Dark soliton interaction in optical time division multiplexed system with randomly varying birefringence and random dispersion map

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Correlated perturbations caused by both randomly varying birefringence and random dispersion map are considered in optical time division multiplexed dispersion-managed dark soliton system, and their effects on soliton interaction are investigated numerically. These perturbations enhance soliton interaction, and their effects relate to the strength of perturbation, separation, and pulse width. The correlation plays an important role and reinforces these effects. Moreover, there is a stochastic limit between two perturbations in the system, where the effect is the largest and the corresponding interaction distance is the shortest.

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Dark solitons propagate in the normal dispersion region. They have robust features such as low intrinsic transmission loss, resistance to perturbations, and weak interaction between neighboring solitons. The amplified spontaneous emission noise in the dark soliton system is only half of that corresponding to the bright counterpart. The techniques for generating and detecting dark soliton pulses have been developed[1].

Because of the asymmetry of the refractive index profile of the core, an optical fiber exhibits birefringence. If the birefringence is randomly varied during the soliton propagation, the traveling pulses will experience waveform distortion, a dispersive wave radiation will appear in the system and result in the decrease of transmission capacity[2-4].

Recently, both bright and dark soliton propagations in dispersion-managed system are considered most attractive for long-haul optical communication, where dispersion-managed soliton is a new type of optical soliton. For these dispersion-managed solitons, the effects such as Gorden-Haus jitter, emission of radiation, and phase-matched four-wave mixing, are strongly reduced[5]. However, the perturbations in dispersion-managed system are harmful to soliton propagation. For example, stochastic perturbations caused by randomly varying birefringence or random dispersion map lead to disintegration of soliton, and degrade the stability of soliton propagation[6,7].

Optical time division multiplexing (OTDM) is a very powerful technique for the ultra-high speed communication. OTDM systems have several advantages such as high bit-rate-to-bandwidth ratio, natural accommodation of higher bit rate payloads, and ease of supervising the multiplexed line if error checking of aggregated streams is feasible. However, the mutual interaction between neighboring pulses becomes a serious obstacle in the OTDM system due to their periodical changes of pulse width[8].

In this letter, correlated perturbations are considered in a dispersion-managed soliton system, their effects on soliton interaction are discussed in an OTDM system, and some novel conclusions are obtained.

In a real transmission system, the envelop of field becomes inhomogeneous because of possible variations in the fiber dispersion and birefringence, then satisfies[2,5]

$$
\begin{aligned}
&i\frac{\partial u}{\partial Z}+i\eta_1(Z)\frac{\partial u}{\partial \tau}-\frac{d(Z)}{2}[1+\eta_2(Z)]\frac{\partial^2 u}{\partial \tau^2}\\
&\quad+(|u|^2+\frac{1}{3}|v|^2)u+\frac{1}{3}v^2u^*\exp(-4i\beta Z)=0,\\
&i\frac{\partial v}{\partial Z}-i\eta_1(Z)\frac{\partial v}{\partial \tau}-\frac{d(Z)}{2}[1+\eta_2(Z)]\frac{\partial^2 v}{\partial \tau^2}\\
&\quad+(|v|^2+\frac{1}{3}|u|^2)v+\frac{1}{3}u^2v^*\exp(4i\beta Z)=0, \qquad (1)
\end{aligned}
$$

where u and v are normalized elliptically polarized components along two orthogonal directions, respectively. $Z=z/z_d$, $z_d=\tau_0^2/|\bar{d}|$, $\tau=t/\tau_0$. τ_0, z_d, $\bar{d}$, and $d(Z)$ are the pulse width, dispersion length, the path-average dispersion, and the dispersion map, respectively. Fluctuations of normalized time delay and dispersion magnitude $\eta_1(Z)$, $\eta_2(Z)$ are thought as the perturbations caused by both randomly varying birefringence and random dispersion map. $\eta_1(Z)$ and $\eta_2(Z)$ can be assumed reasonedly as the stochastic variables, and satisfy

$$
\langle \eta_i(Z)\rangle = 0, \quad \langle \eta_i(Z)\eta_i(Z')\rangle = D_i\delta(Z-Z'), \quad i=1,2, \qquad (2)
$$

where D_i ($i=1$, 2) is the standard deviation.

