changes, the shape of the curve of SNR versus the pump noise self-correlation time τ exhibits a changing process of multiform SR. Therefore, varying the net gain a_0 of laser system can control the SR effectively.

The Langevin equation of amplitude for a loss-noise model of a single-mode laser is given by[16]

$$\frac{\mathrm{d}r}{\mathrm{d}t'} = a_0 r - Ar^3 + \frac{Q}{2r}(1-|\lambda_q|) + r p_R(t') + \varepsilon_r(t'). \quad (1)$$

If we consider the pump noise is modulated by periodic signal $B\cos\Omega t'$, thus the Langevin equation of intensity for a loss-noise model of a single-mode laser with an input signal is given by

$$\frac{\mathrm{d}I}{\mathrm{d}t'} = 2a_0 I - 2AI^2 + Q(1-|\lambda_q|) + 2Ip_R(t')B\cos\Omega t' + 2\sqrt{I}\varepsilon_r(t'), \quad (2)$$

where noises $p_R(t')$ and $\varepsilon_r(t')$ are correlated as

$$\begin{cases} \langle p_R(t')\rangle = \langle \varepsilon_r(t')\rangle = 0 \\ \langle p_R(t')p_R(s)\rangle = \frac{P}{2\tau}\mathrm{e}^{-\frac{|s-t'|}{\tau}} \\ \langle \varepsilon_r(t')\varepsilon_r(s)\rangle = Q(1+|\lambda_q|)\delta(t'-s) \\ \langle p_R(t')\varepsilon_r(s)\rangle = 0 \end{cases}. \quad (3)$$

In Eqs. (1)—(3), a_0 and A represent the net gain coefficient and self-saturation coefficient, respectively; I is the laser intensity; B and Ω are the amplitude and frequency of the periodic signal; $p_R(t')$ is the real part of the pump noise, and $\varepsilon_r(t')$ is the quantum noise of phase locking; P and Q are the intensities of the pump noise and the quantum noise, respectively; τ is the pump noise self-correlation time; λ_q is the cross-correlation coefficient between the real part and the imaginary part of the quantum noise, and $-1 \le \lambda_q \le 1$.

Let

$$I = I_0 + \delta(t'),$$

where $I_0 = \frac{a_0}{A}$ is the deterministic steady-state intensity, and $\delta(t')$ is the perturbational term. We linearize Eq. (2) around the deterministic steady-state intensity I_0, thus the linear equation of the laser intensity is found to be

$$\frac{\mathrm{d}\delta(t')}{\mathrm{d}t'} = -\gamma\delta(t') + 2I_0 p_R(t')B\cos\Omega t' + 2\sqrt{I_0}\varepsilon_r(t') + Q(1-|\lambda_q|),$$

where $\gamma = 2a_0$.

According to the steady-state mean intensity correlation function defined by

$$C(t) = \lim_{t'\to\infty} \overline{\langle I(t')I(t'+t)\rangle} = \lim_{t'\to\infty}\left(\frac{\Omega}{2\pi}\int_{t'}^{t'+\frac{2\pi}{\Omega}}\langle I(t')I(t'+t)\rangle \mathrm{d}t'\right),$$

we can have

$$C(t) = \left(I_0 + \frac{Q(1-|\lambda_q|)}{\gamma}\right)^2 + \left(\frac{2I_0Q(1+|\lambda_q|)}{\gamma} + \frac{I_0^2B^2P(2\pi\gamma^2+\Omega^3)(\Omega^2-\gamma^2+\tau^{-2})}{\tau^2\Omega\gamma(k_1^2+\Omega^2)(k_2^2+\Omega^2)(\gamma^2+\Omega^2)}\right)\mathrm{e}^{-\gamma|t|} + \frac{I_0^2B^2P(\Omega^2+\gamma^2-\tau^{-2})}{\tau(k_1^2+\Omega^2)(k_2^2+\Omega^2)}\mathrm{e}^{-\frac{|t|}{\tau}}\cos\Omega t + \frac{2\Omega I_0^2B^2P}{\tau^2(k_1^2+\Omega^2)(k_2^2+\Omega^2)}\mathrm{e}^{-\frac{|t|}{\tau}}\sin\Omega|t|, \quad (4)$$

where $k_1 = \gamma - \tau^{-1}$ and $k_2 = \gamma + \tau^{-1}$.

