Absorption-amplification response with or without spontaneously generated coherence effect in a four-level atomic system

Jiahua Li (李家华)[1], Wenxing Yang (杨文星)[1,2], and Jucun Peng (彭菊村)[3]

[1] *Department of Physics, Huazhong University of Science and Technology, Wuhan 430074*
[2] *Wuhan Institute of Physics and Mathematics, Chinese Academy of Sciences, Wuhan 430071*
[3] *Department of Physics, Xiaogan Normal University, Xiaogan 432100*

Received January 5, 2004

We discuss and analyze the absorption properties of a weak probe field in a typical four-level atomic system in the presence of a spontaneously generated coherence (SGC) term. The influences of the SGC and a coherent pump field on the probe absorption-amplification are investigated. The results show that the absorption of such a weak probe field can be dramatically enhanced due to the SGC effect. At the same time, the probe-absorption profile exhibits a two-peak structure and the probe-absorption peak gradually decreases as the pump intensity increases. On the contrary, the amplification of such a weak probe field near the line center of the probe transition can be achieved by adjusting the coherent pump field intensity in the absence of the SGC effect.

OCIS codes: 270.1670, 270.6620.

Quantum coherence and interference lead to the appearance of many new effects and techniques in quantum optics and atomic physics, such as non-absorption resonances and electromagnetically induced transparency (EIT), etc.. There have been many theoretical and experimental studies dealing with the absorption and amplification of light[1−12]. In particular, the effects of the spontaneously generated coherence (SGC) on absorption or spontaneous emission spectra have been extensively investigated recently. Menon and Agarwal reported the influences of the SGC on the pump-probe response of a Λ-type atomic system and showed that this coherence can preserve both EIT and coherence population trapping phenomena[10]. Later, Xu *et al.* investigated the effects of this SGC on the transient-absorption process and found that the transient gain (absorption) properties can be greatly affected by the SGC[11]. Recently, Zhu *et al.* showed that a coherent pump field can induce light amplification without or with population inversion in a four-level atomic system[12]. In this letter we analyze and discuss the absorption-amplification properties of the probe field in such a typical four-level atomic system in the presence of the SGC effect.

A closed four-level atomic system coupled by three laser fields is shown in Fig. 1. The transition $|2\rangle \leftrightarrow |3\rangle$ of frequency ω_{32} is driven by a strong coherent coupling laser of frequency ω_c with Rabi frequency $\Omega_c = \vec{\mu}_{32}\cdot\vec{E}_c/(2\hbar)$. A weak probe laser of frequency ω_p with Rabi frequency $\Omega_p = \vec{\mu}_{31}\cdot\vec{E}_p/(2\hbar)$ is applied to the transition $|1\rangle \leftrightarrow |3\rangle$ of frequency ω_{31}. A pump laser of frequency ω_s with Rabi frequency $\Omega_s = \vec{\mu}_{41}\cdot\vec{E}_s/(2\hbar)$ drives the transition $|1\rangle \leftrightarrow |4\rangle$ and controls the population inversion between the states $|1\rangle$ and $|3\rangle$, which determines the probe absorption or amplification.

In the present analysis we use the following assumptions: 1) the Rabi frequencies are real; 2) the dipole moments $\vec{\mu}_{31}$ and $\vec{\mu}_{32}$ are not orthogonal and the angle between them is θ_1, $\vec{\mu}_{41}$ and $\vec{\mu}_{42}$ are also not orthogonal and the angle between them is θ_2. These conditions are necessary for the existence of the SGC effect; 3) we usually ignore the dephasing rate γ_{21} between the levels $|2\rangle$ and $|1\rangle$.