The numerical simulations of pulse propagation are performed in the optical fiber link with randomly varying birefringence and random dispersion map, the fiber segments z_+ and z_- are 50 km, and their dispersions are $d_1=-2.305$ ps/(km · nm) and $d_2=2.258$ ps/(km · nm), respectively. For the soliton with pulse width of 5 ps, the dispersion length is 270 km corresponding the average dispersion $\bar{d}=-0.0235$ ps/(km · nm). The random modulation with Gaussian random deviation is used in time delay and dispersion magnitude of every fiber segment. Uniformly distributed random numbers from the intervals θ_1, $\theta_2\in(0,1)$ were used, and random numbers are produced by the built-in generator Random(x) from the Matlab. Transformation of uniformly distributed random numbers from the intervals θ_1, $\theta_2 \in (0,1)$ into Gaussian random numbers is performed by[9]

$$\eta_i = \sqrt{2D_i} \times \sin(2\pi\theta_i) \times \sqrt{-2\ln\theta_i}, \quad i = 1, 2. \qquad (3)$$

As usual, an average of hundreds of different sequences for random numbers is used, and the fast-varying terms (the last left-terms) of Eq. (1) are neglected in the below numerical simulations.

Interaction between adjacent solitons may induce the soliton collision and the time jitter, thus increase the bit-error-rate, and becomes a serious obstacle in the OTDM system. Moreover, in the OTDM dispersion-managed soliton system with randomly varying birefringence and random dispersion map, the situation is very distinguished.

As described in Ref. [8], the interaction distance of two neighboring solitons is defined as the distance where the timing shifts of neighboring pulses exceed a half of their pulse width. Figure 1 shows the interaction distance versus the standard deviation for different situations, Figs. 2 – 4 are the normalized intensity of two solitons versus the propagation distance for the different standard deviations. The initially input soliton pulses are $\tanh(t - \Delta/2)$ for $t > 0$, and $\tanh(t + \Delta/2)$ for $t \leq 0$, where the separation of solitons is $\Delta = 6$ (corresponding to about 34 Gb/s for 5-ps pulse). The pulse width is 5 ps and standard deviation of every perturbation is chosen. We can see that these perturbations enhance the interaction between solitons, and their effects relate to every standard deviation, the separation, and pulse width. The reason is that the perturbations amplify the sidebands of dark soliton which affect neighbouring solitons, further correlate the two solitons, and enhance their interaction. We also find the combining effects of perturbations are larger than that of each perturbation.

It should be pointed out that multiplicative perturbations are all assumed as independent white Gaussian perturbations in previous models. However more realistic physical models of soliton system require considering the case of colored perturbations, especially when these perturbations are of the common origin, the presence of correlation may change the dynamics of the soliton system. When they partially are of the common origin, the strength of correlation is introduced, and Eq. (2) and the equation below are adopted to demonstrate their

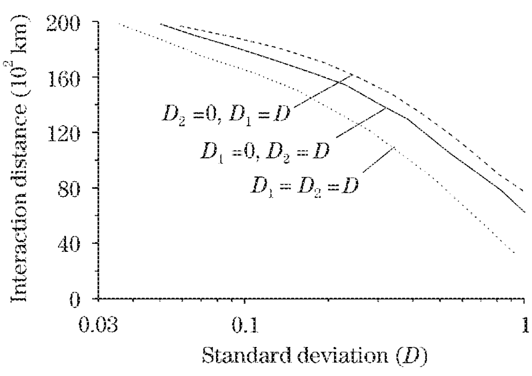

Fig. 1. The interaction distance versus the standard deviation for the different situations (5-ps soliton).

correlation[10,11]

$$\langle \eta_1(Z)\eta_2(Z') \rangle = \langle \eta_2(Z)\eta_1(Z') \rangle = k\sqrt{D_1 D_2}\delta(Z - Z'), \quad |k| \leq 1, \qquad (4)$$

where k is the strength of correlation between two stochastic perturbations.

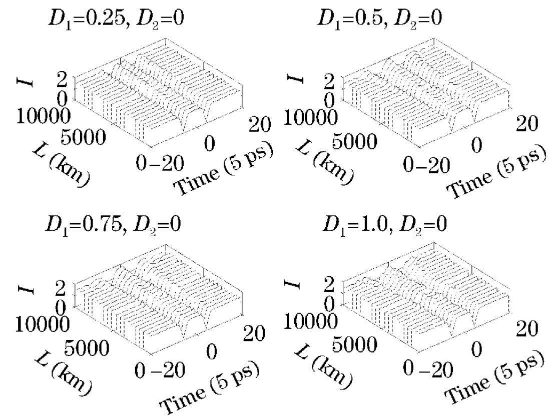

Fig. 2. The normalized intensity I of two solitons versus the propagation distance L for the different standard deviations (5-ps soliton).

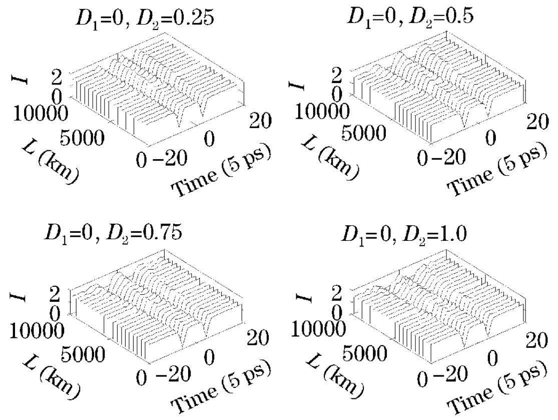

Fig. 3. The normalized intensity I of two solitons versus the propagation distance L for the different standard deviations (5-ps soliton).

