A theoretical method based on Fourier spectrum analysis for the focusing performances of the X-ray compound refractive lenses

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It is important to predict the intensity distribution in focusing plane for designing the X-ray compound refractive lenses. On the basis of analyzing the structure of X-ray compound lenses and comparing it with Fraunhofer diffraction system, it is concluded that the X-ray focusing system can be regarded as a kind of Fraunhofer diffraction system. Therefore, a method based on Fourier spectrum analysis is presented to predict the intensity distribution in the focusing plane for the X-ray lenses. A brief analysis on the relationship between the parameters of X-ray lenses and their focusing performance is also given in this paper.

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The X-ray compound lens is a novel refractive optical device for focusing hard X-rays. The index of refraction for X-ray in material can be written as $\tilde{n} = 1 - \delta + \beta i$, where β is the absorption index and δ is the refractive index decrement. Because $(1 - \delta)$ in the X-ray region is very close to 1, and the absorption is strong, refractive lenses, which have been extensively used in the visible light region, were commonly considered impractical for focusing X-rays. To overcome this problem, the X-ray compound lens, which consists of a linear array of many single lenses made of low-Z materials, was proposed to shorten the focal length to $1/N$ of that of a single lens, where N is the number of the single lenses, ranging from 10 to 300[1-3]. Compared with other X-ray optical devices such as bent-crystal/multilayer X-ray mirrors, capillary optics, and Fresnel and Bragg-Fresnel zone plates[4-6], the X-ray compound refractive lens is the most suitable device for focusing high energy X-rays in one or two dimensions. It does not change the beam path, it has perfect stability under high temperature, and is easy to cool. In addition, its requirement for surface quality is lower, and its structure is more compact. Due to the advantages of the X-ray compound lens, it has been widely studied after its first demonstration[2].

The theoretical design is basic for fabricating focusing lenses with proper optical performance needed in many micro-focus experimental systems[3,7-11]. The conventional theoretical design of X-ray compound refractive lens is based on the Fresnel-Kirchhoff approach[12]. Because the thickness of the lens is much shorter than its focal length, the compound lens can be considered as a diffractive screen. The complex amplitude distribution $U(\rho)$ of a point ρ in the focal plane of an X-ray compound refractive lens can be calculated according to the Fresnel-Kirchhoff integral equation. Moreover, the intensity distribution in the focal plane $I(\rho)$ can be obtained.

Unlike the lens used in the visible light region, the performance of X-ray compound lens is relative to the refraction effect and the absorption. So the diffraction screen $H(r)$ is supposed to consist of two parts, the transmission coefficient $\tau(r)$ and the attenuation coefficient $A(r)$. In conclusion, the diffraction screen $H(r)$ is defined as $H(r) = A(r) \cdot \tau(r)$. Accordingly, the intensity distribution $I(\rho)$ can be calculated from the Fresnel-Kirchhoff integral equation when a monochromatic X-ray plane wave impinges on the lens.

It is very intricate to calculate $I(\rho)$ from the Fresnel-Kirchhoff integral equation[12]. In order to simplify the calculation of $I(\rho)$, a method based on Fourier spectrum analysis is presented to predict $I(\rho)$ for the X-ray compound refractive lenses. On the basis of analyzing the structure of X-ray compound refractive lens and comparing the Fraunhofer diffraction system with the focusing system of the X-ray compound lens, it is concluded that the X-ray focusing system can be regarded as a kind of Fraunhofer diffraction system. Therefore, we analyze the Fourier spectrum of the diffraction screen of the compound lens, and obtain the function of the intensity distribution, which is same with the result obtained by means of Kirchhoff integral within the Fresnel approximation. Our method based on Fourier analyzing is more facile. In the present paper, we describe our method in more detail. Moreover, a brief analysis on the relationship between the parameters of X-ray lenses and their focusing performance is also given.