Under the electro-dipole approximation and the rotating-wave approximation, we can easily obtain the following density matrix equations:

$$\dot{\rho}_{11} = \gamma_{31}\rho_{33} + \gamma_{41}\rho_{44} + i\Omega_p(\rho_{31} - \rho_{13}) + i\Omega_s(\rho_{41} - \rho_{14}), \tag{1a}$$

$$\dot{\rho}_{22} = \gamma_{32}\rho_{33} + \gamma_{42}\rho_{44} + i\Omega_c(\rho_{32} - \rho_{23}), \tag{1b}$$

$$\dot{\rho}_{33} = -(\gamma_{31} + \gamma_{32})\rho_{33} + i\Omega_p(\rho_{13} - \rho_{31}) + i\Omega_c(\rho_{23} - \rho_{32}), \tag{1c}$$

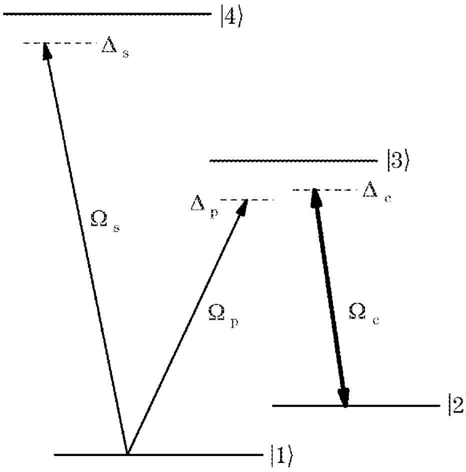

Fig. 1. Schematic diagram of the four-level atomic system under consideration.

$$\dot{\rho}_{44} = -\left(\gamma_{41}+\gamma_{42}\right)\rho_{44} + i\Omega_{\mathrm{s}}\left(\rho_{14}-\rho_{41}\right), \tag{1d}$$

$$\begin{aligned}\dot{\rho}_{12} = {} & i\left(\Delta_{\mathrm{p}}-\Delta_{\mathrm{c}}\right)\rho_{12} + i\Omega_{\mathrm{p}}\rho_{32} + i\Omega_{\mathrm{s}}\rho_{42} - i\Omega_{\mathrm{c}}\rho_{13} \\ & +\sqrt{\gamma_{31}\gamma_{32}}\cos\theta_1\eta_1\rho_{33} + \sqrt{\gamma_{41}\gamma_{42}}\cos\theta_2\eta_2\rho_{44},\end{aligned} \tag{1e}$$

$$\begin{aligned}\dot{\rho}_{13} = {} & -\left[\left(\gamma_{31}+\gamma_{32}\right)/2 - i\Delta_{\mathrm{p}}\right]\rho_{13} + i\Omega_{\mathrm{s}}\rho_{43} \\ & -i\Omega_{\mathrm{c}}\rho_{12} + i\Omega_{\mathrm{p}}\left(\rho_{33}-\rho_{11}\right),\end{aligned} \tag{1f}$$

$$\begin{aligned}\dot{\rho}_{14} = {} & -\left[\left(\gamma_{41}+\gamma_{42}\right)/2 - i\Delta_{\mathrm{s}}\right]\rho_{14} + i\Omega_{\mathrm{p}}\rho_{34} \\ & +i\Omega_{\mathrm{s}}\left(\rho_{44}-\rho_{11}\right),\end{aligned} \tag{1g}$$

$$\begin{aligned}\dot{\rho}_{23} = {} & -\left[\left(\gamma_{31}+\gamma_{32}\right)/2 - i\Delta_{\mathrm{c}}\right]\rho_{23} - i\Omega_{\mathrm{p}}\rho_{21} \\ & +i\Omega_{\mathrm{c}}\left(\rho_{33}-\rho_{22}\right),\end{aligned} \tag{1h}$$

$$\begin{aligned}\dot{\rho}_{24} = {} & -\left[\left(\gamma_{41}+\gamma_{42}\right)/2 + i\left(\Delta_{\mathrm{p}}-\Delta_{\mathrm{c}}-\Delta_{\mathrm{s}}\right)\right]\rho_{24} \\ & +i\Omega_{\mathrm{c}}\rho_{34} - i\Omega_{\mathrm{s}}\rho_{21},\end{aligned} \tag{1i}$$

$$\begin{aligned}\dot{\rho}_{34} = {} & -\left[\left(\gamma_{31}+\gamma_{32}+\gamma_{41}+\gamma_{42}\right)/2 + i\left(\Delta_{\mathrm{p}}-\Delta_{\mathrm{s}}\right)\right]\rho_{34} \\ & +i\Omega_{\mathrm{p}}\rho_{14} + i\Omega_{\mathrm{c}}\rho_{24} - i\Omega_{\mathrm{s}}\rho_{31},\end{aligned} \tag{1j}$$