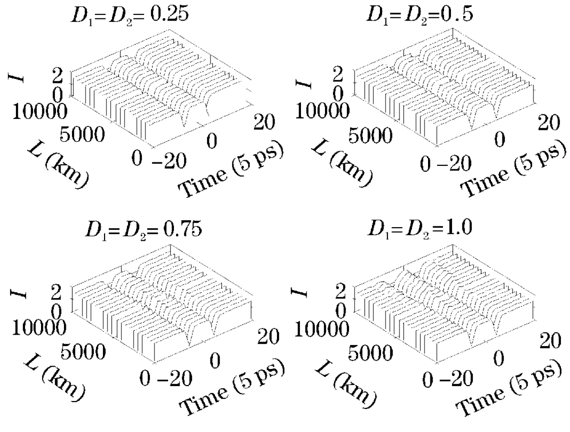

Fig. 4. The normalized intensity I of two solitons versus the propagation distance L for the different standard deviations (5-ps soliton).

When correlation between two perturbations is considered, the transformation of uniformly distributed random numbers from the intervals $\theta_1,\theta_2,\theta \in (0,1)$ into Gaussian random numbers is performed by[9]

$$\begin{aligned}\eta_1 = \sqrt{2D_1} \times [\sqrt{1-k^2}\sin(2\pi\theta_1) \times \sqrt{-2\ln\theta_1} \\ +k\sin(2\pi\theta) \times \sqrt{-2\ln\theta}], \\ \eta_2 = \sqrt{2D_2} \times [\sqrt{1-k^2}\sin(2\pi\theta_2) \times \sqrt{-2\ln\theta_2} \\ +k\sin(2\pi\theta) \times \sqrt{-2\ln\theta}]. \qquad (5)\end{aligned}$$

Figure 5 demonstrates the interaction distance versus the correlation strength, and Fig. 6 is the normalized intensity of two solitons versus the propagation distance for different correlation strengths. The initially input

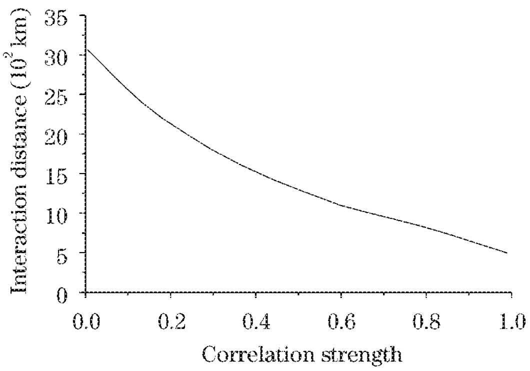

Fig. 5. The interaction distance versus the correlation strength (5-ps soliton).

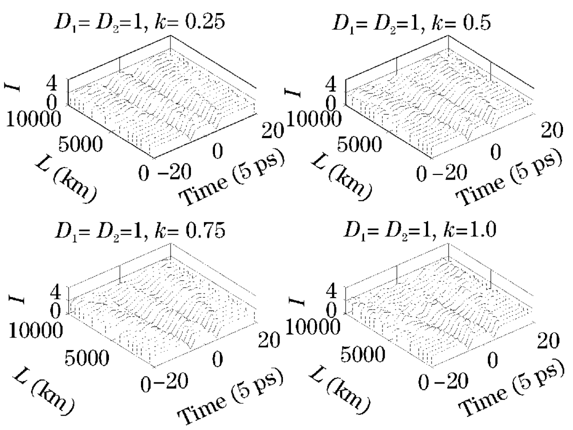

Fig. 6. The normalized intensity I of two solitons versus the propagation distance L for the different correlation strengths (5-ps soliton).

soliton pulses are the same as the before, and the standard deviations are chosen as $D_1 = D_2 = 1$. We find the correlation enhances the interaction between solitons, and its effects relate to every standard deviation, the correlation strength, and the separation between solitons. The reason is that when correlated perturbations are simultaneously present, their effects on soliton system should contain each contribution and the correlated part. In the system, there is a stochastic limit where the effect of these perturbations is the largest on soliton interaction.

In summary, effects of correlated perturbations caused by both randomly varying birefringence and random dispersion map are investigated on soliton interaction with numerical simulation. When correlated perturbations are simultaneously present, their effects on soliton system should contain each contribution and the correlated part. These perturbations enhance the interaction . The correlation plays an important role, and boosts up these effects. Furthermore there is a stochastic limit between two perturbations in the system, where their effect is the largest, and the corresponding interaction distance is the shortest.

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