Thus, translate Eq. (4) into the power spectrum $S(\omega)$ by Fourier transform

$$S(\omega) = S_1(\omega) + S_2(\omega), \quad (5)$$

where $S_1(\omega)$ and $S_2(\omega)$ are output power spectra of the signal and the noise, respectively.

The SNR is defined as the ratio of the output power of the signal to the broadband noise output at $\omega = \Omega$ (only the spectrum of $\omega > 0$ is kept). We have

$$R = \frac{P_s}{S_2(\omega=\Omega)}. \quad (6)$$

Inserting $P_s = \int_0^\infty S_1(\omega)\mathrm{d}\omega$ and $S_2(\omega=\Omega)$ into Eq. (6), we can get the output SNR

$$R = \frac{\pi I_0B^2Pk_3(k_3-\Omega^2)}{4\tau Q(1+|\lambda_q|)(k_1^2+\Omega^2)(k_2^2+\Omega^2)} + \frac{\Omega k_3^2(k_3-\tau^{-2})}{4\gamma^2k_2(k_1^2+\Omega^2)} + \frac{\pi k_3^2(k_3-\tau^{-2})}{\Omega^2k_2(k_1^2+\Omega^2)} - \frac{\Omega k_3^2(k_3-\tau^{-2})}{4\gamma^2k_1(k_2^2+\Omega^2)} - \frac{\pi k_3^2(k_3-\tau^{-2})}{\Omega^2k_1(k_2^2+\Omega^2)} \quad (7)$$

where $\tau \neq 1/2a_0$.

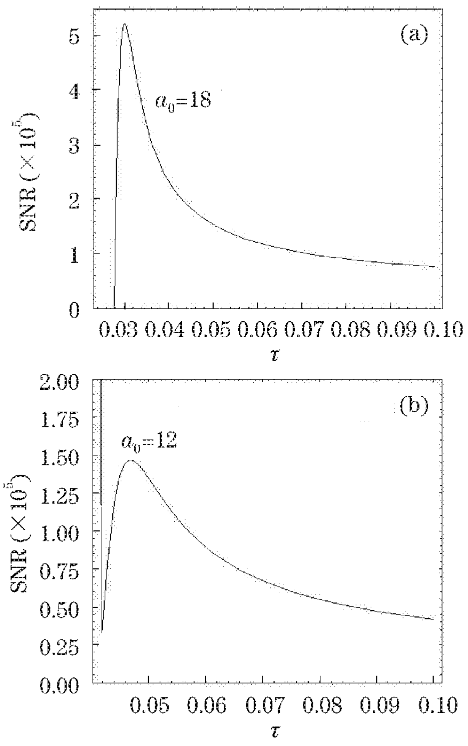

Fig. 1. The SNR as a function of the self-correlation time τ for the different values of the net gain a_0. The values of the other parameters are $A = 1$, $B = 20$, $\lambda_q = 0.5$, $P = 0.001$, $Q = 0.001$, and $\Omega = 3$.