where γ_{ij} denotes the spontaneous decay rate from level $|i\rangle$ to $|j\rangle$. $\Delta_{\mathrm{c}} = \omega_{32} - \omega_{\mathrm{c}}$, $\Delta_{\mathrm{p}} = \omega_{31} - \omega_{\mathrm{p}}$, and $\Delta_{\mathrm{s}} = \omega_{41} - \omega_{\mathrm{s}}$ are the corresponding coupling, probe, and pump detunings, respectively, as shown in Fig. 1. $\sqrt{\gamma_{31}\gamma_{32}}\cos\theta_1\eta_1\rho_{33}$ $\left(\sqrt{\gamma_{41}\gamma_{42}}\cos\theta_2\eta_2\rho_{44}\right)$ represents the quantum interference effect resulting from the cross coupling between spontaneous emissions $|3\rangle \to |1\rangle$ ($|4\rangle \to |1\rangle$) and $|3\rangle \to |2\rangle$ ($|4\rangle \to |2\rangle$). If levels $|1\rangle$ and $|2\rangle$ lie so closely that the SGC effect has to be taken into account, then η_1 $(\eta_2) = 1$, otherwise η_1 $(\eta_2) = 0$. In this paper, the parameters $\Omega_{\mathrm{c,p,s}}$, $\Delta_{\mathrm{c,p,s}}$, and γ_{ij} are in units of γ. It is worthwhile to point out that only for the small energy spacing between the two lower levels $|1\rangle$ and $|2\rangle$ there will be the remarkable SGC effect, however for the large energy spacing, the rapid oscillation in ρ_{12} will average out such an effect.

Closure of this atomic system requires that $\rho_{ij} = \rho_{ji}^{*}$ and $\rho_{11}+\rho_{22}+\rho_{33}+\rho_{44} = 1$. By a straightforward semiclassical analysis, the above matrix element can be used to calculate the total linear complex susceptibility χ of the probe transition

$$\chi = \frac{N_0|\mu_{31}|^2\rho_{13}}{2\hbar\varepsilon_0\Omega_{\mathrm{p}}}, \tag{2}$$

where N_0 is the atomic number density. Therefore, the absorption-amplification coefficient for the probe laser coupled to the transition $|1\rangle \leftrightarrow |3\rangle$ is proportional to $\mathrm{Im}\left(\rho_{13}\right)$. If $\mathrm{Im}\left(\rho_{13}\right) < 0$, the probe field will be absorbed. On the contrary, the probe field will be amplified. In the following, we investigate the absorption properties of such a weak probe field by numerically solving the above density matrix equations 1 (a) – 1 (j) in the steady state with or without the SGC effect via a simple MATHEMATICA code.

In the following analysis, for the coherent coupling and pump fields we only consider the case of resonant excitations, i.e., $\Delta_{\mathrm{c}} = \Delta_{\mathrm{s}} = 0$. Equation (1) is numerically solved in the steady state under a variety of conditions. In Figs. 2(a) and (b), we plot the absorption-amplification coefficient $\mathrm{Im}\left(\rho_{13}\right)$ of the probe field versus the probe detuning Δ_{p} based on Eq. (1) for the different pump field intensities Ω_{s}. The calculations show that for the fixed Ω_{c} and Ω_{s} values, the probe field is dramatically absorbed in the presence of the SGC effect. For this case, as the coherent pump intensity increases, the probe absorption gradually decreases and the probe-absorption profile takes on the two-peak structure. When the coherent pump intensity is strong enough (up to $\Omega_{\mathrm{s}} = 2\gamma$), the probe field can be amplified only near the line center of the probe transition, as shown in Fig. 2(a). In contrast, in the absence of the SGC effect, the amplification of the probe field can be achieved by using strong coherent pump field and increase with increasing the coherent pump field intensity. Thus the probe-amplification profile exhibits the two-peak structure, as shown in Fig. 2(b). In view of the above two cases, the double-peak structure can be clearly understood in terms of the dressed states. When $\Omega_{\mathrm{s}}/\Omega_{\mathrm{c}}$ is large, the double peaks are located at $\Delta_{\mathrm{p}} \approx \pm\Omega_{\mathrm{s}}$ corresponding to the energy separation of the dressed states $|+\rangle = 1/\sqrt{2}\left(|4\rangle + |1\rangle\right)$ and $|-\rangle = 1/\sqrt{2}\left(|4\rangle - |1\rangle\right)$ due to the dynamic Autler-Townes splitting.