A Fraunhofer diffraction equipment is shown in Fig. 1. Suppose a planar monochromatic X-ray wave impinges on it, the Kirchhoff integral within the Fraunhofer approximation can be expressed as

$$U(x', y') = C e^{ikL_0} \iint H(x, y) \exp[\frac{-ik}{z}(xx' + yy')] \mathrm{d}x \mathrm{d}y, \quad (1)$$

where C stands for a constant, $H(x, y)$ is the function of diffraction screen, L_0 is the optical path from the center of diffraction screen to the point in the focal plane (in Fig. 1, L_0 is the optical path from point O to point

P). Meanwhile, the Fourier transform of $H(x,y)$ can be expressed as

$$H(\xi,\eta)=\iint H(x,y)\exp[-i2\pi(\xi x+\eta y)]\mathrm{d}x\mathrm{d}y. \quad (2)$$

Under the condition: $2\pi(\xi,\eta)=\frac{k}{z}(x',y')$,

$$\begin{aligned}|\,U(x',y')\,|^2&=|\,C\mathrm{e}^{ikL_0}\cdot F\{H(x,y)\}\,|^2\\&=|\,H(\xi,\eta)\,|^2, \quad (3)\end{aligned}$$

where $F\{H(x,y)\}$ stands for the Fourier transform of $H(x,y)$. From Eq. (3), it can be seen that if we can get Fourier transform of the diffraction screen, we can then get the intensity distribution in the focal plane, because e^{ikL_0} in Eq. (3) does not change the intensity distribution.

Because of the absorption of the X-ray, X-ray compound refractive lens is unique from those refractive lenses used in visible light region. A single X-ray refractive lens is shown in Fig. 2, where its structure parameters are also shown. The function of the X-ray refractive lens is refracting as well as absorbing the incident X-ray radiations.

According to the Lambert's law, the absorption effect of the X-ray compound refractive lens shown in Fig. 3 can be expressed as

$$\begin{aligned}A_N(r)&=A^N(r)\\&=\exp\left[-\frac{4\pi N\beta\, d}{\lambda}\right]\exp\left[-\frac{2\pi N\beta\, r^2}{\lambda R}\right], \quad (4)\end{aligned}$$

where λ is the wavelength of the X-ray. The absorption effect can commonly be neglected for the refractive lenses used in visible light region.

According to the thin lens theory, the refraction effect of the X-ray refractive lens shown in Fig. 3 can be expressed as

$$\begin{aligned}\tau_N\,(r)&=\tau^N(r)\\&=\exp\left[i\frac{2\pi N}{\lambda}\,(t-\delta\, d)\right]\exp\left[-i\frac{\pi N\delta\, r^2}{\lambda R}\right], \quad (5)\end{aligned}$$

where t denotes the thickness of the edge of the lens. It is analogy to the refractive lenses used in visible light region. The shape of the X-ray compound lens is typically concave, which is different to the lens for visible light. It is because that in X-ray region the refractive index is smaller than 1, while in visible light region the refractive index is larger than 1. Based on thin lens formulae, the focal length of the X-ray compound refractive lens is $F_N=R/\delta N$.

From the above analysis, it is concluded that an X-ray compound lens can be equated to the structure shown in Fig. 4, where the absorption effect $A_N(r)$ is considered as a diffraction screen, the refraction effect $\tau_N(r)$ is regarded as a phase transformation. Therefore, the system shown in Fig. 3 is similar to the Fraunhofer diffraction equipment shown in Fig. 1, where $A_N(r)$ is the equivalent of $H(x,y)$, the function of $\tau_N(r)$ is analogy to the convex lens shown in Fig. 1 for focusing the incident X-ray radiations. In conclusion, the focusing system of the X-ray compound lens shown in Fig. 3 could be considered as a Fraunhofer diffraction equipment. Based on the relationship of Fraunhofer diffraction field with Fourier transform, it is practical to predict the intensity distribution in the focal plane through analyzing Fourier spectrum of the diffraction screen $A_N(r)$.