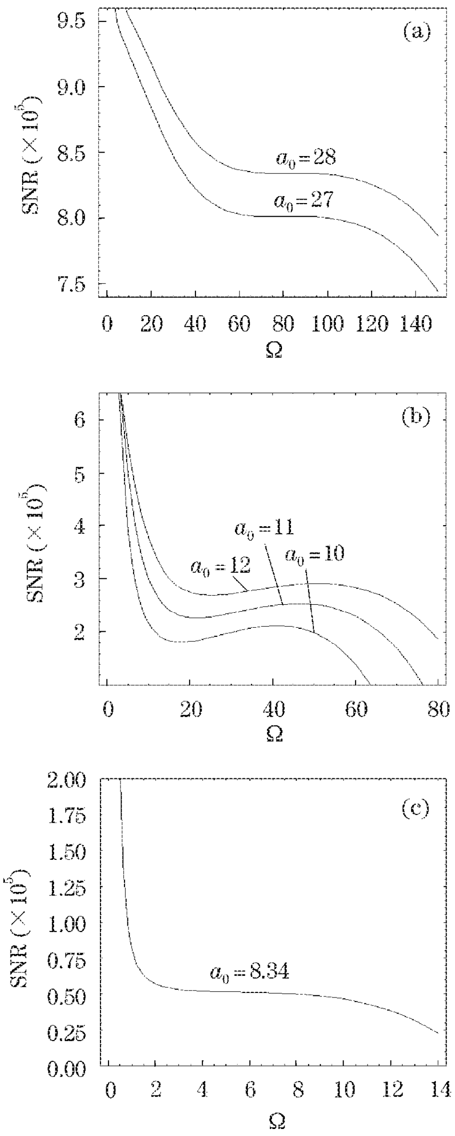

Fig. 2. The SNR as a function of the signal frequency Ω for the different values of the net gain a_0. The values of the other parameters are $A = 1$, $B = 60$, $\lambda_q = 0.5$, $P = 0.001$, $Q = 0.001$, and $\tau = 0.06$.

In this paper, we discuss the laser operating above the threshold ($a_0 > 0$) only. By choosing a_0 as a parameter, the curve of SNR versus the self-correlation time τ is plotted with Eq. (7), as shown in Fig. 1. It exhibits a changing process as follows. 1) In Fig. 1(a), there is a maximum in the curve, that is, the system appears single-peak stochastic resonance. 2) In Fig. 1(b), when a_0 decreases, the curve exists simultaneously in resonances and suppressions. Hence, the curve experiences from single-peak SR to the simultaneous existence of resonances and suppressions as a_0 decrease.

By choosing a_0 as a parameter, the curve of the SNR versus the signal frequency Ω is plotted with Eq. (7), as shown in Fig. 2. The curve exhibits a changing process as follows. 1) In Fig. 2(a), in the range of $27 \leq a_0 < 50$, when a_0 decreases, the curve descends monotonically, and the whole curve falls down. 2) In Fig. 2(b), in the range of $8.34 < a_0 < 27$, when a_0 decreases, the curve exhibits the simultaneous appearances of a maximum and a minimum, and the whole curve falls down, the position of the maximum and the minimum moving towards the descent direction of Ω. 3) In Fig. 2(c), when a_0 decreases to critical point ($a_0 = 8.34$), the maximum and the minimum of the curve disappear simultaneously and the curve exhibits a monotonous descending again. So we have found that decreasing a_0 can lead to repeated changing process of the curve from monotonous descending to the simultaneous appearances of a maximum and a minimum, and finally to monotonous descending.

By choosing a_0 as a parameter, the curve of the SNR versus λ_q is plotted with Eq. (7), as shown in Fig. 3. By virtue of Fig. 3, we can see that, the resonant peak appears at $\lambda_q = 0$, and when a_0 decreases, the peak becomes higher and the position of the peak unchange.

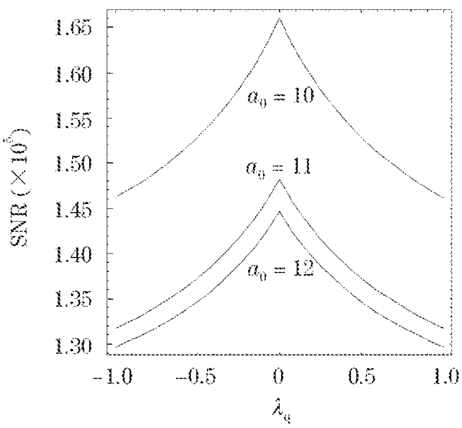

Fig. 3. The SNR as a function of the cross-correlation coefficient between the real and imaginary parts of the quantum noise λ_q for the different values of the net gain a_0. The values of the other parameters are $A = 1$, $B = 10$, $P = 0.001$, $Q = 0.001$, $\tau = 0.06$, and $\Omega = 3$.

It is noted that, 1) in practical application, in order to prevent distortion for a modulation signal, the laser is required to operate in linear region. Therefore, the adoption of linear approximation method is according with practical situation. 2) The net gain a_0 is a main physical parameter of reflecting the operational state of the laser. In our study, we found that when a_0 changes, the system exhibits SRs of various forms. This proposes a new method and theoretical basis for the application of SR to design and optimize optical communication system. 3) In Eqs. (1) and (7), we have applied the unified colored noise approximation and linear approximation method. Therefore, in order to ensure that the obtained results satisfy the demand of the two approximation conditions, the valid range of all parameters are specially noticed in this paper.

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