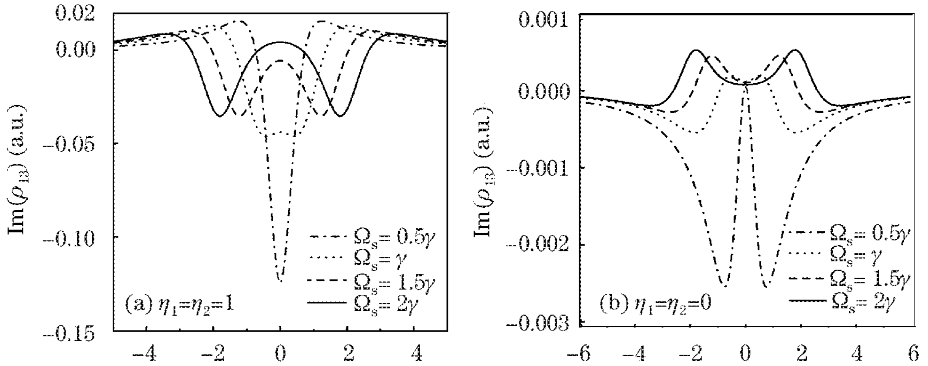

Fig. 2. Absorption-amplification coefficient $\mathrm{Im}\left(\rho_{13}\right)$ of the probe field as a function of the probe detuning Δ_{p} for the different pump field intensity Ω_{s}. Other fixed parameters is $\Omega_{\mathrm{c}} = 0.5\gamma$, $\Omega_{\mathrm{p}} = 0.01\gamma$, $\Delta_{\mathrm{c}} = \Delta_{\mathrm{s}} = 0$, $\gamma_{41} = \gamma_{42} = 1.2\gamma$, $\gamma_{31} = \gamma_{32} = \gamma$, and $\theta_1 = \theta_2 = \pi/4$.

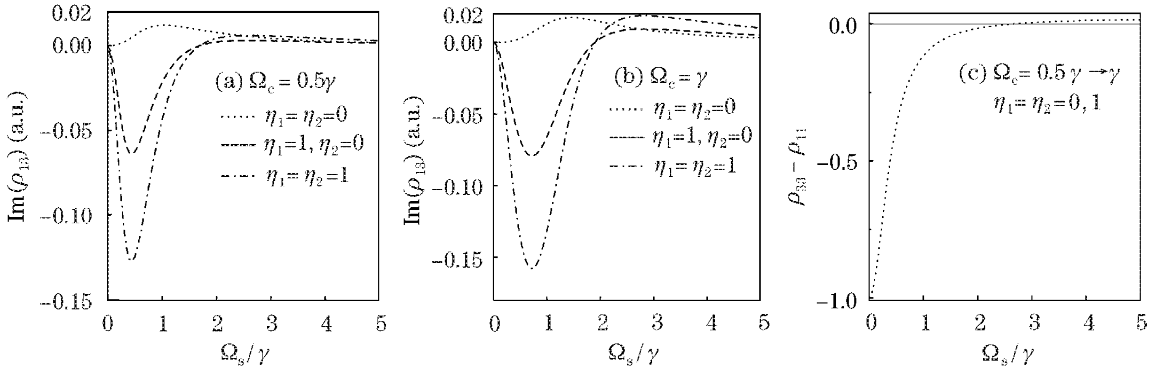

Fig. 3. (a) and (b): Absorption-amplification coefficient $\mathrm{Im}\,(\rho_{13})$ of the probe field as a function of the coherent pump field intensity Ω_s at the line center of the probe transition $\Delta_p = 0$ with or without the SGC effect; (c): population inversion $\rho_{33} - \rho_{11}$ as a function of the coherent pump field intensity Ω_s at the line center $\Delta_p = 0$. Other fixed parameters is $\Omega_p = 0.01\gamma$, $\Delta_c = \Delta_s = 0$, $\gamma_{41} = \gamma_{42} = 1.2\gamma$, $\gamma_{31} = \gamma_{32} = \gamma$, and $\theta_1 = \theta_2 = \pi/4$.