Since $A_N(r)$ is circularly symmetrical, it is proper to get the Fourier transform of $A_N(r)$ based on Fourier-Bessel transform. Denote the integral relation

$$\int_0^\infty J_0(\alpha x)\cdot\exp(-px^2)x\mathrm{d}x=\frac{1}{2p}\exp(-\frac{\alpha^2}{4p}), \quad (6)$$

and define $\rho'=k\rho/2\pi F_N$, $F_N=R/\delta N$, where J_0 denotes the Bessel function of zero order, ρ' is the polar coordinate in the Fourier transform plane. The intensity distribution of point ρ in the focal plane of the X-ray compound lens thus can be concluded as

$$\begin{aligned}I_N(\rho)=\frac{1}{4}\cdot|C|^2\cdot\exp\left(-\frac{8\pi\beta\, Nd}{\lambda}\right)&\\\cdot\left(\frac{\lambda R}{N\beta}\right)^2\cdot\exp\left[-\frac{\pi N\delta^2}{\lambda R\beta}\cdot\rho^2\right].& \quad (7)\end{aligned}$$

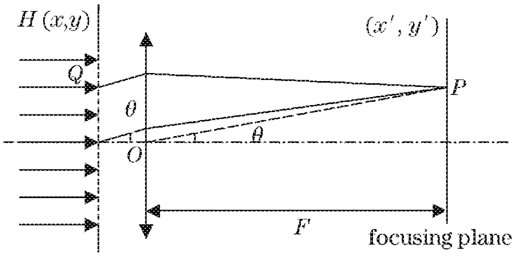

Fig. 1. Fraunhofer diffraction equipment. The receiving plane is at the focusing plane.

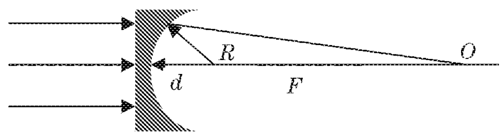

Fig. 2. Focusing scheme of a single X-ray lens.

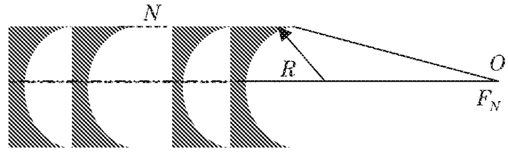

Fig. 3. Scheme of a compound X-ray lens.

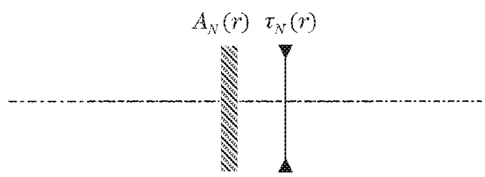

Fig. 4. The equivalent of a compound X-ray lens.

From Eq. (7), we can see the intensity distribution of the X-ray compound refractive lens in the focal plane is a Gaussian distribution, and the optical performances of X-ray compound refractive lens will be impacted by the parameters including N, R, d, δ, β, and λ. The focusing performance of the lens is better when the ratio δ^2/β is larger, the number of elementary lenses N is greater, and the radius R of the concave surface is smaller. However, this implies a smaller numerical aperture when R becomes smaller. Furthermore, the intensity of the focus becomes lower when R becomes smaller and N becomes larger. Therefore those parameters have to be carefully designed in order to obtain better focusing performances and transmission efficiency, or to obtain proper optical performances to meet special applications. In the fabrication of the X-ray compound refractive lens, we use quasi-LIGA (Lithograpie, Galvanoformung, Abformung) technique, and it allows the parameter d to be very close to zero. Therefore, the intensity of the X-ray compound refractive lens at the focal point is mainly determined by $(\lambda R/N\beta)^2$.

To demonstrate whether the Fourier spectrum analysis is proper in concluding the intensity distribution of X-ray compound refractive lens in the focal plane, it is appropriate to compare Eq. (7) with the result that concluded from Fresnel-Kirchhoff integral equation[12]. It is found that the method based on Fourier spectrum analysis is reasonable and proper to be used in the theoretical analysis of X-ray compound refractive lens. On some occasion, it is simple and efficient to get intensity distribution with this method. Furthermore, it is possible to help to design the needed diffraction screen, or to change the optical performance in the focal plane to meet new applications. In addition, with the developments of the fabrication techniques, some novel X-ray focusing devices are emerging, this method may also be used to analyze and design X-ray compound refractive-diffractive lenses according to the binary optic theory.

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