In order to show explicitly the dependence of the probe absorption and amplification on the pump field intensity, we plot the absorption-amplification coefficient $\mathrm{Im}\,(\rho_{13})$ of the probe field versus the coherent pump field intensity Ω_s for the two different coupling field intensities $\Omega_c = 0.5\gamma$ and $\Omega_c = \gamma$ as Figs. 3(a) and (b). We find that, in the presence of the SGC effect, the probe absorption at the line center $\Delta_p = 0$ approaches a maximum value at a suitable pump field intensity Ω_s due to the quantum constructive interference between the two dressed states. As the coherent pump field intensity continues to increase, the absorption of the probe field can be considerably suppressed and reach a saturation value. Specially, for the two cases of $\eta_1 = 1$, $\eta_2 = 0$, and $\eta_1 = \eta_2 = 1$, the absorption profiles are distinct. For the former, the magnitude of the probe absorption is small compared with the latter. Moreover, to approach the saturation value for the former is faster than for the latter, as shown in Figs. 3(a) and (b). It should be noted that when the intensity of the coupling field increases to $\Omega_c = \gamma$, the probe field for the latter can be amplified with the assistance of sufficiently strong pump field. In the absence of the SGC effect, the probe field will be amplified. First, the probe amplification increases monotonically, then decreases gradually with increasing the coherent pump field intensity while the population inversion approaches the saturation value. That is to say, in order to achieve the amplification of the probe field, we should eliminate the SGC effect. Figure 3(c) gives the population inversion $\rho_{33} - \rho_{11}$ of the probe field versus the coherent pump field intensity Ω_s for the coupling field intensity ranging from $\Omega_c = 0.5\gamma$ to $\Omega_c = \gamma$ with or without the SGC effect. The results show that all the curves of the population inversion completely overlap under the given conditions and when $\Omega_s > 2.7\gamma$ is satisfied, the population inversion arises ($\rho_{33} > \rho_{11}$). This means that under the condition of strong coupling field Ω_c, the SGC effect cannot affect the population inversion of the probe field.

In conclusion, we discussed and analyzed the absorption-amplification response of the probe field in a typical four-level atomic system with or without the spontaneously coherence effect. Our results showed that, in the absence of the spontaneously coherence effect, the amplification of the probe field can be achieved by adjusting the coherent pump field intensity. In contrast, in the presence of the spontaneously coherence effect, the probe field was dramatically absorbed under the same pump field intensity. As the coherent pump field intensity increases, the absorption-amplification profile exhibits the two-peak structure and the probe-absorption peak gradually decreases.

The authors would like to thank Dr. Y. Wu for many stimulating discussions. This work was supported in part by the National Natural Science Foundation of China (No. 90103026 and 10125419). J. Li's e-mail adress is huajia_li@163.com.

References

1. X. M. Hu and Z. Z. Xu, J. Phys. B **35,** 4527 (2002).
2. J. Y. Gao, S. H. Yang, D. Wang, X. Z. Guo, K. X. Chen, Y. Jiang, and B. Zhao, Phys. Rev. A **61,** 023401 (2000).
3. M. Yan, E. G. Rickey, and Y. F. Zhu, Phys. Rev. A **64,** 043807 (2001).
4. Y. Wu and X. Yang, Opt. Lett. **28,** 1793 (2003).
5. L. Deng, M. Kozuma, E. W. Hagley, and M. G. Payne, Phys. Rev. Lett. **88,** 143902 (2002).
6. G. X. Li, J. S. Peng, and G. M. Huang, J. Phys. B **33,** 3743 (2000).
7. Y. Wu, J. Saldana, and Y. E. Zhu, Phys. Rev. A **67,** 013811 (2003).
8. Y. Wu, L. L. Wen, and Y. F. Zhu, Opt. Lett. **28,** 631 (2003).
9. J. Wang, Y. F. Zhu, K. J. Jiang, and M. S. Zhan, Phys. Rev. A **68,** 063810 (2003).
10. S. Menom and G. S. Agarwal, Phys. Rev. A **57,** 4014 (1998).
11. W. H. Xu, J. H. Wu, and J. Y. Gao, Phys. Rev. A **66,** 063812 (2002).
12. H. Kang, L. Wen, and Y. Zhu, Phys. Rev. A **68,** 063806 (2003).